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## INPUT SHAPING FOR UNDERDAMPED SYSTEMS

## TVAROVÁNÍ VSTUPU PRO PODTLUMENÉ SYSTÉMY

### Abstract

The main aim of this paper is to provide possibility to test and validate multiple damping and input shaping methods for underdamped systems. Mathematical model for specific development and demonstration was created in Matlab Simscape environment. Additional Matlab Simulink system was modelled to achieve proper data acquisition and comparison in contrast to plain system without any special damping methods.

### Abstrakt

Hlavním cílem této práce je poskytnout možnost testování a validace více metod tlumení a vstupních tvarů u podtlumených systémů. Matematický model pro specifický vývoj a demonstraci byl vytvořen v prostředí Matlab Simscape. Další systém Matlab Simulink byl modelován tak, aby dosáhl správného získávání a srovnání dat v porovnání s běžným systémem bez zvláštních metod tlumení.

### Keywords

Underdamped Systems, Input Shaping, Crane, Matlab, Simulink, Simscape, Animation

## 1 INTRODUCTION

Write Design of controlled systems is bond with physical properties of individual components used in such a system. In order to ensure proper methods all system requirements should be known in significant detail. Among some important, acceleration, maximum speed, resonant frequency, maximum overshoot, settling time and response time are. Bridge cranes with high loads are particular example for this issue. In such a system the proper handling of load is crucial in order to ensure stability, safety and performance of a whole system.

This article deals with special methods to approach underdamped systems in order to improve their transient responses. Bridge crane, both inverted and normal pendulum or harmonic clutch could be the examples of most used underdamped systems.

Damping is defined as draining of kinetic energy from system or material which is subject to stress. Damping happens inside of material compounds, where individual layers and particles rub within each other.

In the field of automation theory, the property of oscillation of a system is defined by damping ratio. The damping ratio is a parameter, usually denoted by  $\zeta$  (zeta), that characterizes the frequency response of a second order ordinary differential equation. The damping ratio provides a mathematical means of expressing the level of damping in a system relative to critical damping. General Systems can be divided into few types according to damping ratio:

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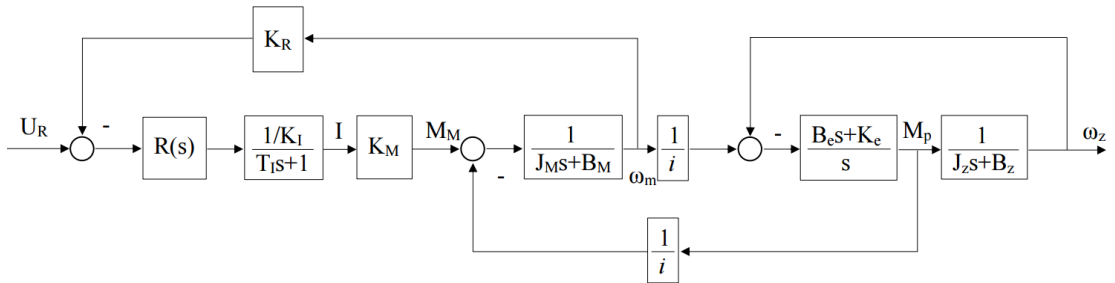
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- Undamped (in case of  $\zeta > 0$ )
- Underdamped (in case of  $\zeta < 1$ )
- Overdamped (in case of  $\zeta > 1$ )
- Critically Damped (in case of  $\zeta = 1$ )

It is obvious that undamped systems are not real in our world since the amplitude will always drop with every oscillation due to internal and system friction.

## 2 PROPER PROPORTIONAL GAIN SETTING METHOD

Among other methods how to lower oscillation of underdamped system the proper proportional gain setting method is one of the basic ones. This method provides the possibility to increase damping ratio from values close to undamped system to  $\zeta = 0.2$ . The improvement itself is not significant, however in contrast to systems with damping ratio really close to undamped system this could still be a major improvement. The other set back of this method is that this method may not meet every requirement thanks to oscillating property of such a system. The main advantage of this method is represented by no need for additional sensors in case of closed loop system. This method is typical for servomotors which are always already connected in closed loops with feedback of position.



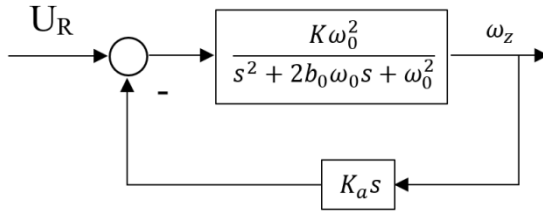
**Fig. 1** - Block diagram of servomotor.

