Transactions of the VŠB – Technical University of Ostrava, Mechanical Series
No. 2, 2016, vol. LXII
article No. 2016

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STIFFNESS MODELS OF NOVEL FORCE/DISPLACEMENT SENSORS

MODELOVANIE TUHOSTI NOVÝCH SNÍMAČOV SILY/POSUNUTIA

Abstract

Miniaturization in field of robotics leads to use of elastic deformation where whole robotic device (precise positioning device, micro-gripper, etc.) is build from one piece of material. The disadvantage of such specific robotic devices is complicated approach to measure of their movement and acting forces. Application of influence of electromagnetic field with parallel resonating circuit seems as suitable method for sensing small deflections. This paper describes mentioned method of wireless measurement of small deflections of compliant robotic structure. Two structures of one and two-component force/displacement sensor are presented as examples using this approach. In the paper we are focused to mathematical description of stiffness models which provide basic static and dynamical properties of such structures.

Abstrakt

Miniaturizácia v oblasti robotiky vedie k využitiu princípov elastickej deformácie kde zvyčajne z jedného kusu materiálu je vytvorené celé robotické zariadenie (polohovadlo, uchopovač, atď.). Nevýhodou týchto špecifických robotických zariadení je zložitejší spôsob snímania ich pohybov resp. pôsobiacich síl. Ako vhodná metóda snímania malých deformácií sa javí využitie pôsobenia elektromagnetického poľa v kombinácii s paralelným rezonančným obvodom umiestneným na poddajnej robotickej štruktúre. Tento článok popisuje uvedenú metódu bezkontaktného snímania malých deformácií pružných kompaktných štruktúr. Ako príklady využitia tejto metódy sú uvádzané návrhy jedno a dvoj-zložkového snímača síl resp. posunutia pri ktorých sa zameriavame na matematický popis modelu tuhosti, pomocou ktorého je možné určiť základné statické a dynamické vlastnosti skúmanej mechanickej štruktúry.

Keywords

Stiffness, compliant structures, load cell, force sensor, micro-robotic devices

1 INTRODUCTION

Some robotic tasks connected with manipulation of small objects (dimension of few μm) can not enable to use classic constructions of robot mechanics based on assembly from discrete mechanical parts, or they are hardly realizable. The solution lies in designing devices which use principles of elastic deformation. Compact designs of mechanisms made from the one piece of elastic material enable to miniaturize dimensions and to make such structures in small or micro scale range [1]. Such types of devices have many advantages like as very high positioning accuracy (better then 1μm), small or limited volume of the operation space, high frequency of motions (hundreds

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displacements per second), high stiffness, vacuum compatibility, clean room compatibility with no backlash and friction, etc. Between most known applications of compact compliant devices belong various types of precise positioning devices, micro-grippers (for manipulation with e.g. optic fibers, molecules etc.), mechanical amplifiers (mainly in combination with actuator) and sensors' deformable parts (like as, accelerometers, gyroscopes and force/displacement sensors (include measurement of torques or pressures)). These all advantages enable to build high precision sensors with wide range of measured data. As example could be mentioned multi-axis force sensor for micro-robotics applications [2] they can measure forces with resolution up to 5μN with range to 0.5N. Only optimal design of deformable part of sensor will not suffice to get sensor with high sensitivity and it necessary to include suitable electronic in signal processing and signal calibration. An example is high sensitive accelerometer with accuracy 10^{-7} m/s^2 and with range $\pm 10^{-1}$ m/s^2 [3].

In general, information about states in mechatronic devices are required for control system, to improve selected properties. These are gained from wide range of sensors. Unfortunately, in the case of compliant structures, the information about movement (displacement) or acting load are gained by single-purpose sensors usually by strain-gauges or by piezoelectric elements [4]. Novel method based on electromagnetic principle seems as suitable and universal solution for measurement of small displacements or acting loads. In this paper is given basic information about such approach with formulation input requirements to design compliant micro-robotic structures. To verification of such approach had been designed two simple structures of one and two-component force/displacement sensors. Those were chosen for their simple realization, and work on this same principle as micro-robotic device. In the paper we have focused to mathematical description of their stiffness, with aim to get main static, dynamic and modal properties of proposed structures.

