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DETERMINATION OF MEASUREMENT UNCERTAINTY OF STRAIN AND STRESS USING STRAIN GAGES

STANOVENÍ NEJISTOTY MĚŘENÍ PŘETVOŘENÍ A MECHANICKÉHO NAPĚTÍ ZÍSKANÉ Z ODPOROVÝCH TENZOMETRŮ

Abstract

The article deals with the determination of measurement uncertainty of strain and stress using resistance strain gages. You can find two methods to define the uncertainty in the article, GUF and MMC, and both are applied for measurements carried out with resistance strain gages. Definition of the measurement uncertainty was set for the strain measured by uniaxial and biaxial strain gages. The uncertainty of the stress was defined for linear strain gages, T-rosettes and Rosettes. There were universal mathematic-technical models defined to measure strain and stress. These models can be used either for standard and special measurements i.e. high-temperature, or for measurements in radiation field. Each part of the strain and stress uncertainty is analysed from the point of view of a size and shape of probability function error that strain and stress can adopt. The maximum focus was dedicated to the errors influencing measured strain like strain gage properties, installation and operating influences, external influences, time effects and the influence of the measured object. There are two errors influencing the mechanical stress described and analysed in the thesis, the error of the Young's modulus of elasticity and the error of the Poisson's ratio.

Abstrakt

Práce se zabývá stanovením nejistoty měření přetvoření a mechanického napětí pomocí odporových tenzometrů. V práci jsou uvedeny dvě metody pro stanovení nejistoty měření. Metody GUF a MMC jsou následně aplikovány na měření prováděná odporovými tenzometry. Stanovení nejistoty měření bylo provedeno pro přetvoření měřené jednoosými a dvojosými tenzometry. Nejistota mechanického napětí byla stanovena pro jednoosé tenzometry, kříže a růžice. Pro měřené přetvoření a vypočtené mechanické napětí byly vytvořeny universální matematicko–technické modely, které lze následně aplikovat na standardní i speciální měření, jako jsou vysokoteplotní nebo měření v radiačním poli. Jednotlivé dílčí složky nejistoty přetvoření a mechanického napětí jsou rozebrány z hlediska velikosti chyby a tvaru pravděpodobnostní funkce, které mohou nabývat. Největší důraz byl kladen na chyby ovlivňující měřené přetvoření, jako jsou vlastnosti tenzometru, instalace a provozní vlivy, vnější a časové vlivy a vliv měřeného objektu. Chyby ovlivňující mechanické napětí jsou uvedeny a rozebrány dvě, a to chyba modulu pružnosti a chyba Poissonova čísla.

Keywords

Strain gage, strain, stress, uncertainty of measurement, GUF and MMC method

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1 INTRODUCTION

Resistance tensometry is the key measurement method of the experimental mechanics that belongs to the scientific-applicational discipline of the mechanics of bodies. The strain gages are the most used sensors for measuring strain (stress), especially for easy-to-get, accuracy, low cost and applicability. These days the tensometry is used in most of industries and technical sciences. The role of tensometry especially in engineering and civil engineering is irreplaceable. Each measurement represents an error and uncertainty of measurement. It is not possible to avoid measurement errors in total, but it is required to reduce them on an acceptable value. To define the uncertainty of measurement it is necessary to define the main sources of errors and then to quantify them. From the practical point of view, it is necessary to consider the error and the uncertainty of measurement for the integral part of the measurement and, in most cases, the knowledge of measurement error is more important than the result itself. Nowadays the strain and stress are analysed applying numerical modelling. The standard of calculation modelling is very high and, with HW and SW development, it has become almost a routine process. This technical approach has got several advantages. A higher technical standard and effectiveness of processes together with lower cost of research, construction, production and control activities are the key ones. Nevertheless, it is necessary to take in mind that no matter how good your calculation is, it is a subject to a degree of inaccuracy, and that is why the calculation cannot fully substitute an experiment. If we do not verify our results obtained by numerical modelling by an experiment, we cannot consider our results credible.

2 SITUATION OVERVIEW

The current level of knowledge of the resistance tensometry is very high, including the strain gages error (i.e. article [12] and [13]) and measurements [1], [2], [3]. We can say the same in case of the area of the uncertainty of measurement [4], [5], [6], [8]. Nevertheless, it is necessary to remind that the resistance tensometry is rather old technical discipline, which roots go back to the beginning of the 20th century and it keeps developing. On the other hand, the theory of uncertainty of measurement is relatively young technical discipline. The uncertainty of strain measurement is partially analysed and described in the articles [10] and [11].

Nowadays the experimental analysis of strain and stress (engineering, civil engineering constructions) plays an irreplaceable role. Even the numerical modelling of strain and stress is applied more and more, the data obtained via experiments are also important, especially in case of verification of complex calculations. In certain technical areas it is better to perform an experiment than calculations. Among these we can name especially prediction of strength, lifetime, operational reliability of constructions and devices, and also monitoring of operations and operating status of constructions and devices. However, it is necessary to take in mind that each experiment is a subject to a degree of inaccuracy. The degree of inaccuracy of measurement can be defined mathematically as the uncertainty of measurement that specifies the interval of occurrence of the actual value with a certain probability. If we want to use the data collected in experiments to verify the numerical modelling methods, it is necessary to define the uncertainty of measured value first (the strain and stress in our case). And only after that, it will be possible to carry out a certain interaction of numerical modelling and experimenting. The need for knowledge of uncertainty of measurement is fully obvious in the other activities as well. If we do not know the value of measurement uncertainty of strain, we can hardly declare that the intended prediction or lifetime is credible.

