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**MATHEMATICAL MODELS FOR OPTIMIZING AIRLINE OFFERINGS IN TERMS OF
INTERNATIONAL REGIONAL PUBLIC AIRPORTS**

**MATEMATICKÉ MODELY PRO OPTIMALIZACI NABÍDKY LETECKÝCH LINEK
V PODMÍNKÁCH MEZINÁRODNÍCH REGIONÁLNÍCH VEŘEJNÝCH LETIŠŤ**

Abstract

In recent decades, air transport has become one of the most dynamically developing modes of transport. The number of passengers using air transport is constantly increasing, aircraft technical equipment is improving, and technical and infrastructure support for air transport is being developed. However, the dynamics of traffic development at international airports are not the same. While traffic to major airports in the States is growing rapidly, growth rates are generally very slow at regional airports. Most regional airports are currently trying to create or expand their existing destination portfolio with new scheduled services to transit destinations, from where passengers can continue. The right choice of transit destination is very important for the development of a regional airport, mainly because the demand for transport from a regional airport is growing with the increasing offer of one-way transfer as far as possible around the world.

The paper deals with the design of a mathematical model. On the basis of this solution, it is possible to determine the optimal portfolio of transit destinations to which it is appropriate to operate from a given regional airport.

Abstrakt

V posledních desetiletích se letecká doprava stala jedním z nejdynamičtěji se rozvíjejícím druhům dopravy. Počet cestujících, kteří leteckou dopravu využívají, neustále narůstá, zdokonaluje se technické vybavení letadel, rozvíjí se i technická a infrastrukturní podpora letecké dopravy. U mezinárodních letišť však dynamika rozvoje provozu není stejná. Zatímco provoz na nejvýznamnějších letištích států narůstá rychle, na regionálních letištích je zpravidla tempo růstu velmi pomalé. Většina regionálních letišť se v současnosti snaží vytvořit nebo rozšířit své stávající destinační portfolio o nové pravidelné linky do tzv. tranzitních destinací, odkud mají cestující možnost pokračovat dále. Správný výběr tranzitní destinace je pro rozvoj regionálního letiště velmi důležitý především proto, že s rostoucí nabídkou možnosti přepravovat se s jedním přestupem pokud možno do celého světa roste zájem o přepravu z regionálního letiště.

Článek se zabývá návrhem matematického modelu, na základě jehož řešení je možno určit optimální portfolio tranzitních destinací, do kterých je vhodné provozovat letecké spojení z daného regionálního letiště.

Keywords

Regional airports, mathematical models, optimizing.

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1 INTRODUCTION

Regional international public airports in the Czech Republic and Slovakia (hereinafter referred to as “regional airports”) are currently mostly used during the seasonal operation, when charter flights intended for the transportation of holidaymakers to/from holiday destinations are directed to these airports. In the off-season, it is often the case that the scale of passenger air transport at regional airports is declining considerably and thus the airport's infrastructure and staffing potential remain unused. At the same time, considerable investment funds have to be invested in regional airports, as these airports usually also serve as backup airports of the main airports of the state. Therefore, in order to perform a backup function, they must also be adequately prepared to receive all types of aircraft operating at major airports.

For this reason, airport managers are motivated to look for a complement of seasonal charter destinations to offer year-round destinations that provide at least partial use of regional airports off-season. Such use is mainly ensured by the offer of destinations used by other groups of passengers, such as merchants, investors, managers (hereinafter referred to as "business clients"). However, given their geographically broadly defined interest activities, these categories of passengers do not require the offer of point-to-point destinations, but the offer of destinations to major catchment areas (in transport and logistics, the system is referred to as the hub & spoke system) Fig. 1.

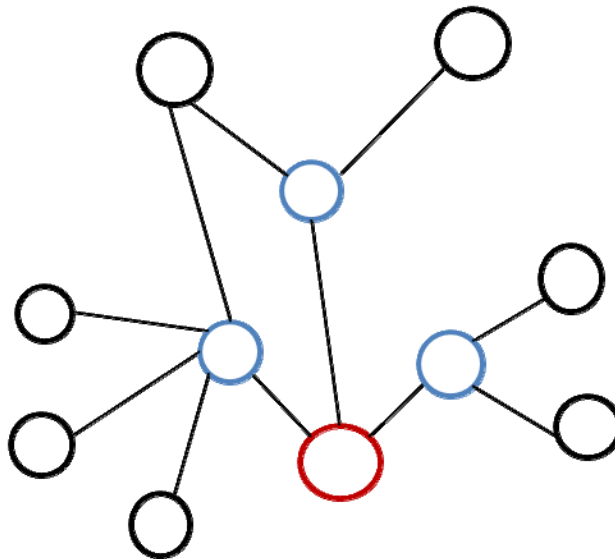


Fig. 1 Schematic representation of the hub & spoke system

The red marked peak in Fig. 1 represents the position of the regional airport throughout the system.