2 MEASUREMENT OF SMALL DEFORMATIONS

The information about small deformations in micro-robotic devices expressed as variable of displacement, or as function depends on stiffness and acting load (see Eq. (1)) is usually required by control system. The compliant structures work as precise positioning devices require strong control system. In comparison with conventional manipulators [5], it is not possible to use/analyze so huge numbers of data from sensors e.g. located in each joint. Integration of sensors in to compliant structure is possible only in limited way. If we are focused to devices in millimeters scale, the application of strain-gauges leads to acceptable results. In other side miniaturization lead to application of piezoelectric elements integrated directly in structure [4]. Other approach to “measure” all movements in structure is based on optimal design of kinematics with reducing number flexure joints. Then the pseudo-rigid system is fit into the elaborated framework of multi-body dynamics, in particular pre-control in combination with a feedback controller [6]. Between most known methods to measure small deformations of micro-robotic devices belong optical sensors based on triangular principle. Such sensors are very often used in some experimental platforms, because the diameter of laser tip is e.g. 0.75mm. In the case of smaller devices, only CCD cameras are suitable. In other side is required image processing and optic lens with minimal deformation of image.

Developing method of measure small deformations of micro-robotic devices is based on electromagnetic principle. The global idea of measurement is expressed like as: Small micro-robotic device works under influence of electromagnetic field. The movement (deformation) of structure leads to change of electromagnetic field parameters. Such parameters are measured and are depend on size of deformation of compliant structure. To verify such approach in our laboratories, the method was simplified. This one is based on integrating parallel resonation LC circuit to the compliant structure. Such simplification leads to decreasing frequency of peaks (see Fig. 1) what represent change of selected parameter of electromagnetic field.

The satisfaction of some requirements is demanded in design of structures suitable for novel approach of contactless and wireless measurement of small deformations. There are two main requirements. Material of whole structure should be dielectric with the highest ratio between Yield strength and Young's modulus and the output / flexural displacement (observed deformation) should correspond to parameters of LC circuit (it should be strong parallel).

Precise positioning devices and deformable parts of sensors work on this same principle. Consequently that, as suitable devices for verification of our method will be proposed one and two axes force/displacement sensors. Physical model of prototype for initial tests of such type of sensor is shown on the Fig. 1. Main parts of proposed sensor are: flexure (in middle, elliptical shape), field emitter (construction around flexure) and suitable evaluation electronic. Other components used in initial tests are micro-positioning screw (on top, produces input displacement/acting load) and calibration force sensor (bottom). Such method of contactless measurement of flexure structure displacement with some simulations is described in [7] and [8].

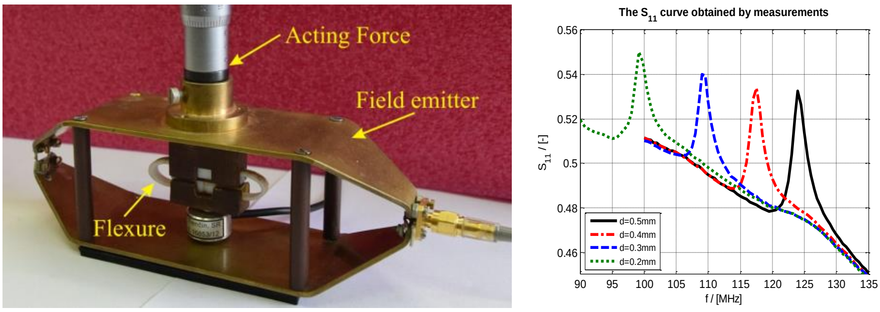

Fig. 1 Physical model of prototype of novel force/displacement sensor (left), the S_{11} curve obtained by measurements (right)

3 STIFFNESS MODELING

Devices works on principles of elastic deformation are modeled and simulated by substitution of flexures (joints, hinge and links) by mass-spring-damping system. In miniaturization process the influence of mass is minimized. On the other side damping usually depend on properties of used material of structure. Only compliance/stiffness depends upon known material properties (like as Young's modulus and Poisson ratio) and geometry of flexure (dimensions and shape).

