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SIMPLE STATICALLY INDETERMINATE TRUSS (LINEAR, NONLINEAR AND STOCHASTIC APPROACH)

STATICKY NEURČITÁ PŘÍHRADA (LINEÁRNÍ, NELINEÁRNÍ A STOACHASTISKÝ PŘÍSTUP)

Abstract

This contribution deals with simple planar and statically indeterminate pin-connected truss. This truss contains 3 members. The ways and methods of derivations and solutions according to theories of 1st and 2nd order are shown. There are applied linear and nonlinear approaches and their simplifications via Maclaurin's series. Programming connected with stochastic Simulation-Based Reliability Assessment Method (i.e. direct Monte Carlo Method) is used for determination of probabilistic reliability assessment (i.e. calculation the probability that plastic deformation occur in members of truss). Finally, the errors of all approaches are evaluated and compared.

Abstrakt

Tento článek se zabývá jednoduchou, rovinnou a staticky neurčitou příhradovou konstrukcí. Příhrada se skládá z 3 členů. Způsoby a metody odvození a řešení dle teorie prvního a druhého řadu jsou uvedeny. Jsou využity lineární a nelineární přístupy a jejich zjednodušení přes Maclaurinovy řady. Programování spojené se stochastickou metodou Simulation-Based Reliability Assessment (tj. přímá metoda Monte Carlo), je využito pro určení pravděpodobnostního posudku spolehlivosti (tj. vypočet pravděpodobnosti výskytu plastické deformace v příhradě). Nakonec byly porovnány a vyhodnoceny chyby všech přístupů.

Keywords

planar truss, theories of 1st and 2nd order, nonlinearities, force and thermal loading, elasticity, plasticity, Simulation-Based Reliability Assessment (SBRA) Method, probabilistic reliability assessment, error estimation

1 INTRODUCTION

Planar (i.e. 2D) truss structures appear to be the easiest ways of introducing, explaining and solving geometrical and material nonlinearities; see [1], [2] and [3]. In mechanics, for small deformations, tasks of this type (displacements, strains and stresses etc.) can be solved according to the simple 1st order (linear) theory or the more precise but more demanding 2nd order (nonlinear) theory. The application of 1st and 2nd order analysis depends upon the deformation of the structure and/or its components under loading. If the effects of deformations of the structure under loadings are negligible with respects to the equilibrium of external and internal forces, 1st order analysis can be applied. Else if the effects of deformations on equilibrium equations are non-negligible, the response (i.e. solution) should be determined using 2nd order analysis.

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The 2^{nd} order theory always leads to a nonlinear equation or nonlinear equations which can be solved via several numerical methods. However, there are some possibilities for simplifying it, for example via a Maclaurin series etc. It can then be solved easily and directly with small acceptable error.

Hence, if there are some suitable possibilities to obtain simple solutions of complicated problems, the stochastic approach (such as direct Monte Carlo Method, Simulation-Based Reliability Assessment (SBRA) Method, probabilistic assessment etc.) can also be easily applied. The SBRA Method is a fairly popular and modern trend in mechanics. Hence, a probabilistic reliability assessment can also be performed. For more information see [4], [5], [6] and [7].

This article presents a solution of a simple (2D) statically indeterminate pin-connected truss consisting of three members (i.e. derivation according to the 1^{st} and 2^{nd} order theories, possible simplifications, ways of solution, error estimation) together with their probabilistic inputs, outputs (histograms) and reliability assessment (i.e. calculating the probability that plastic deformation will occur in members of the truss). Finally, the errors of both approaches are evaluated.

2 SIMPLE PIN-CONNECTED TRUSS CONSISTING OF THREE MEMBERS (STATICALLY INDETERMINATE)

The simple pin-connected planar truss consisting of three members is loaded by vertical force F [N] and by temperature increasing $\Delta_t = t_1 - t_0 > 0$ [K] or [°C]; see Fig. 2.1. The material of the members is isotropic, linear and elastic. The truss is loaded in a force-controlled and temperature-controlled manner.

Initially, members "1" and "2" of the truss are in an ideal horizontal position with initial temperature t_0 [K] or [^{o}C], and the deformed shape is caused by added force F and temperature t_1 [K] or [^{o}C]; see Fig. 2.1.