On the previous picture, there is a block diagram of closed loop system of servomotor with elastic property. Individual components are described as follows:

$U_R$  – Signal of requested value of speed,  $R(s)$  – transfer function of speed controller,  $K_I$  – current gain of control loop,  $T_I$  – current time constant of control loop,  $I$  – control current,  $K_M$  – moment constant of the motor,  $M_M$  – The torque moment generated by the motor,  $J_M$  – Inertia moment of the motor,  $B_M$  – viscous friction coefficient of the motor,  $i$  – gear transmission coefficient,  $K_R$  – speed sensor gain,  $B_e$  – dissipative damping coefficient of the gearbox,  $K_e$  – stiffness coefficient of gearbox,  $J_z$  – load moment inertia,  $B_z$  – viscous friction coefficient of the load,  $\omega_z$  – angle speed rotation of the load  $\omega_m$  – angle speed of the motor shaft.

## 3 APPROPRIATE TYPE OF DAMPING FEEDBACK

Another damping method for naturally undamped system is the appropriate type of damping feedback method. This active method is best used for systems which could be described as second order systems with low value of damping coefficient  $b_0$ . By adding specific negative feedback loop transfer function can be adjusted as follows:



**Fig. 2** - Acceleration feedback control loop structure.

$$F(s) = \frac{\omega_z(s)}{U_R(s)} = \frac{K\omega_0^2}{s^2 + 2b\omega_0s + 1} \quad (1)$$

$$b = b_0 + K_a \frac{K\omega_0}{2} \quad (2)$$

From previous relations is obvious that by proper adjustment of gain  $K_a$  the final coefficient  $b_0$  can be adjusted.

#### 4 INPUT SHAPING FOR BRIDGE CRANE MODEL

One of many typical problems of underdamped systems is bridge crane. It is one of the most used system for heavy load transport within production section in general industry. Apart of this bridge cranes are commonly used in harbors, transshipment areas, logistical units and other establishments where it is required to move heavy loads from one place to another. Over time multiple bridge crane subsystems, which ensured faster and safer load handling and brought more precise positioning systems where added upon basic bridge crane systems. It is essential to count with oscillation of load, crane transfer speed and settling time while moving heavy load from place A to place B. To achieve a proper control of underdamped systems such as bridge crane it is appropriate to introduce input shaping for control signals of the system.

There is multiple methods and approaches how to create input shaped signals for underdamped systems. These systems provide different system properties which vary in response times, settling time or robustness. One of the proper variants for bridge crane systems is Zero Vibration Shaper or Z-V Shaper. This method is based on series of two impulses which creates shaped input by their convolution. The main idea is to divide reference input into two smaller input with first one slightly higher and the second one shifted in time by half of natural period of system's oscillation. The longer the time in-between these two impulses the higher the response time will become. The pair of impulses for specific system is stated as:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 1 & K \\ 1+K & 1+K \\ 0 & 0,5T \end{bmatrix} \quad (3)$$

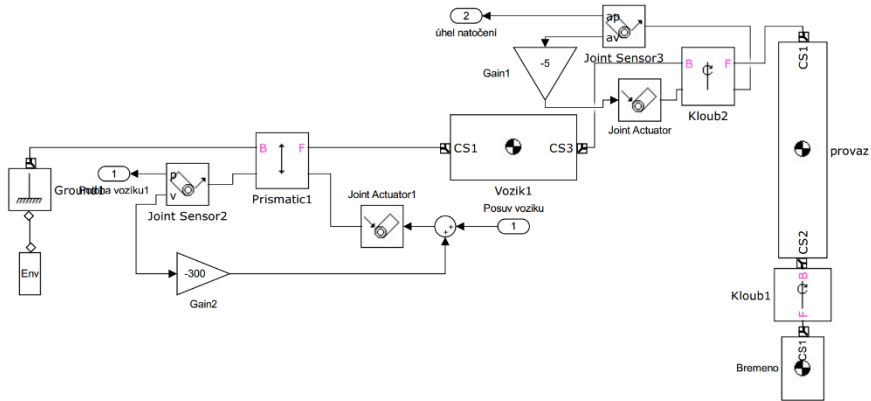
Where:

$$T = \frac{1}{\omega_0 \sqrt{1 - \zeta^2}}$$

$$K = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

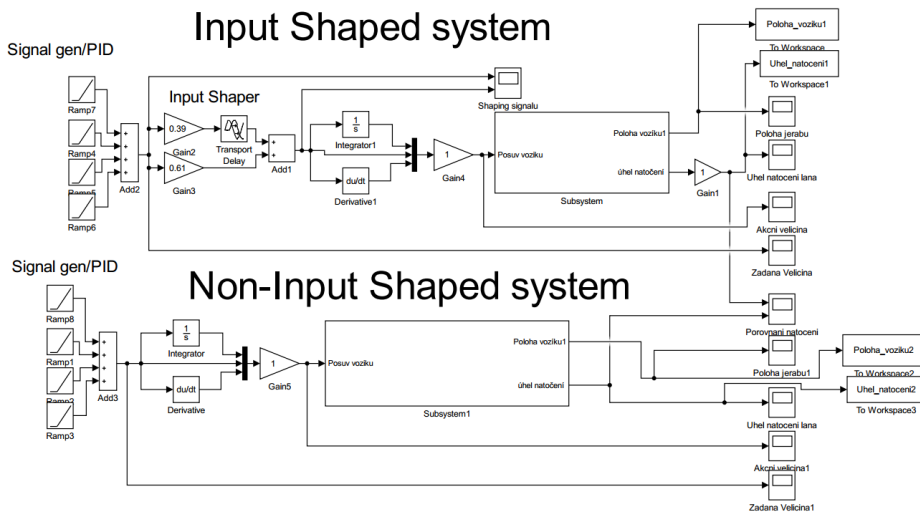
In order to properly set and test this method against standard underdamped system the Matlab SimMechanics model was created. Firstly, proper validation of model and identification of modelled

system was done and right after the exact determination of damping coefficient and natural frequency of the system were found. The bridge crane model is displayed below:



**Fig. 3 - Bridge crane SimMechanics model**

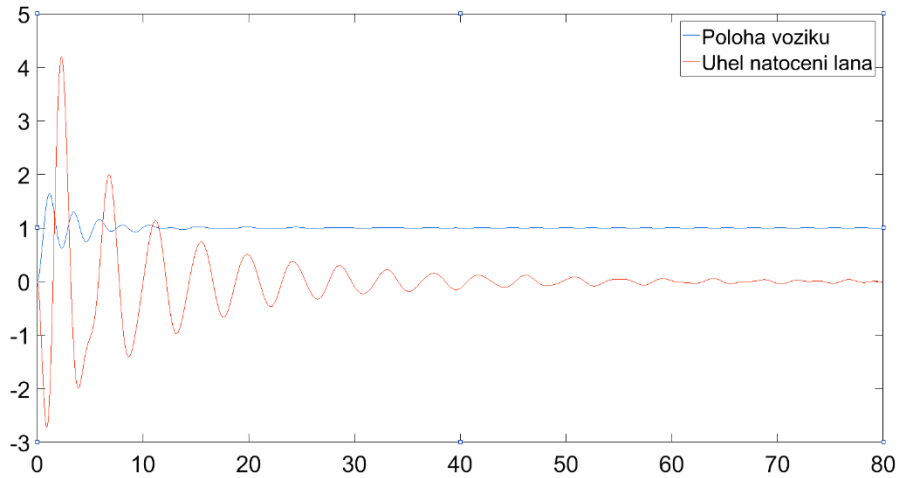
Individual block setting required exact parameters and every bond was properly adjusted to meet real-like properties. With working model of bridge crane second Matlab Simulink model could be created to meet measurement and comparison needs for final evaluation of a method. Apart of system enhanced with Z-V input shaping method plain underdamped system was added to express differences in both systems over time. Matlab Simulink model of such a system is on the following picture:



**Fig. 4 - Matlab Simulink data acquisition and validation model**

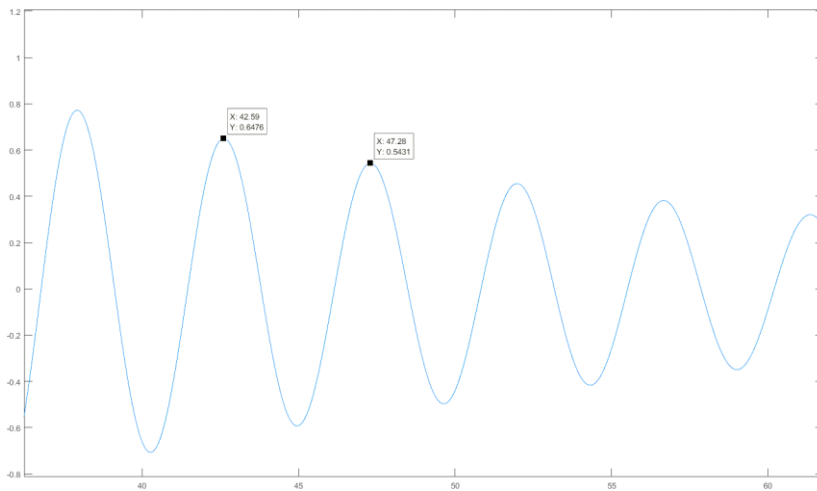
In order to properly set specific gain and time period for Z-V Input shaping method the natural frequency and damping coefficient had to be determined with maximum allowed error around 10%. To achieve a state from which all important specifications of the system could be acquired high proportional gain was introduced to closed loop proportional control system. With this approach both the oscillation and damping decrement could be easily measured and stated for additional calculations.

Following picture shows the final state of a model from which the measurement was obtained.



**Fig. 5** - Critical oscillation of bridge crane model

Natural frequency of the system was measured from series of individual periods at time when systems oscillates around equilibrium position. Weighted average was then used in order to account for measurement errors.



**Fig. 6** - Natural frequency determination

The damping ratio which is also needed for Z-V Input Shape Method is obtained from logarithmic decrement of damping. The first pair of damped amplitudes was count with to define this number.

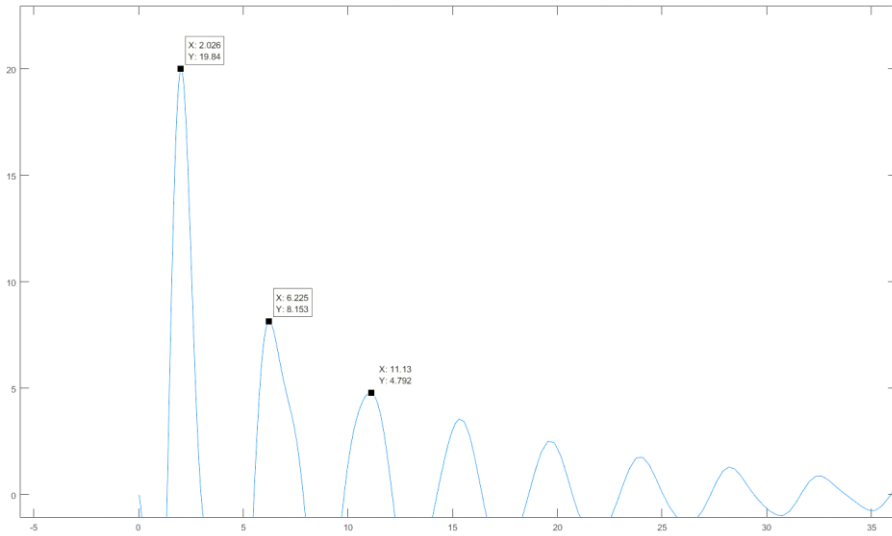
The following data were acquired from graphs:

- natural frequency  $f=0.476 \text{ s}^{-1}$
- Two amplitude peaks  $x_1 = 19.84$  and  $x_2 = 8.153$

$$\vartheta = \ln \frac{x_1}{x_2} = \ln \frac{19.84}{8.153} = 0.8893 \quad (4)$$

$$\zeta = \frac{\vartheta}{\sqrt{4 \cdot \pi^2 + \vartheta^2}} \quad (5)$$

$$\zeta = \frac{0.8893}{\sqrt{4 \cdot \pi^2 + 0.8893^2}} = 0.1401 \quad (6)$$



**Fig. 7** - Logarithmic damping value acquisition

With damping ratio properly calculated only time period is needed for Z-V Input Shaping Method.

$$T = \frac{1}{\omega_0 \sqrt{1 - \zeta^2}} = \frac{1}{0.476 \cdot \sqrt{1 - (0.1401)^2}} = 2.128 \quad (7)$$

$$K = \frac{-\zeta \pi}{\sqrt{1 - \zeta^2}} = \frac{0.1401 \cdot \pi}{\sqrt{1 - 0.1401^2}} = 0.64 \quad (8)$$

With all needed variables calculated Z-V matrix could be created.