To establish a full stiffness model modeling approach in view of deformations (in terms of bending, torsion, and tensile/compression) of each component of the mechanism can be executed. Such model can be derived effectively by the matrix method under the assumption of Hooke's law for the material [9] and [10]. The basic dependence between external load and deflection is expressed as

$$\mathbf{u} = \mathbf{C}\mathbf{F} \rightarrow \mathbf{F} = \mathbf{K}\mathbf{u} \tag{1}$$

where:

u - vector of deflection $u=[u_x, u_y, u_z, \theta_x, \theta_y, \theta_z]^T$ (in case of in-plane deflection $u=[u_x, u_y, \theta_z]^T$)

C - compliance matrix, what is inverse matrix of stiffness K, $K=C^{-1}$

F - vector of external load $F=[F_x, F_y, F_z, M_x, M_y, M_z]^T$

Initial step of stiffness models is calculation of the compliance/stiffness of elastic components like as joints, hinges and links. It is expected that such flexures are connected by rigid bodies. There are several approaches to calculate compliance of flexure element [11] but the common approach is use of Castigliano's theorem [12]: when a body is elastically deformed by a system of loads, the deflection at any point *P* in any direction *u* is equal to the partial derivative of the strain energy with respect to a load at *F* in the direction *u*.

$$\mathbf{u} = \frac{\partial U}{\partial \mathbf{F}} \tag{2}$$

The compliance matrices for most often used compliant hinges are expressed in [11] and [12]. These references take into account variable thickness of the joints and its specific shape.

The compliance matrices of separate flexures are expressed to specific end point (its mean that matrix is related to local coordinate system). For calculation of compliance/stiffness matrices of whole structure is necessary to transform it to global coordinate system. The transformation matrix expressed point located in first coordinate system (B) to second coordinate system (H) is

$$\mathbf{T}_{BH} = \begin{bmatrix} \mathbf{R}_{BH} & -\mathbf{R}_{BH}\mathbf{P}_{BH} \\ 0 & \mathbf{R}_{BH} \end{bmatrix} \tag{3}$$

where:

R_{BH} - rotation matrix between coordinate systems

P_{BH} - position matrix of point in B expressed in reference coordinates H, and it is expressed as

$$\mathbf{P}_{BH} = \begin{bmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix} \tag{4}$$

According to configuration of compliant structure the whole compliance/stiffness matrix can be calculated by relations where (5) is for serial and (6) for parallel configuration.

$$C_1 = \sum_n {}^iT_1^* C_i \left({}^iT_1^*\right)^T \tag{5}$$

$$K_1 = \sum_n {}^iT_1^* K_i \left({}^iT_1^*\right)^T \tag{6}$$

3.1 Stiffness of proposed load cell

The novel method of contactless measurement of small deformations requires design of structures with specified restrictions and requirements. One of them is connected with output - flexural displacements in particular directions should correspond to parameters of LC circuit, mainly the capacitor plates should move strongly in parallel. Consequently that stiffness model has to describe movement of point located on the moving plate of capacitor. The dimensions and shape of flexure is sketch on the Fig. 2.

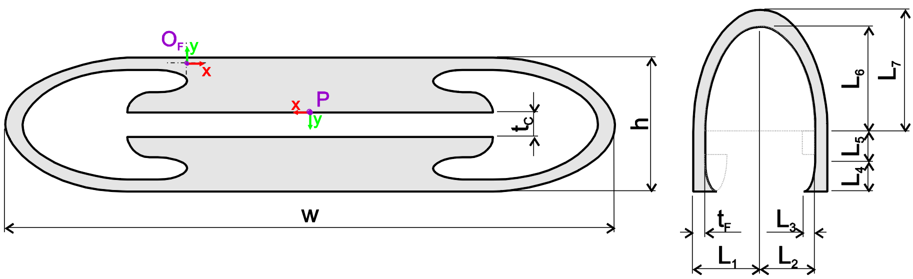

Fig. 2 Dimensions of deformable part of load cell

For simplification of calculations, all lengths are depended on height *h* and width *w* of proposed load cell. The variables t_C and t_F are thicknesses between capacitor plates and minimal thickness of flexure respectively. The thickness of structure will be labeled as t_S. Dimension of flexure are: L_1=h/2; L_2=h/2-t_F; L_3=h/12; L_4=0.0484w; L_5=0.05w; L_6=w/5-1.4t_F; L_7=w/5.