This article deals with determination and verification of mathematical-technical model of measurement uncertainty of strain and stress. There were two methods of uncertainty definition formed using strain gages. The Method of Gum Uncertainty Framework (GUF), which is an analytical method using the law of spreading uncertainty (see equations 1 and 2) and the method Monte Carlo (MMC) based on numerical simulation. The article includes definition of measurement uncertainty of strain and stress using linear and biaxial gages (T-rosettes and Rosettes). The article also includes the error analysis and analysis of strain and stress measurement accuracy.

3 UNCERTAINTIES OF MEASUREMENTS

"The uncertainty of measurement is a parameter related to the result of the test (measurement) that characterizes interval of values in which the real value is with a certain probability" [8]. The measurement uncertainty is part of the measurement result. The technical term (terminus technicus) "*measurement uncertainty"* is a globally recognized expression for describing a value of quality result of measurement or calibration in all fields of human activities (i.e. engineering, medicine, chemistry atc.). Currently the measurement uncertainty can be defined analytically applying the method GUF (see Fig. 1) or numerically by the MMC method (see Fig. 2). From the practical point of view, it is irrelevant whether the measurement uncertainty is defined by the GUF or MMC method. It is important, however, that the measurement model is compiled well and the characteristics of considered sources, esp. considered extreme limits and probability distribution pattern (probability function).

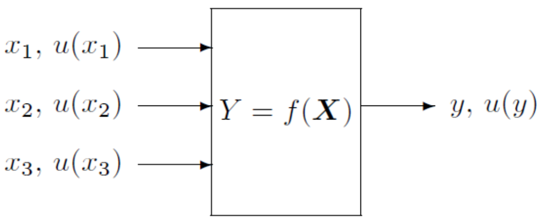

Fig. 1 The GUF method principle [4]

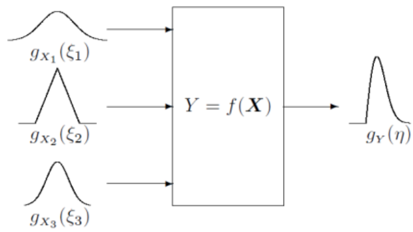

Fig. 2 The MMC method principle [5]

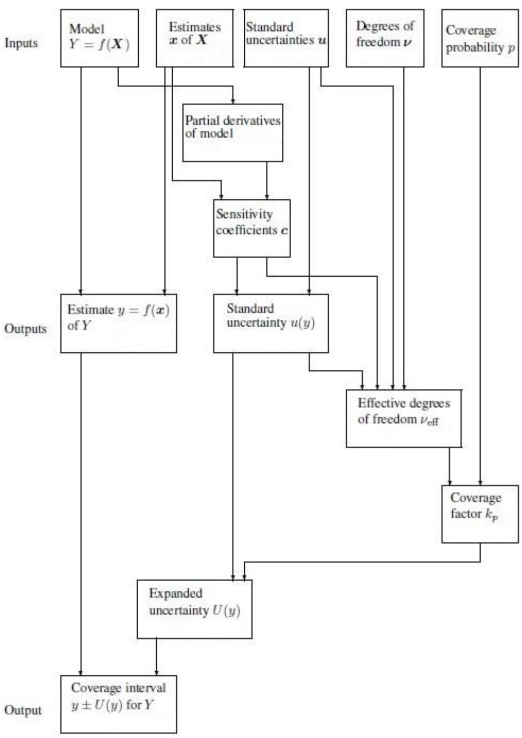

Fig. 3 Algorithm of the GUF method [7]

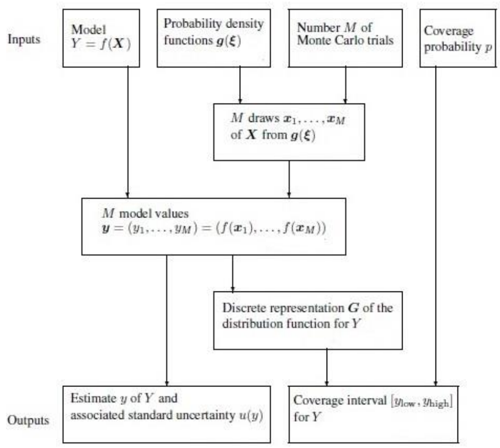

Fig. 4 Algorithm of the MMC method [7]

3.1 Uncertainty – GUF method

The standard uncertainty defined by the GUF method are divided according to the way of obtaining the uncertainty into type A (u_A) and type B (u_B). Both methods are equivalent. The evaluation of the uncertainty type A is made by statistic processing of the results of repeated direct measurements. The evaluation of the uncertainty type B is based on a qualified estimate based on all information about the measured quantity available. To merge both types A and B we obtain the combined standard uncertainty (u_C). We create an expanded measurement uncertainty U to extend the interval of actual measured quantity. It indicates an interval with bigger probability of coverage of actual value. The procedure of defining the measurement uncertainty is different for directly and indirectly measured quantities [8].

