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SIMULATION OF A TWO DEGREES OF FREEDOM CONTROLLER WITH A LIQUID TANK

SIMULACE REGULACE 2DOF REGULÁTORU S VODNÍ NÁDRŽÍ

**Abstract**

The article is dealing with the control of liquid tank system. For the process control, it is implemented two degrees of freedom controller. The system is simulated in Matlab and the results are compared with a control system with standard controller.

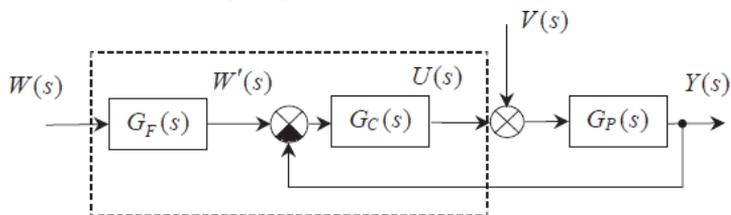
Tento článek se zabývá řízením systému nádrže na vodu. Pro řízení nádrže je navržen regulátor se dvěma stupni volnosti. Systém nádrže je simulován v Matlabu a výsledky regulace jsou porovnány s regulací se standardním regulátorem.

**Keywords**

Controller, 2DOF Controller, Liquid tank, Level Control, Simulation.

**1 INTRODUCTION**

The 2DOF controllers are used mostly for integral system to achieved process control without overshoot. It is also used for ensuring precision of required set point or disturbances entering to the system. [1]-[3] Standard PID controller is not always able to control system with an integral character without overshoot and oscillations in step response. [1]-[4]



**Fig. 1** Basic scheme of 2DOF Controller

The 2DOF controller is described by relation (1) [1]-[3] and scheme of 2DOF controller is shown on fig. 1, where

$G_F$  – input filter,

$G_C$  – controller,

$G_P$  – plant,

$V(s)$  – disturbance variable

$W(s)$  – desired variable,

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$W'(s)$  – filtered desire variable

$Y(s)$  – controlled variable

$$U(s) = K_p \left\{ bW(s) - Y(s) + \frac{1}{T_I s} [W(s) - Y(s)] + T_D s [cW(s) - Y(s)] \right\}, \quad (1)$$

where:

$b$  – the variable weight of the proportional part,

$c$  – the variable weight of derivative part,

$K_p$  – the proportional gain,

$T_I$  – the integral time constant,

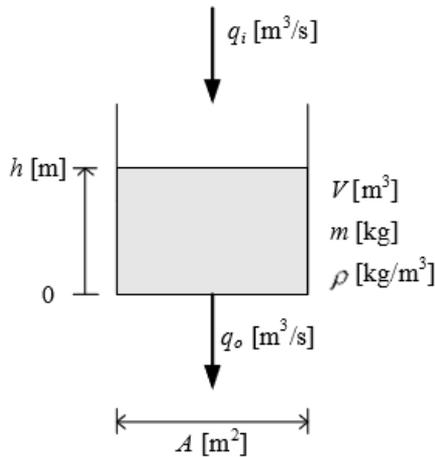
$T_D$  – derivative time constant,

$W(s)$  – the desired variable,

$Y(s)$  – the controlled variable

## 2 CONTROL SYSTEM - LIQUID TANK

The system of level control is based on liquid tank with inflow and outflow, where the inflow can be controlled, and outflow can't.



**Fig. 2** Liquid tank [5]

The description of the liquid tank is shown on fig. 2, where

$h$  – liquid level,

$q_i$  – volumeric inflow,

$q_o$  – volumeric outflow,

$A$  – cross sectional area,

$V$  – liquid volume,

$m$  – mass,

$\rho$  – density.

The variables are parameters  $q_i, h, m$  in the tank. The constant value has the parameters  $A, \rho, q_o$ .

The mathematical model of this system is based on mass balance for the mass in tank [5]-[6]

$$\frac{dm(t)}{dt} = \rho q_i(t) - \rho q_o(t). \quad (2)$$

The mass can be expressed by

$$m(t) = \rho V(t) = \rho A h(t), \quad (3)$$

than we insert the equation (3) into the mass balance differential equation (2), which then becomes

$$\rho A \frac{h(t)}{dt} = \rho q_i(t) - \rho q_o(t), \quad (4)$$

where  $A, \rho$  are parameters which we moved on other side the equation (4) and get following form for the level  $h(t)$ , which is output variable of the control system

$$\frac{h(t)}{dt} = \frac{1}{A} [q_i(t) - q_o(t)], \quad (5)$$

The input value is the change of inflow, outflow is assumed as disturbance

$$u(t) = q_i(t). \quad (6)$$

Now, we apply the Laplace-transformation to get a transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{s}, \quad (7)$$

where  $K = \frac{1}{A}$ .

The parameters of the liquid tank are:

- maximum change of liquid level is  $\Delta h = 0.25\text{m}$ ,
- outflow in the tank is  $\Delta q = 0.2\text{m}^3/\text{s}$ ,
- the size of cross sectional area is  $A = 0.25\text{m}^2$ .

### 3 TUNING OF 2DOF CONTROLLER

The liquid tank is an integral system and has transfer function (7). For the tuning of PI controller, it was chosen model-based method SIMC.