For solving stiffness of proposed load cell to point P is necessary to calculate stiffness of flexure (in this case it is curved beam with variable thickness) and then, by (6) express stiffness of whole structure to point P. Only in-plane deformations will be taking into account.

The simplified function described radius of curved beam *R (ϕ)* is

$$R(\phi)=\begin{cases}\dfrac{L_1-0.5t_F}{\cos(\phi)} & -\arctan\dfrac{L_4+L_5}{L_1-0.5t_F}\le\phi<0\\ \dfrac{(L_1-0.5t_F)(L_7-0.7t_F)}{\sqrt{(L_1-0.5t_F)^2\sin^2(\phi)+(L_7-0.7t_F)^2\cos^2(\phi)}} & 0<\phi\le\pi\\ \dfrac{-L_1+0.5t_F}{\cos(\phi)} & \pi<\phi\le\pi+\arctan\dfrac{L_4+L_5}{L_1-0.5t_F}\end{cases} \quad (7)$$

Function *R (ϕ)* is piecewise function compound from two straight lines and half of ellipse. Distribution of forces and torques in flexure is show in Fig. 3. The plot (Fig. 3 (right)) shown change of size of thickness depend on angle *ϕ*. For simplify symbolic calculation of stiffness will be function $t_F(\phi)$ constant. ($t_F(\phi)=1.2t_F$)

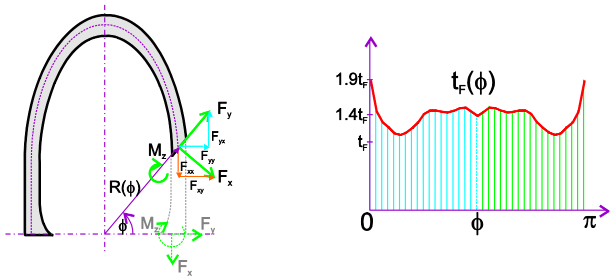

Fig. 3 The curved beam element with forces and torque distribution (left), dependence of flexure thickness on angle *ϕ*

The internal in-plane forces and the moment at any point on *R(ϕ)* may be expressed as follows

$$\begin{aligned}&\sum F_x=0:\quad F_x(\phi)=F_x\sin(\phi)-F_y\sin(\phi)\\&\sum F_y=0:\quad F_y(\phi)=-F_x\cos(\phi)+F_y\cos(\phi)\\&\sum M_z=0:\quad M_z(\phi)=M_z+F_xR(\phi)\cos(\phi)+F_yR(\phi)\cos(\phi)+F_xR(\phi)\sin(\phi)-F_yR(\phi)\sin(\phi)\end{aligned} \quad (8)$$

The strain energy in the beam can be expressed as

$$U=\frac{1}{2E}\int_{-\arctan\frac{L_4+L_5}{L_1-0.5t_F}}^{\pi+\arctan\frac{L_4+L_5}{L_1-0.5t_F}}\left(\frac{F_x(\phi)^2}{A_k}+\frac{F_y(\phi)^2}{A_k}+\frac{M_z(\phi)^2}{I_{zk}}\right)d\phi \quad (9)$$

where:

A_k - area of cross-section $A_k = 1.2t_Ft_S$

I_{zk} - second moment of area about *z* axis, $I_{zk} = (1.2t_Ft_S^3)/12$

Using Castigliano’s theorem (2), the deformation components can be obtained from (9) as

$$u_x=\frac{\partial U}{\partial F_x(\phi)};\quad u_y=\frac{\partial U}{\partial F_y(\phi)};\quad \theta_z=\frac{\partial U}{\partial M_z(\phi)} \quad (10)$$

Using equations (8) to (10) and carrying out the indicated partial differentiation and integration, the following equation (like (1)) in matrix form is obtained

$$\begin{pmatrix}u_x\\u_y\\\theta_z\end{pmatrix}=\begin{pmatrix}C_{xFx}&C_{xFy}&C_{xMz}\\C_{yFx}&C_{yFy}&C_{yMz}\\C_{\theta zFx}&C_{\theta zFy}&C_{\theta zMz}\end{pmatrix}\begin{pmatrix}F_x\\F_y\\M_z\end{pmatrix} \quad (11)$$