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 1 & K \\ 1 + K & 1 + K \\ 0 & 0.5T \end{bmatrix} \quad (9)$$

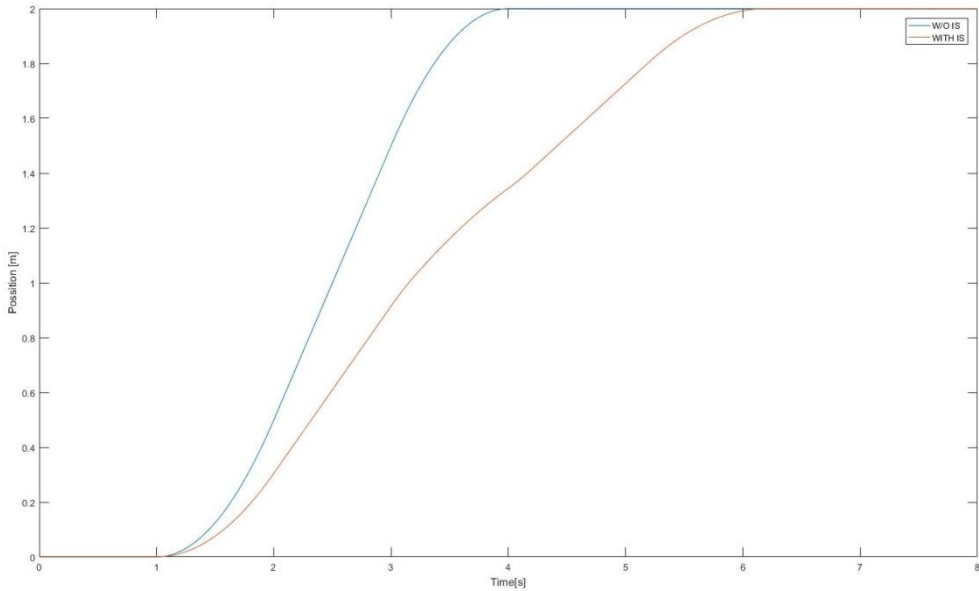
$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 0.61 & 0.39 \\ 0 & 1.064 \end{bmatrix} \quad (10)$$

The Z-V Input Shape Method in laplace domain has following shape:

$$ZV(s) = A_1 + A_2 e^{-s \frac{T}{2}} \quad (11)$$

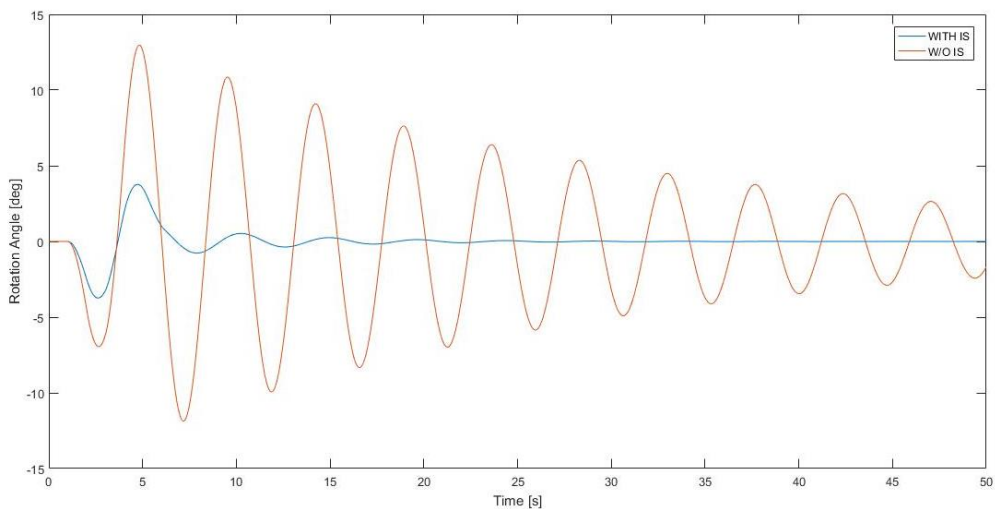
## 5 MEASUREMENTS AND COMPARISON

During measurement multiple behaviors of both systems were compared to each other. Since there is noted disadvantage of longer response time for Z-V Input Shape Method this system property was closely observed. The difference between both models is displayed on the graph below:



**Fig. 8 -** Response times comparison

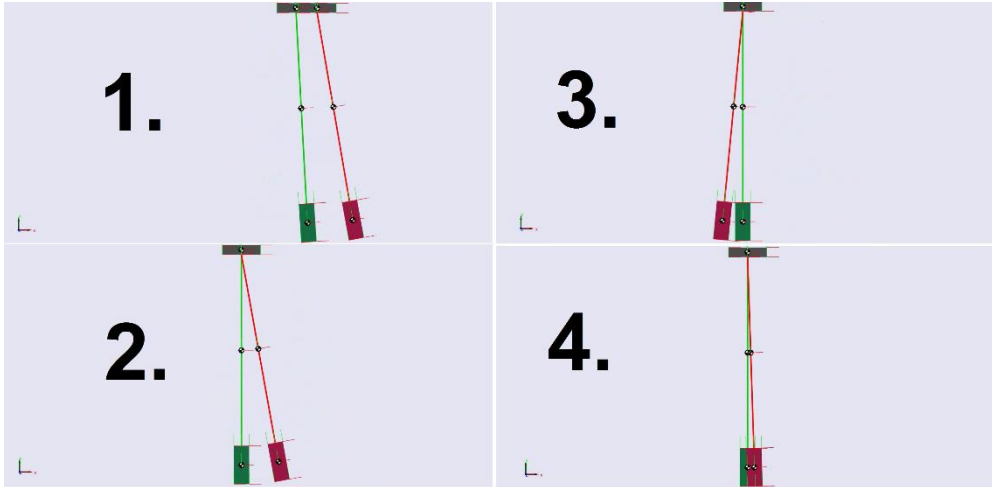
It is clearly visible that Z-V Input Shaping Method extend the response time around one period which was about 2 seconds in this case. Since this method employs two signals which are time shifted to each other it is logical to expect longer response time. Some input shaping method can include even more individually shaped input signals which will extend response time even further, but will prove to be even more functional. Additionally, response time setback should be balanced out with extremely faster settling time of the load and final time to move a crane with load from the place A to the place B.



**Fig. 9 -** Measured results of Z-V Input Shaping Method

From the measured results displayed in the graph above it is obvious to see that Z-V Input Shaping Method is working with expected results. The settling time is significantly lower even with high loads attached to bridge crane.

To visualize the results and be able to evaluate Z-V Input Shaping Method even further 2D animation was rendered.



**Fig. 10** - 2D model crane animation screens

On the following screenshots you can see underdamped system of bridge crane without input shaping method represented by red body and bridge crane system with input shaping represented by green body. Individual shots were taken around 3 seconds after each other and after introduction of control signal into both of the systems. On the first picture the two seconds response time delay, caused by Z-V method's time shifted pair of input signal impulses, can be observed. The second screen describes enormously faster settling time of the green body system which is almost immediately in its equilibrium after reaching desired position. On the other hand, the red body system still oscillates around its equilibrium dozens of seconds after green system is on its position and ready to move again.

## 6 CONCLUSIONS

My paper deals with specially shaped input signals for underdamped systems. From simulations and measurements, it is obvious that input shaping methods are providing significant results over previous state. However, it is fair to say that this method will bring not negligible response time delay. For the purposes of bridge crane it is well balanced out with extraordinary results of settling time even for high loads. This important fact should be taken in account when thinking about input shaping methods. Some systems could not benefit so greatly from settling time advantages.

Other very important aspect of this method is the crucial need to determine mathematical model of a controlled system with maximum allowed error around 10%. Natural frequency of the system and damping ratio are among key components of input shaping method algorithm.

In case of bridge crane systems every variation in weight of a load will result in different natural frequency of a system and also a bit different damping ratio. Above this, the length of a rope of the crane will also introduce changes in above mentioned system variables. This fact can be dealt with by lookup tables which could be set for multiple variation ranges in order to meet the 10% error



requirement. With this extension it could be fairly easy to control bridge crane for different loads and different lengths of a rope which carries the load.

Additionally, my work covers modelling of bridge crane in Simscape Multibody with Matlab Simulink simulation for measurements and validation. With this additional work the model can be easily changed and automatically recalculated for future works and adaptations. Added to this the work features with simple and clean animation which can be used for teaching purposes or detailed observation of load's behavior.

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