The combined standard uncertainty determined by the GUF method is defined, based on the law of spreading uncertainty, by the equation 1 for the correlated values and by the equation 2 for uncorrelated values.

$$u_c^2(y) = \sum_{j=1}^{N} \left[\frac{\partial f}{\partial x_i} \right]^2 \cdot u^2(x_i) = \sum_{i=1}^{N} [c_i \cdot u(x_i)]^2 \equiv \sum_{i=1}^{N} u_i^2(y) \tag{1}$$

$$u_c^2(y) = \sum_{i=1}^{N} c_i^2 \cdot u^2(x_i) + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_i c_j u(x_i) \cdot u(x_j) \cdot r(x_i, x_j) \tag{2}$$

The algorithm (see Fig 3) for defining the combined standard uncertainty u_C according to the GUF method of direct measurements is the following [8]:

1. potential sources of uncertainties $Z_1,\dots Z_i$ of the measured value X are guessed
2. estimate standard uncertainty X_i of the measured value X is defined (max. tolerance Z_{max})
3. the standard uncertainty $u(x_i)$ is defined from the aquation 3

$$u(x_i) = \frac{Z_{i\max}}{\chi} \tag{3}$$

4. the sensitivity coefficients c_i (x_i) are defined
5. correlation between defined standard uncertainties $u(x_1)$ – $u(x_i)$ is considered and the correlation coefficients r are determined if required
6. in case of more than 3 independent measurements, the type B uncertainty u_A and other additional procedures are defined, see [4]
7. individual standard uncertainties $u(x_i)$ (contributions) are merged into the resulting value, and the type B uncertainty $u_B(x)$ is defined
8. the combined uncertainty $u_c(x)$ is defined according to the equation 1 or 2
9. the combined standard expanded uncertainty U of required probability given by the equation 4 is defined

$$U = k \cdot u_c \tag{4}$$

Where:

k coverage factor [-]

k = 1 corresponds to the probability of 68.27%

k = 2 corresponds to the probability of 95.45%

k = 3 corresponds to the probability of 99.70%

The mentioned coeffients are valid for normal probability distribution.

The algorithm of the combined uncertainty u_c of the GUF method for indirect measurements is following:

1. the measurement model Y and its functional relation of value Y with values X_1, X_n is defined by equation 5; then the estimate y of the output (measured) quantity is given by equation 6

$$Y = f(X_1, X_2, ..., X_n) \tag{5}$$

$$y = f(x_1, x_2, ..., x_n) \tag{6}$$

2. the standard uncertainty $u(x_n)$ – GUF method for direct measurement of the input values X_1, X_n is defined according to the points 1 – 8
3. the sensitivity coefficient $c_n[u(x_n)]$ is defined
4. correlations between individual standard uncertainties $u(x_1) - u(x_n)$ is assessed and the correlation coefficients r are defined
5. individual standard uncertainties $u(x_i)$ (contributions) are merged into the resulting value; the type B uncertainty $u_B(y)$ is defined
6. the combined standard uncertainty $u_c(y)$ is defined by equation 1 or 2
7. the combined expanded uncertainty U of the required estimate is defined by equation 4

3.2 The uncertainty – MMC method

The determination of the uncetainty MMC method is based on a general procedure of the numerical solution of physical models carried out by Random repetitive tests. It is a stochastic simulation method, which result is processed statistically. The principle of the MMC method is generating random or pseudorandom numbers according to the density probability of input values, where the output is discrete form of the probability function of output quantity. The phases of determining the uncertainty MMC are: defining the mathematical model, simulations, evaluating and summarizing the obtained values. Algorithm (see Fig 4) of defining the standard uncertainty u_c of the MMC method is the following [5]:

1. forming the mathematical model Y = f(X), where Y is a scalar output variable and X represents n of input value. Each X_i is a random value of a density probability $g(\xi_i)$, where ξ_i is a value of the given quantity. Y is a random variable of a potential value η and a density probability $g(\eta)$
2. setting number of repetitions *M* and the coverage probability *p* of MMC
3. generating M of random vectors x_r, r = 1,…M by density probability
4. putting the generated values into the model $y_r = f(x_r)$
5. sorting the values y_r in a non-decreasing order. Then the discrete distribution function G is defined out of the order of values
6. the calculation of arithmetic average (average value) $\bar{y}$ and of the experimental standard deviation given by equations 7 and 8

$$\bar{y} = \frac{1}{M}\sum_{r=1}^{M} y_r \tag{7}$$

$$u(\bar{y}) = \sqrt{\frac{1}{M-1}\sum_{r=1}^{M}\left(y_r - \bar{y}\right)^2} \tag{8}$$

7. defining the coverage interval for Y determined from discreate form of G. We define the interval by calculating q = pM first, then [y_{low}, y_{high}] is 100% of the coverage interval for Y, where y_{low} and $y_{high} = y_{(r+q)}$. The probabilistically symmetrical interval is r = (M-q)/2, where y_{low}, y_{high} is coverage interval [-]

The uncertainty u(y) of the probability function of output value Y, with normal distribution, is defined for the coverage probability of 68.27 as experimental standard deviation by the equation 8. In other cases, where the probability distribution of output variable is different but normal, there is necessary to calculate the shortest coverage interval of the required coverage probability (i.e. p = 68.27%, p = 95.45% etc.) [5], [9].

In most cases the repetition value $M = 10^6$ should be sufficient for the coverage interval of 95%. The no. of repetitions should not drop under $M = 10^4$. In case of more difficult calculations that might take too much time, the adaptive MMC method can be applied. It follows the convergence of uncertainty u(y) of the output quantity (see [6]).

4 MEASUREMENT UNCERTAINTY OF STRAIN

Strain ε is a very small deformation of surface of a material, caused by loading the body. The strain gages are used to measure the deformation at or around the point. The deformation measurement using strain gages assumes that the deformation of the tested object is transferred to the strain gage without loss. In resistance tensometry, the transferred deformation causes a measurable change of electrical resistance. The principle of resistance strain gages can be summarized as follows [2]: “If a material deforms when loaded, the deformation will also occur on the surface of the material. The change is then transferred on the strain gage that changes its resistance, which is linearly proportional to the elongation (strain) on the material surface”.