The numerical expression of compliance matrix is

$$C_F = \begin{pmatrix} 0.00994269 & -0.00122075 & 0.544313 \\ 0 & 0.00997269 & -0.544313 \\ -0.544313 & -0.544313 & 227.694 \end{pmatrix}$$

Now, it is possible to calculate stiffness/compliance of whole load cell with respect of coordinate system in the point P. It is necessary to use two transformation matrices – first only rotation matrix (12) where compliance matrix of flexure would transform about π/2, and second which transform end point of flexure to point P (13).

$$T_{0F} = \begin{bmatrix} R_z\left(-\frac{\pi}{2}\right) & 0 \\ 0 & R_z\left(-\frac{\pi}{2}\right) \end{bmatrix} \tag{12}$$

$$T_{FP} = \begin{bmatrix} R_z(\pi) & -R_z(\pi)\left(P_x\left(\frac{w}{2} - L_4 - L_5 - L_7\right) + P_y\left(\frac{-h + t_C + t_F}{2}\right)\right) \\ 0 & R_z(\pi) \end{bmatrix} \tag{13}$$

The compliance matrix to point P is

$$C_P = T_{FP} T_{0F} C_F T_{0F}^T T_{FP}^T \tag{14}$$

where:

C_F - compliance matrix of flexure (see (1) and (11))

Consequently that load cell is build from two flexures connected parallel, the stiffness in point P should calculated by (6) as

$$K_P = 2C_P^{-1} \tag{15}$$

3.2 Stiffness model of proposed xy-force sensor

The design of proposed structure of xy force sensor consists of two independent parallelograms that deflect independently in two directions (see Fig. 7).

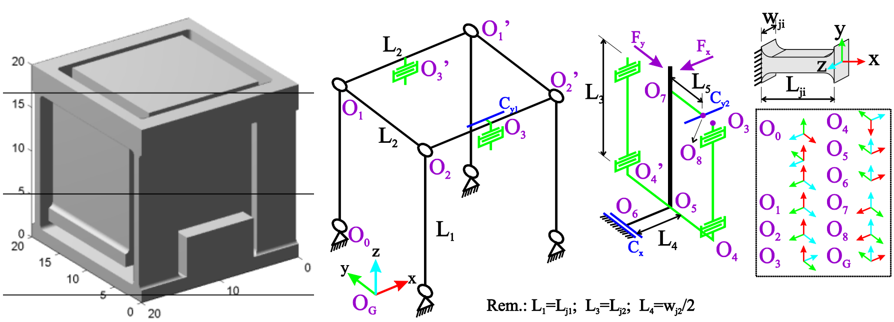

Fig. 4 Proposed compliant structure of sensor (left), geometry of the final design, and orientation of local coordinate system in observed points

Transformation matrices between particular local frames to that the compliance matrices were calculated are as follows:

$$\mathbf{T}_{01}=\begin{bmatrix}\mathbf{R}_z\left(\frac{\pi}{2}\right)\mathbf{R}_x\left(\frac{\pi}{2}\right) & 0\\ 0 & \mathbf{R}_z\left(\frac{\pi}{2}\right)\mathbf{R}_x\left(\frac{\pi}{2}\right)\end{bmatrix};\ \mathbf{T}_{12}=\begin{bmatrix}\mathbf{I} & -\mathbf{P}_z(L_2)\\ 0 & \mathbf{I}\end{bmatrix};\ \mathbf{T}_{23}=\begin{bmatrix}\mathbf{R}_x\left(\frac{\pi}{2}\right) & -\mathbf{R}_x\left(\frac{\pi}{2}\right)\mathbf{P}_y\left(\frac{L_2}{2}\right)\\ 0 & \mathbf{R}_x\left(\frac{\pi}{2}\right)\end{bmatrix} \quad (16a)$$

$$\mathbf{T}_{34}=\begin{bmatrix}\mathbf{R}_z(\pi) & 0\\ 0 & \mathbf{R}_z(\pi)\end{bmatrix};\ \mathbf{T}_{45}=\begin{bmatrix}\mathbf{R}_y\left(\frac{\pi}{2}\right) & -\mathbf{R}_y\left(\frac{\pi}{2}\right)\mathbf{P}_y\left(\frac{L_2}{2}\right)\\ 0 & \mathbf{R}_y\left(\frac{\pi}{2}\right)\end{bmatrix};\ \mathbf{T}_{56}=\begin{bmatrix}\mathbf{I} & -\mathbf{P}_x(-L_4)\\ 0 & \mathbf{I}\end{bmatrix} \quad (16b)$$