The decision-making situation from the regional airport perspective can be clearly represented by the hierarchical graph shown in Fig. 2.

The hierarchical graph in Figure 2 contains three levels. The top level is represented by regional airports, the middle level includes transit destinations (whose set will be marked with the I symbol in the mathematical models below), which are considered by regional airport management to link a regional airport to an existing network airline line. The lowest level includes the target destinations, which represent possible areas of interest of the business clientele (their set will be denoted by J in the mathematical models below). It should be added that the transit destination may also be a destination for some travellers.

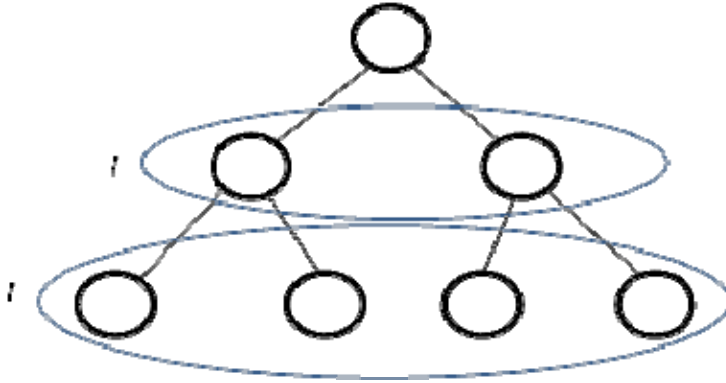


Fig. 2 Schematic representation of the decision-making problem at regional airport level

As a result, it may not be very effective to operate an air connection with each destination. It would be very attractive to passengers, but it would not be effective for carriers, because existing demand could be spread in several directions, aircraft usage would be reduced and air carriers could lose interest in operating flights to transit destinations, possibly would increase the "subsidy intensity" for airport founders.

If we mark m the number of transit destinations eligible for operations serving regional airports, then $|I|=m$ and if the airport management considers flights to a maximum of p transit destinations, then the number of possible solutions that come in Consideration can be mathematically expressed by

$$\binom{m}{1} + \binom{m}{2} + \dots + \binom{m}{p} = \sum_{i=1}^p \binom{m}{i}$$

It is also very important to note that a maximum of 1 transfer per transit destination is generally acceptable for passengers wishing to use the regional airport.

2 ANALYSIS OF THE CURRENT STATE OF KNOWLEDGE

The models that are available today to solve the above-defined task can be divided into two categories:

1. according to the level of knowledge of potential passenger demand,
2. according to the level of availability of final destinations from transit sessions.

Depending on the level of knowledge of potential passenger demand, existing models can be divided into models for decision-making situations with known demand and models for decision-making situations without known demand.

Depending on the level of availability of final destination from transit sessions, the existing models can be divided into models with disjointly served final destinations (each destination is only available with one intended transit destination) and general models (there are final destinations available from multiple considered transit destinations). An example of disjointly served destination destinations is shown in Fig. 2, an example of the general situation of served destinations is shown in Fig. 3.

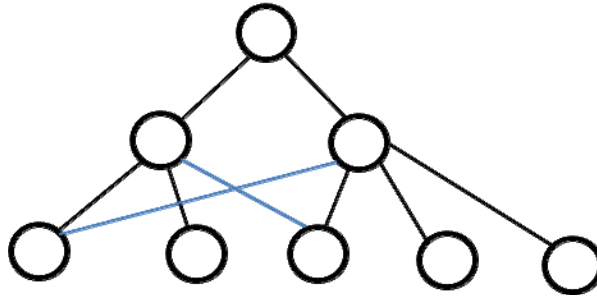


Fig. 3 Availability of finish destinations from multiple transit destinations

In each optimization task, the first must be formulated optimization criterion, according to which the value of the individual permissible solutions is evaluated.

In task with known demand, the optimization criterion is the number of passengers to finish destinations whose demand through the transit destinations is satisfied (one transit destination is meant). The number of passengers which traveling from transit destinations to final destinations, if necessary, the number of passengers requesting transport to transit destinations (transit destinations are their final destinations).

In tasks with unknown demand, the number of available final destinations, possibly the number of transit destinations, may be a suitable optimization criterion.

However, for general mathematical model variants (final destinations available from multiple transit destinations), when designing the model, account must be taken of the fact that the available destination must only be counted once to the optimization criterion, since the number of available final destinations due to the availability of multiple transit operations destinations.

In the mathematical models described above, it has been found that there are some reserves, respectively. Unresolved pages remain important from the point of view of real passenger decision-making, which is mainly related to the non-inclusion of some very important limiting factors affecting the values of selected types of input data (eg variable demand during individual days of the week, but also during a specific day of the week).