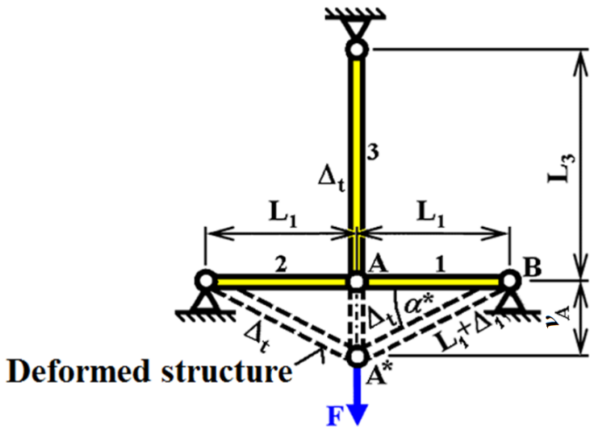


Fig. 2.1 Simple pin-connected truss (statically indeterminate) consisting of three members (loaded by force F and by uniform temperature increasing Δ_t)

Expressions are derived for angle α^* [rad], normal forces N_i [N], axial stresses σ_i [Pa], elongations Δ_i [m] in all members i = 1, 2 and 3 and vertical displacement v_A [m] according to the theory of small deformations for 1^{st} and 2^{nd} order analyses. The given inputs are force F, length of members L_1 and L_3 [m], modulus of elasticity $E_1 = E_2$ and E_3 [Pa] of the material of the members, area of the cross-sections $A_1 = A_2$ and A_3 [m^2] of the members, global temperature increasing Δ_t and coefficient of thermal expansion $\alpha_{t1} = \alpha_{t2}$, α_{t3} [K^{-1}] or [$^oC^{-1}$].

Hence, the angle α^* is unknown and is connected with the deformed structure. By applying the Method of Joints at point "A*" of the deformed structure (2nd order theory; see Fig.2.2b), the equations for normal forces can be derived as

$$\left.\begin{aligned} & N_1 \cos\alpha^* - N_2 \cos\alpha^* = 0 \\ & \Rightarrow \quad N_1 = N_2\,, \\ & N_1 \sin\alpha^* + N_2 \sin\alpha^* + N_3 - \mathrm{F} = 0 \\ & \Rightarrow \; 2N_1 \sin\alpha^* + N_3 - \mathrm{F} = 0\,. \end{aligned}\right\} \qquad (2.1)$$

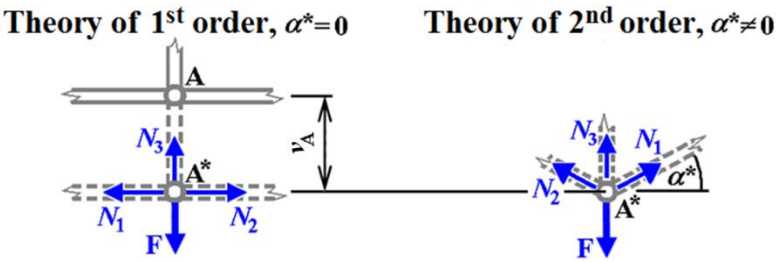


Fig. 2.2 Normal forces (theory of 1st and 2nd order)

Let is the value of angle α^* small. Hence $\alpha^* = 0$ (i.e. the angular changes are neglected, $\sin\alpha^* = 0$ and $\cos\alpha^* = 1$; see Fig.2.2a; is substituted in eq. (2.1) the simple formulas for the **theory of 1st order** can be derived. Hence, $N_3 = \mathrm{F}$ and the members "1" and "2" do not change their length (i.e. deformation boundary condition $\Delta_1 = \Delta_2 = \dfrac{N_1 \mathrm{L}_1}{E_1 A_1} + \alpha_{t1}\Delta_t \mathrm{L}_1 = 0 \quad \Rightarrow$ $N_1 = N_2 = -\alpha_{t1}\Delta_t E_1 A_1$).