4.1 Accuracy of strain measurement

In the resistance tensometry, the strain ε is a designation for very small deformations. The accuracy of the strain measurement by resistance strain gages is based on a fact that the strain gage becomes a true sensor (a finished product) only when installed on the measured object. It is extended by a fact that the characteristics of the “made sensor” (strain gage) are influenced by the measured object, environment in which the object is located, wiring and processing of the sensor signal and the influence of time when external influences effect the measured object. To carry out the measurement successfully with required accuracy, it is necessary that the user understands well the function of the measured object and its properties, select a proper strain gage, design correctly the measuring chain and identify all influences during the measurement. Prior to each measurement it is necessary to analyse accuracy of measurement, based on which to carefully examine potential errors (disturbances) to be eliminated or compensated subsequently. In case it is impossible to eliminate or compensate the errors, it is necessary to include them into the measurement uncertainty.

Frequent phenomenon of the strain measurement is a relatively low required accuracy of the measurement. And especially when it is sufficient to measure with error in order of % (sometimes even in tens of %!). Another frequent phenomenon is that due to finance it is not possible to make more accurate measurements (especially in high-temperature tensometry). Finally, there is a time factor, when the experimenter cannot make measurements (processing the acquired data) with the required accuracy for time reasons.

Nevertheless, no matter if the measurement is made with low or high accuracy, it is always necessary to include all potential errors into the overall uncertainty, both in compensated and uncompensated (objectively negligible effects) form.

4.2 Strain uncertainty

The strain uncertainty u_ε can be divided into 5 sub-sources of uncertainties (u_ε-A till u_ε-E) which are based on signal origins. The main sources of uncertainties can be further divided into individual sub-sources based on errors and they have the basic probability distribution. See the scheme on Fig. 5. The value of the error and the probability distribution that the errors can have are shown in Tab. 1.

We meet a merge of the term "uncertainty" and "error" in the articles [10], [11], and [12]. The philosophical-technical approach, that is rather different between authors, is another specific factor concerning the uncertainty of tensometry (the strain and stress measurement). Some authors include even experimenter's errors into the strain measurement uncertainties (see [12] and [13]), which belongs, according to my opinion, into the category of gross errors. The errors of real location and shape of deformation (tension) on the measured object are other errors included into measurement uncertainty (see [10]). The error of experimenter and real state of deformation is rather difficult to quantify. Although it is possible, in certain cases, to carry out validation experiments or comparison with FEM simulation, it is very difficult to correct the mentioned errors during the real measurements on complex components.

The author of the article [12] states that there were more than 70 sources of uncertainties when measuring with strain gages. He divides the uncertainties into four main groups: gage, method, environment and operator. The author connects most of uncertainties with the experimenter. The errors caused by human factor are listed in the Article [13]. The authors of the article [10] describe only 5 sources of uncertainties (errors), meaning: recording device, transverse sensitivity, temperature effect, strain gage misalignment (deflection) and non-linearity of Wheatstone Bridge. The strain uncertainty is defined applying the GUF method. The article [11] states the methodology applying the Monte Carlo method to estimate measurement uncertainties. There are the following sub-uncertainties considered in the mathematical model: the uncertainty of location (deflection from the main axis), integrational effect and the transverse sensitivity. The article also includes the errors and properties of welding strain gages. The MEBU methodology (numerical Method for the Estimation of Biases and Uncertainties) stated in the article enables to define an error (uncertainty) caused by the integration effect, the transverse sensitivity and an error caused by the welding method of strain gage.