3 EXTENDING EXISTING MODELS

In this chapter will be described as a new model that takes into account variable demand throughout the day. It is, therefore, a model that will work with transport demand values. The changing demand during the day will be taken into account through the different values of demand in the defined time periods for which individual days will be spread. Combined with changing demand throughout the day, it will also be required to fly to the same transit destination every day for the same time period. The model will be created for a case where there are disjointly served destination destinations.

Problem formulation

Is a given regional airport, a set of transit destinations I , where is again $|I| = m$, a set of finish destinations J with disjunct subsets, a set of days per week K (consider $|K| = 7$), and a set of time periods L , where $|L| = r$ (it is assumed that the number of time periods during each day of the week will be the same). The demand value d_{ikl} is known for each transit destination $i \in I$, day of week $k \in K$, and time period $l \in L$. The values of demand in individual periods are obtained according to formulas

$$d_{ikl} = \sum_{j \in J_i} \delta_{jkl} \quad (1)$$

where δ_{jkl} is the demand for the destination $j \in J$, on the day of the week $k \in K$ and the time period $l \in L$.

If on the day of the week $k \in K$ and the time period $l \in L$ generates the demand as well as the transit destination $i \in I$, then mark it as σ_{ikl} , then for value d_{ikl} transit destination then apply:

$$d_{ikl} = \sum_{j \in J_i} \delta_{jkl} + \sigma_{ikl} \quad (2)$$

The task is to design a model whose solution will be a plan of flights to selected $2 \leq p < m$ transit destinations every day of the week for the same time period (during the day, there will be just 1 year in each selected transit destination $i \in I$). The goal of optimization is to maximize the total number of passengers whose demand will be met by the supply of transit destinations.

Two groups of variables will be introduced into the task z_{ikl} and y_i . The variables in both groups will have the definitions fields 0 and 1. They will be bivalent variables. It will be true that when after the optimization calculation $z_{ikl} = 1$, then a transit destination will be operated in the destination portfolio $i \in I$ on the day of the week $k \in K$ and time period $l \in L$. It will be true that when after the optimization calculation $z_{ikl} = 0$, then a transit destination will be not operated in the destination portfolio $i \in I$ on the day of the week $k \in K$ and time period $l \in L$. It will be true that when after the optimization calculation $y_i = 1$, then will be transit destination included in the destination portfolio regional airport. It will be true that when after the optimization calculation $y_i = 0$, then will not be transit destination $i \in I$ included in the destination portfolio regional airport.

The mathematical model will be:

$$\max f(y, z) = \sum_{i \in I} \sum_{k \in K} \sum_{l \in L} d_{ikl} z_{ikl} \quad (3)$$

under conditions

$$\sum_{l \in L} z_{ikl} \leq 1 \quad \text{pro } i \in I; k \in K \quad (4)$$

$$z_{ikl} \leq y_i \quad \text{pro } i \in I; k \in K; l \in L \quad (5)$$

$$\sum_{i \in I} y_i \leq p \quad (6)$$

$$z_{i1l} = z_{i2l} = \dots = z_{i7l} \quad \text{pro } i \in I; l \in L \quad (7)$$

$$y_i \in \{0; 1\} \quad \text{pro } i \in I \quad (8)$$

$$z_{ikl} \in \{0; 1\} \quad \text{pro } i \in I; k \in K; l \in L \quad (9)$$

The expression (3) represents purpose function expressing the overall demand for destinations on all days and in all time periods. The limiting condition (4) ensures that will be every day in a week realization maximum 1 flight to each destination $i \in I$. The limiting condition (5) ensures that whenever a flight is operated to each destination $i \in I$ on any day of the week and its time period, then the corresponding variable y_i will be fixed at 1 and the condition (6) will reduce the number of free transit destinations. A group of limiting conditions (7) ensures that when flights to a transit destination are scheduled $i \in I$, then it takes place every day of the week at the same time period. A group of limiting conditions (8) and (9) define the domains of the variables that appear in the model.

4 CONCLUSIONS

The mathematical models available today to address the issue of introducing a new airline can be divided into models for decision-making situations with known demand and models for decision-making situations without known demand. Furthermore, these models can be divided into models with disjunctively served final destinations (each destination is only available from one considered transit destination) and general models (there are final destinations available from multiple transit destinations).

The paper presents a further extension of the spectrum of models by a model enabling to work with changing demand during the day and week. The basic idea is based on the discretization (division) of the time period into multiple sub-intervals (time periods), in which a significant change in the value of demand occurs.

At the same time, the model allows scheduling flights to transit destinations so that they take place on the same days of the week. The optimization criterion is the overall satisfactory demand of passengers.

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