#### 3.1 SIMC method

The method SIMC is basic and very efficient method to controller tuning method. The rules are analytically derived, and from the chosen form of control process and dynamic behavior of the controlled system, the PI and PID controller settings are possible easily to find. [1],[2],[5]. The formula for SIMC method with integral system is following [1],[2]

$$K_p^* = \frac{1}{2KT_d}, \quad (8)$$

$$T_I^* = 8T_d. \quad (9)$$

The time delay is needed in the control system to use the formulas of SIMC method (8), (9), but the liquid tank has no time delay. In that case, it has to be chosen or determined a system constant  $T_w$  and substitute the time delay with it [5].

$$T_w = \frac{A \cdot \Delta h}{\Delta q} \quad (10)$$

The variable weight of the input filter will be calculated from closed - loop transfer function.

$$G'_{w'y}(s) = \frac{G_o(s)}{1 + G_o(s)} = \frac{k_1 K_p (T_I s + 1)}{T_w T_I s^3 + T_I s^2 + T_I k_1 K_p s + k_1 K_p}, \quad (11)$$

which it is modified to the representation

$$G'_{w'y}(s) = \frac{T_I s + 1}{\frac{T_w T_I}{k_1 K_p} s^3 + \frac{T_I}{k_1 K_p} s^2 + T_I s + 1} \quad (12)$$

The equation for variable  $b$  of the filter is based on real pole  $s_1$  of the characteristic polynomial from transfer function (12)

$$b = \frac{1}{-s_1 T_I}. \quad (13)$$

### 3.2 Parameters of 2DOF controller

The system constant is

$$T_w = \frac{A \cdot \Delta h}{\Delta q} = \frac{0.25 \cdot 0.25}{0.2} \doteq 0.31$$

the tuning parameters of the controller are following

$$K_P^* = \frac{1}{2KT_w} = \frac{1}{2 \cdot 4 \cdot 0.31} \doteq 0.40,$$

$$T_I^* = 8T_d = 8 \cdot 0.31 = 2.50.$$

The transfer function of the control process is

$$G'_{w'y}(s) = \frac{2.50s + 1}{0.49s^3 + 1.56s^2 + 2.50s + 1}.$$

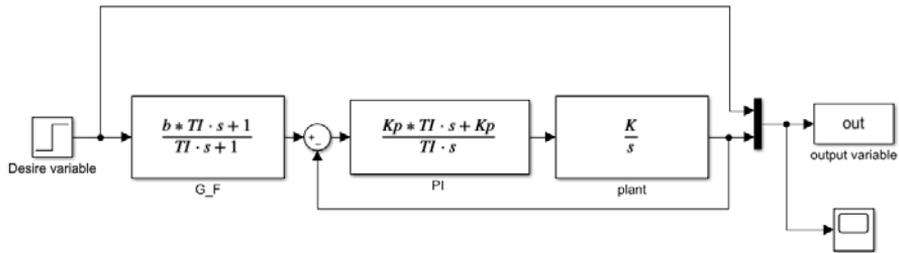
The values of the real pole and  $b$  parameter are

$$s_1 = -0.56,$$

$$b = \frac{1}{0.56 \cdot 2.5} \doteq 0.71.$$

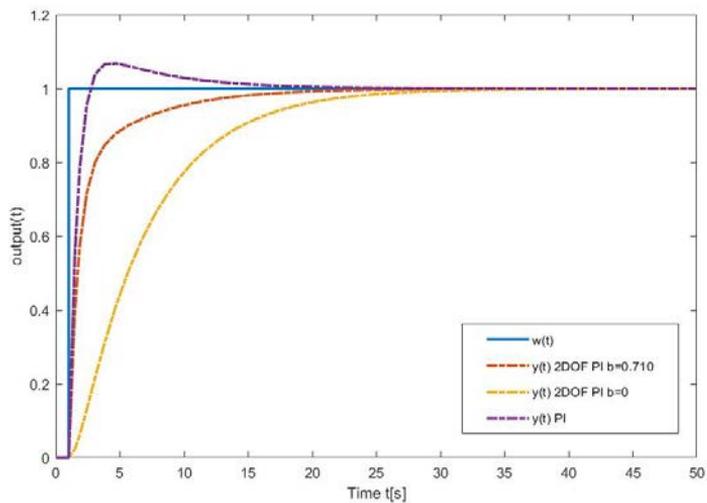
## 4 SIMULATION

The Simulation scheme of the closed-loop control system is created in Matlab Simulink program for comparing process control of standard and 2DOF controller, see fig. 3



**Fig. 3** Simulation scheme

The results of the simulation are shown on fig.4. There the process control is compared between the result with standard PI controller and 2DOF controllers with  $b=0$  and  $b=0.71$ , particularly it is chosen to settle time and overshoot. Their values are written in table 1.



**Fig. 4** Simulation of process control

## 5 CONCLUSIONS

In the article is described mathematical model based on a mass balance of a liquid tank, which is an integral system. The goal of the article is to use 2DOF controller for level control of integral system. I choose the SIMC method and determine the internal constant of the tank to tune the PI controller. I used 3 types of 2DOF controller, where the variable weight  $b$  of the proportional part for PI 2DOF controller I calculate from transfer function, then I use the common setting  $b=0$ , to compare it. For the last type, I choose  $b=1$ . In that case, the PI 2DOF controller has behaviour of standard PI controller.

**Tab. 1** Results of simulation

Controller	Settling time	Overshoot
Standard PI $b=1$	6.68 s	6.9%
2DOF PI $b=0.71$	7.71 s	0%
2DOF PI $b=0$	16.67 s	0%

The results of the simulation are shown on fig. 4 and in tab. 1, where the standard PI controller has around 7% overshoot, but the settling time (with 5% tolerance) is the shortest comparing with other results. Good result is gotten with PI 2DOF controller, which has the variable weight  $b = 0.71$ . The process control is without overshoot and settling time is just 1s longer than the settling time of the standard PI controller.

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