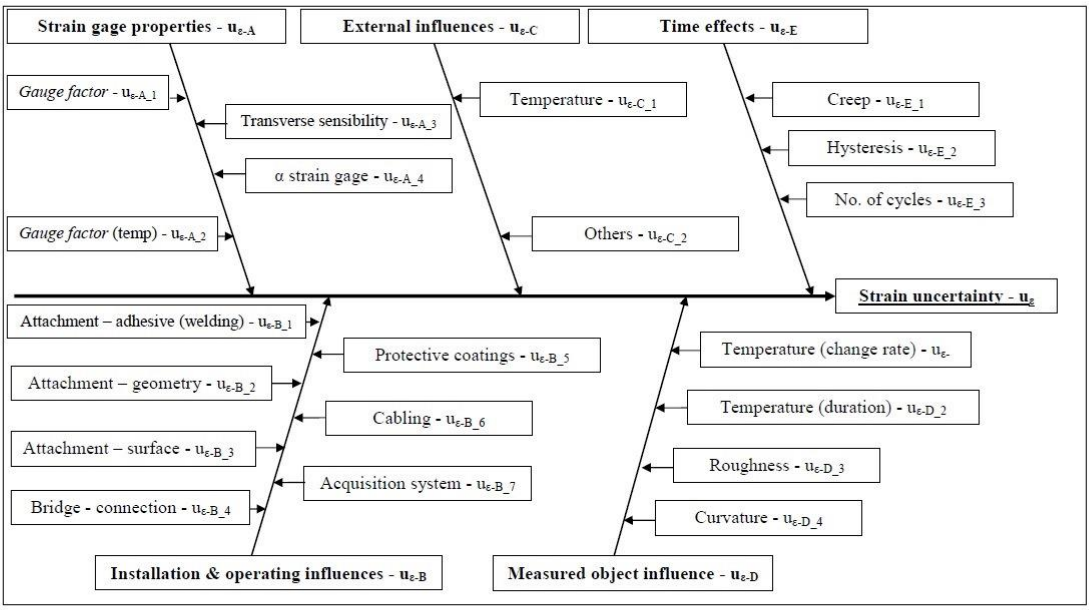

Fig. 5 Uncertainty sources of measured strain

Tab. 1 Properties of uncertainty sources of measured strain - u_ε

	Uncertainty source	Error		Distribution - χ
A	**Strain gage properties**			
1	*Gauge factor*	δ_K	According to the used strain gage (see datasheet)	Uniform
2	*Gauge factor* (temperature)	δ_{K100}		Uniform
3	Transverse sensitivity	δ_Q		Normal
4	α strain gage	δ_α		Triangular
B	**Installation and operating influences**			
1	Attachment – adhesive	δ_L	0,5 – 2.5 μm/m	Uniform
	Attachment – welding		10 - 100 μm/m	
2	Attachment – geometry	δ_G	1 - 5 μm/m for deflection of 5° is equal to 1.5 % of measured value	Normal
3	Attachment – surface	δ_D	1 - 4 μm/m	Uniform
4	Bridge – connection	δ_B		
5	Protective coating	δ_O	0 - 3 μm/m	Uniform
6	Cabling	δ_C	0 - 10 μm/m (0 - 5 μV/V)	Normal
7	Acquisition system	δ_{MJ} ($\delta_{MJ\text{-}1\text{ - }4}$)	1 - 2 μV/V, 1 - 6 μV/V 4 - 10 μV/V, 10 - 20 μV/V	Normal Uniform
C	**External influences**			
1	Temperature	δ_T ($\delta_{\varepsilon app}$)	5 - 20 μm/m (20 - 100 μm/m)	Triangular
2	Others	δ_P	According to external environment strain gage is located; 1 - 20 μm/m	Uniform
D	**Influence of measured object**			
1	Temperature – rate of change	δ_M	Estimate 0 - 100 μm/m	Triangular
2	Temperature – effect duration	δ_N		Uniform
3	Roughness	δ_R		Normal
4	Curvature	δ_Z		Normal
E	**Time effects**			
1	Creep	δ_V	5 - 20 μm/m 0.5 - 2% of measured strain	Uniform
2	Hysteresis	δ_H	0.25 – 0.5% of measured strain	Normal
3	No. of cycles	δ_U	10 μm/m	Uniform

5 MEASUREMENT UNCERTAINTY OF STRESS

The stress σ and shear stress τ in the form of stress tensor T_σ describe a tension around the general point of the body. The body tension depends on shape, load and material properties of the body. We divide the tension, according to the number of non-zero stresses in the tensor, on uniaxial, biaxial and triaxial.

The stress is calculated from the measured strain applying the constitutional equations (physical relations). The method of the stress calculation utilizes the Hooke's law (equation 9 [1]) to calculate the stress. The law is valid only in the field of elastic deformations of material, which are bounded by the yield of strength of the specific material R_e ($R_{p0.2}$). The law is valid only for the uniaxial tension.

$$\sigma = \varepsilon \cdot E \tag{9}$$

Mechanical behaviour of isotropic material in the area of elastic deformations is described by two independent constants – elasticity modulus E (Young's module) and Poisson's ratio (coefficient of transverse deformation) μ[16], [17].

There are mainly uniaxial and biaxial tension in the field of experimental analysis of strain. The uniaxial tension is rather rare, we mostly meet the biaxial tension in two forms:

1. directions of principal stresses are known (biaxial stress with familiar directions) or they are at least assumed
2. directions of principal stresses are not known (biaxial stress with unknown directions)

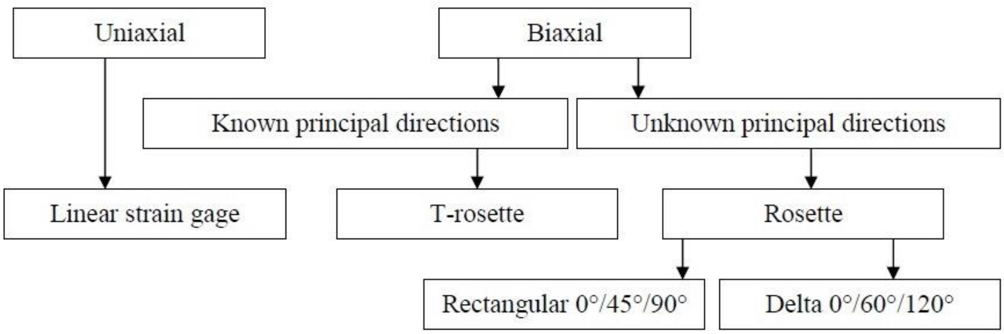

Fig. 6 Stress division of experimental analysis of stress and strain gages

The principal stress σ_1 and σ_2 gained from the T-rosettes is defined from the measured main deformations ε_1 and ε_2 by the equation 10 and 11. The principal stress (strain) for Rectangular Rosettes is defined from the measured deformations in three directions ε_a, ε_b and ε_c by the equation 12 and 13 [1].

$$\sigma_1 = \frac{E}{1-\mu^2}\left(\varepsilon_1 + \mu \cdot \varepsilon_2\right) \tag{10}$$

$$\sigma_2 = \frac{E}{1-\mu^2}\left(\varepsilon_2 + \mu \cdot \varepsilon_1\right) \tag{11}$$

$$\varepsilon_{I,II} = \frac{\varepsilon_a + \varepsilon_c}{2} \pm \sqrt{\left(\frac{\varepsilon_a - \varepsilon_c}{2}\right)^2 + \left(\frac{2 \cdot \varepsilon_b - \varepsilon_a - \varepsilon_c}{2}\right)^2} \tag{12}$$

$$\sigma_{I,II} = \frac{E}{1-\mu} \cdot \frac{\varepsilon_a + \varepsilon_c}{2} \pm \frac{E}{\sqrt{2} \cdot (1+\mu)} \cdot \sqrt{\left(\varepsilon_a - \varepsilon_b\right)^2 + \left(\varepsilon_c - \varepsilon_b\right)^2} \tag{13}$$

5.1 Accuracy of mechanical stress measurement

The accuracy of stress measurement is given by the combination of knowledge of a real value of measured strain and the independent material properties of measured object (see Fig. 7).