$$\mathbf{T}_{57}=\begin{bmatrix}\mathbf{R}_z(\pi) & -\mathbf{R}_z(\pi)\mathbf{P}_z(L_3)\\ 0 & \mathbf{R}_y(\pi)\end{bmatrix};\ \mathbf{T}_{78}=\begin{bmatrix}\mathbf{I} & -\mathbf{P}_y(L_5)\\ 0 & \mathbf{I}\end{bmatrix};\ \mathbf{T}_{j2}=\begin{bmatrix}\mathbf{I} & -\mathbf{P}_y(-L_2)\\ 0 & \mathbf{I}\end{bmatrix} \quad (16c)$$

Rem.: Orientation of local frame in point O_0 is equal with orientation of coordinate system to which is calculated compliance matrix of flexure hinge.

Compliance/stiffness matrix of whole structure to point O_7 is expressed as

$$\mathbf{C}_7=\mathbf{T}_{57}\mathbf{T}_{45}\left(\mathbf{T}_{34}\left(2\mathbf{T}_{23}\left(\left(\mathbf{T}_{01}\mathbf{C}_{j1}\mathbf{T}_{01}^T\right)^{-1}+\left(\mathbf{T}_{12}\mathbf{T}_{01}\mathbf{C}_{j1}\mathbf{T}_{01}^T\mathbf{T}_{12}^T\right)^{-1}\right)\right)^{-1}\mathbf{T}_{34}^T+\left(\mathbf{C}_{j2}^{-1}+\mathbf{T}_{j2}\mathbf{C}_{j2}^{-1}\mathbf{T}_{j2}^T\right)^{-1}\right)\mathbf{T}_{45}^T\mathbf{T}_{57}^T \quad (17)$$

From the (7) it is possible to express relevant stiffness coefficients required to calculate output displacement of both capacitor plates. It is necessary to remind, that output in y direction is difference of displacements between coordinate systems located in points O_8 (main movement) and O_3 (parasitic displacement). Output displacement in x direction is simply relevant displacement of point O_6. In the Fig. 5 is show deformation of proposed xy-force sensor in y direction. Output displacement in direction y is

$$u_y=u_{yO8}-u_{yO3} \quad (18)$$

Numerically expressed (from Fig. 5) is output displacement in y direction of proposed force/displacement sensor equal to u_y= -1.0551 - -0.0565 = 0.9986mm. From displacement u_y is clear that potential neglecting of parasitic deformations in point O_3 can lead to increased inaccuracy of measured results.

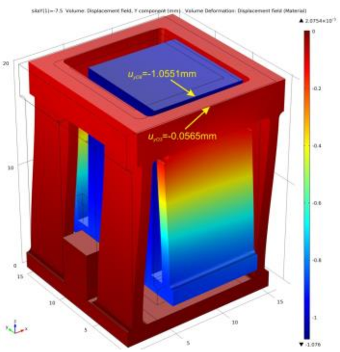

Fig. 5 Displacement of proposed xy sensor in y-direction

4 CONCLUSIONS

Novel method of contactless measurement of small distances requires development of new structures of force/displacement sensors. Such structures should be designed with respect to careful analysis based on various types of mathematical models. In case of sensors of mechanical quantities (forces, pressures, distances, etc.) is the knowledge of stiffness very important, because it is in relationship with wide range of sensor properties (deformations, working frequency, dynamical properties, etc.). The stiffness models of proposed force/displacement sensors have been derived. Such models will be used to other design steps like optimization of dimensions, calculation of dynamical characteristics etc. The dependence between acting load (force, torques) stiffness and displacement of compliant structure has been indicated. It was shown that parasitic deformations of deformable part of sensor have important influence to output displacement, and consequently can not be neglected.

ACKNOWLEDGMENTS

This work was supported by the Slovak Research and Development Agency under the contract No.: APVV-14-0076 – "MEMS structures based on load cell" and by the national scientific grant agency VEGA under project No.: 2/0154/16 – "Network management of heterogeneous multi-agent systems"

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