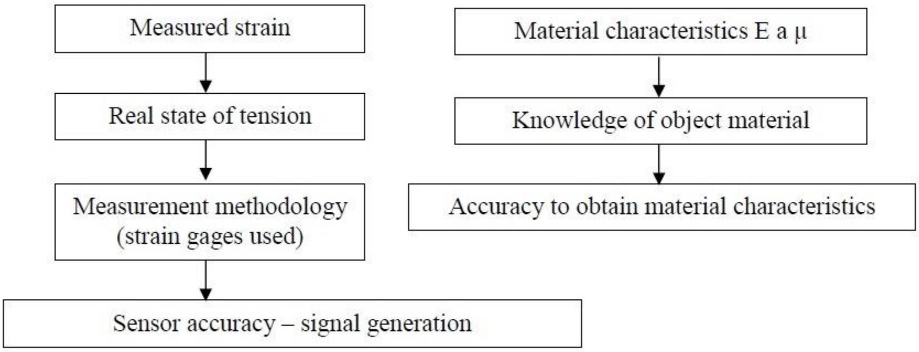

Fig. 7 Influencing factors of measurement accuracy of the stress

Defining the stress from measured strain we often assume that we know, or at least we suppose a tension on the measured object. Based on the mentioned presumption a type of strain gage is selected and then installed on the measured object. In case of measuring on simple objects, where forces on object are known, we can assume that the reduction of measurement accuracy do not occur due to an action of another state of stress. In case of measuring on complex objects, it could occur that the selected methodology (strain gage) does not correspond to the state of stress; then the measurement accuracy may decrease. The accuracy may also decrease if, for whatever reason, we know what the state of stress will be, but from technical, financial or time reasons we will not measure with adequate strain gages.

To define the stress the knowledge of material properties of the measured object is expected. The elasticity modulus significantly influences the actual chemical composition of the object or the thermochemical processing of steel. In practice, we usually meet the situation when the elasticity modulus is defined for steel grades and not for individual melting, etc. In case of lower steel grades (i.e. 10 and 11) the material composition has rather wide range of tolerance. The situation in case of thermochemical processing is also complicated. It is not standard to determine actual values of material properties after processing, i.e. quenching or annealing. The above mentioned experience is generally known and applied in constructions and using devices. It follows from these findings that the elasticity modulus of the measured object is almost always loaded with a certain degree of inaccuracy (error).

The accuracy of defining the elasticity modulus is given by an error of the determination method. The elasticity modulus defined by the tensile-pressure test (or by other methodology) is interpreted as a single number (straight line) in the field of elastic deformations. In a fact, such an interpretation is only a simplification of the real situation. The difference between methods of measurement can reach 3 to 5%.

In the field of Experimental Stress Analysis defining stress from the measured strain, it is contemplated that the values of the elasticity modulus and Poisson's ratio assigned to equations 9 to 13 are average values of a certain predicted interval. There is an exception in case of situations when, to specify the measurement results, the material properties are defined separately from the measured object (of the used material).

5.2 Error of the elasticity modulus and Poisson's ratio

The error of the elasticity modulus oscillates between 3 and 15%. The error δ_E cannot drop under the mentioned 3%, by principle of determination. It is very unlikely that the elasticity modulus (and the associated error) of the measured object exceeds 9% considering the amount of information available about materials in engineering, especially steel and alloys. An average value of the elasticity modulus is E = 205 000 MPa and the deviation from low-carbon up to high-alloy steel is stable and reaches an average value of ±15 000 MPa, which represents ±7.32%. The drop of the elasticity modulus on 50°C represents 1.4 up to 2.1%. These values express the fact, that we can work with an average error of ±8 to ±9% considering we have a basic material knowledge of the measured object made of steel, which temperature oscillates in the interval of ±25°C. The more information about the elasticity modulus of the measured object we have, the minor error of the mentioned ±8 to ±9% we can consider. And vice versa, if the material of the measured object is heat-treated, its elasticity modulus changes much more than the mentioned ±8 to ±9%. Then, it is necessary to consider the elasticity modulus for individual heat treatment and, if the information is not known, to extend the error of the elasticity modulus on ±13 to ±14%. [14], [15].

Let's consider a situation that we know the material of the measured object, its heat treatment and temperature. Then the elasticity modulus is determined from the relevant charts or the producer's certificate (in case of a higher quality steel). Then the modulus error will depend on its determination (if done) or the "authenticity" of material. Let's consider then that the error of modulus is approximately ±5 to ±6%.

An average value of the Poisson's ratio for steel is $\mu = 0.285$ and it is found in the interval 0.27 to 0.30. Based on that we can state that the error of Poisson's ratio $\delta\mu$ is approximately ±5.3%. That is why we can assume that the error of Poisson's ratio will be smaller.

5.3 Stress uncertainty

The uncertainty of linear stress (the linear strain gage) $u_{\sigma n}$ is formed by two sources of uncertainty. The strain uncertainty u_ε and the uncertainty of elasticity modulus u_E. The scheme is shown on the Fig. 8. The stress uncertainties $u_{\sigma 1}$, $u_{\sigma 2}$ (T-rosettes) and $u_{\sigma I}$ and $u_{\sigma II}$ (Rosettes) are formed by three sources of sub-uncertainty. The strain uncertainty u_ε, the uncertainty of elasticity modulus u_E and the Poisson's ratio uncertainty u_μ. The scheme is shown on the Fig. 9 and Fig. 11. The strain uncertainty of the main straight lines $u_{\varepsilon I}$ and $u_{\varepsilon II}$ and the uncertainty of angle $u_{\varphi H}$ (Rosettes) is formed by one sub-source, the strain uncertainty u_ε. The scheme is shown on the Fig. 10.

The T-rosettes and Rosettes belong to a group called the multiple strain gages. It means that there are two or three measuring grids placed on a pad. It is necessary then to define the strain uncertainty for each direction separately. Simultaneously, it is necessary to realize that some sources of strain uncertainties shown in Fig 5 are related to the measuring grid ($u_{\varepsilon\text{-}A\_1,2,3}$ and $u_{\varepsilon\text{-}B\_4,6,7}$) and other part is related to the strain gage as a sensor. Then the strain uncertainty can be divided into several strain uncertainties and each of the adequate direction consists of two parts. The part of the sub-uncertainties related only to the measured direction and the part of the sub-uncertainties common for all directions (for sensor as a whole). The resulting division of probability will be slightly different for each strain uncertainty of the corresponding direction.

The uncertainty of the elasticity modulus u_E is given by the error δ_E for which we consider a uniform distribution. The Poisson's ratio uncertainty u_μ is given by the error δ_μ for which we consider a uniform distribution.

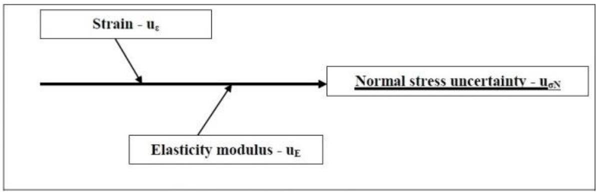

Fig. 8 Uncertainty sources – linear strain gage (σ_N)

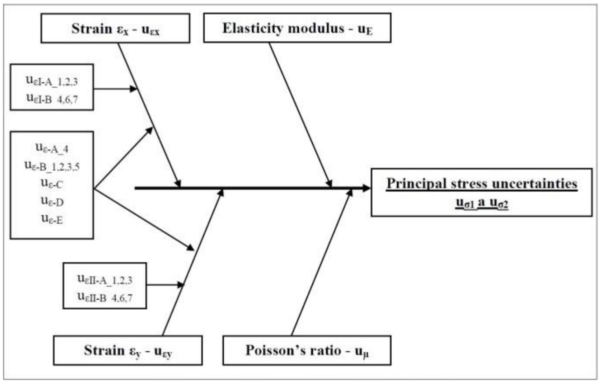

Fig. 9 Uncertainty sources – T-rosettes (σ_1 and σ_2)

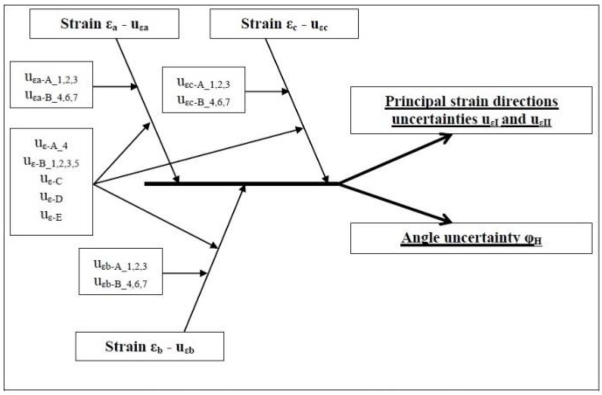

Fig. 10 Uncertainty sources - Rosettes (ε_I, ε_{II} and angle φ_H)

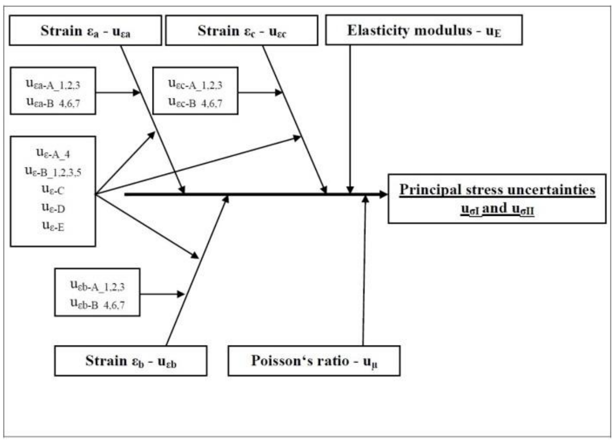

Fig. 11 Uncertainty sources – Rosettes (σ_I and σ_{II})

6 MEASUREMENT UNCERTAINTY OF LINEAR STRAIN - REAL CASE

Let's consider laboratory measurement (at room temperature T_0) on the steel shaft 12040 (E = 210 000 MPa) that is not heat treated. The shaft was loaded in the axial direction up to the yield strength ($R_{p0.2}$ = 370 MPa). There was a strain gage HBM type: 1-LY11-3/120 applied, connected into the quarter-bridge. Then the uncertainty of normal stress $u_{\sigma N}$ can be defined applying GUF or MMC method. To define the uncertainty, we assume the measured strain ε = 1668.29 μm/m, which corresponds to σ_N = 350.34 MPa and the following input parameters:

Tab. 2 The input parameters to define measurement uncertainty u_σ

Quantity	Estimate (error)	Standard uncertainty	Probability distribution
X_i	x_i (δ)	u (x_i)	χ
ε	-	11.47 μm/m	Normal
E	9%	-	Uniform

To define the uncertainty of normal stress u_σ type B of the uniaxial strain it is necessary to define sensitivity coefficients C (equation 14 and 15) based on the Hooke's law (see equation 9).

$$c_\varepsilon = \frac{\partial \sigma_N}{\partial \varepsilon} = \frac{\partial(\varepsilon \cdot E)}{\partial \varepsilon} = E \text{ [MPa]} \tag{14}$$

$$c_E = \frac{\partial \sigma_N}{\partial E} = \frac{\partial(\varepsilon \cdot E)}{\partial E} = \varepsilon \text{ [μm/m]} \tag{15}$$

6.1 GUF calculation

The standard strain uncertainty u_ε was defined directly by selecting sub-uncertainties (see Fig. 5 and Tab. 1).

$$u_{B_\varepsilon} = u_\varepsilon \cdot c_\varepsilon = 11.47 \cdot 210000 = 2.41 \text{ [MPa]} \tag{16}$$

$$u_{B_E} = \frac{z_{\max}}{\chi} \cdot c_E = \frac{18900}{\sqrt{3}} \cdot 1668.29 = 18.20 \text{ [MPa]} \tag{17}$$

$$u_{B_{\sigma N}} = \sqrt{u_{B_\varepsilon}^2 + u_{B_E}^2} = \sqrt{2.41^2 + 18.20^2} = \pm 18.36 \text{ [MPa]} \tag{18}$$

$$U_{\sigma_N} = k \cdot u_{B_{\sigma N}} = 2 \cdot \pm 18.36 = \pm 36.72 \text{ [Mpa]} \tag{19}$$

6.2 MMC calculation

Tab. 3 Calculated values (statistic parameters) obtained by the MMC simulation

Average value (y)	y	350.35 MPa
Standard deviation – standard uncertainty	u(y)	18.36 MPa
Shortest coverage interval for 95,45% - low limit	y_{low}	319.81 MPa
Shortest coverage interval for 95,45% - high limit	y_{high}	380.78 MPa
Shortest coverage interval for 95,45%	s. c. i.	60.96 MPa
Shortest coverage interval for 95,45%	-	[-30.54 ; 30.43]

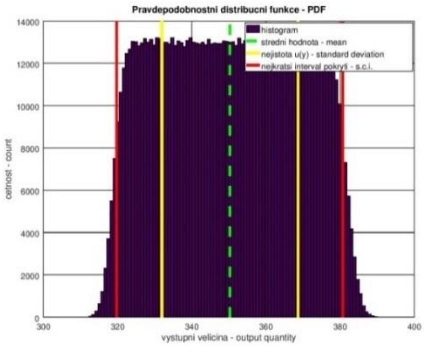

Fig. 12 Probability density function

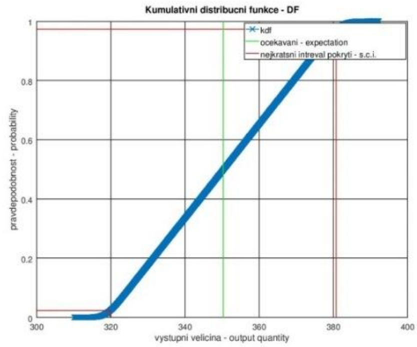

Fig. 13 Cumulative distribution function

Where:

PDF - probability density function

CDF - cumulative distribution function

6.3 Comparisinon of GUF and MMC

σ_N (GUF) = 350.34 MPa [-36.72 ; 36.72]$_{95.45\%}$

σ_N (MMC) = 350.35 MPa [-30.54 ; 30.43]$_{95.45\%}$

Comparing the results of both methods it is obvious, that the coverage interval y ± U for σ_N results 17% less in case of MMC than in case of GUF method. It is given by a choice of the probability distribution. The standard uncertainty u(y) is equal for both methods.

Nevertheless, it is necessary to remind, that the coverage interval and course of PDF and CDF will differ according to the chosen model and measured value of strain. It can be assumed, however, that, from the physical point of view, the resulting value of PDF and CDF of the stress will be close to the uniform distribution of probability.

8 CONCLUSIONS

The presented article provides the analysis and procedure to determine the measurement uncertainty of strain and stress by resistance strain gages. There is a mathematical-technical model created in the article based on the GUF analytical method (uncertainty type B) and the MMC numerical method (Mote Carlo). The created procedure and the information about individual uncertainties provide a manual that is enough technically reliable and credible to define measurement uncertainty of strain and stress applying the linear strain gages, T-rosettes and Rosettes. The basic idea of defining the measurement uncertainty of strain and stress is a combination of understanding the measurement accuracy of resistance tensometry, determination of errors and physical properties of quantities affecting the tensometric measurement and the subsequent implementation of the acquired values according to the theory of measurement of uncertainties.

The MMC method seems to be a very effective method to define the measurement uncertainty of stress. The analysis of a real case of linear strain proved that the results obtained by the GUF and MMC method are almost identical, and therefore it is irrelevant which of the method an experimenter chooses. More, in case of the MMC method the course of probability functions (PDF and CDF) is known, so the resulting behaviour of the measured object can be predicted. Another advantage of MMC is the mentioned higher efficiency in comparison with GUF method, especially in case of T-rosettes and Rosettes (equation 10 – 13), because there is no determination of sensitivity coefficients, which are defined by partial derivations of each variable. Another advantage of MMC method is a simultaneous calculation of the stress and measurement uncertainty.

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