The elongation of member "3" is equal to movement of point "A". Thus $v_\mathrm{A} = \Delta_3 = \dfrac{N_3 \mathrm{L}_3}{E_3 A_3} + \alpha_{t3}\Delta_t \mathrm{L}_3 = \left(\dfrac{N_3}{E_3 A_3} + \alpha_{t3}\Delta_t\right)\mathrm{L}_3$.

According to the **theory of 2nd order**, for the solution of this statically indeterminate structure, two deformation boundary condition should be added. These conditions follows from right-angled triangle $\mathrm{A}, \mathrm{A}^*, \mathrm{B}$; see Fig. 2.3.

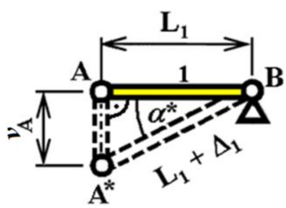


Fig. 2.3 Deformation boundary conditions (theory of 2nd order)

Hence $\cos\alpha^* = \dfrac{\mathrm{L}_1}{\mathrm{L}_1 + \Delta_1} = \dfrac{\mathrm{L}_1}{\mathrm{L}_1 + \dfrac{N_1 \mathrm{L}_1}{E_1 A_1} + \alpha_{t1}\Delta_t \mathrm{L}_1} \Rightarrow$

$$N_1 = N_2 = \frac{E_1 A_1 \left[1 - (1 + \alpha_{t1} \Delta_t) \cos \alpha^*\right]}{\cos \alpha^*} \tag{2.2}$$

and $$\tan \alpha^* = \frac{v_A}{L_1} = \frac{\Delta_3}{L_1} = \frac{\frac{N_3 L_3}{E_3 A_3} + \alpha_{t3} \Delta_t L_3}{L_1} \Rightarrow$$

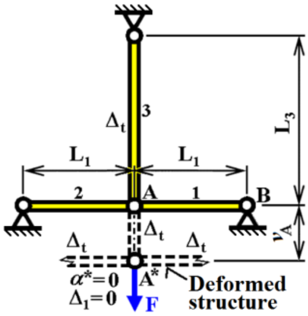
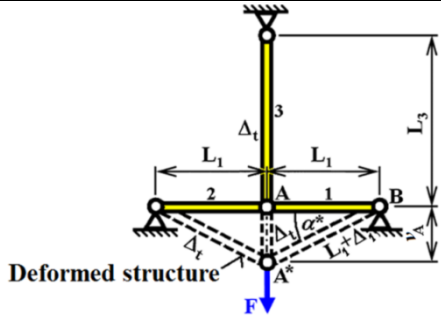
$$N_3 = E_3 A_3 \left(\frac{L_1}{L_3} \tan \alpha^* - \alpha_{t3} \Delta_t \right). \tag{2.3}$$

Equations (2.2) and (2.3) can be substituted into (2.1). Hence, after simplification, the following nonlinear dependence can be derived; see eq. (2.4).

$$\left(2E_1 A_1 \left[1 - (1 + \alpha_{t1} \Delta_t) \cos \alpha^*\right] + \frac{E_3 A_3 L_1}{L_3} \right) \tan \alpha^* - F - \alpha_{t3} \Delta_t E_3 A_3 = 0 \tag{2.4}$$

Finally, the solution according to the 1st order theory and 2nd order theory is given in the Tab. 2.1.

Tab. 2.1 Results of the theory of small deformations (1st and 2nd order theory)

1st order theory	**2nd order theory**
	
Linear solution $\alpha^* = 0$, $N_3 = F$, $N_1 = N_2 = -\alpha_{t1} \Delta_t E_1 A_1$, $\sigma_1 = \sigma_2 = \frac{N_1}{A_1}$, $\sigma_3 = \frac{N_3}{A_3}$, $\Delta_1 = \Delta_2 = 0$.	**Nonlinear solution** $\left. \begin{aligned} &\left(2E_1 A_1 \left[1 - (1 + \alpha_{t1} \Delta_t) \cos \alpha^*\right] + \frac{E_3 A_3 L_1}{L_3} \right) \tan \alpha^* + \\ &- F - \alpha_{t3} \Delta_t E_3 A_3 = 0 \end{aligned} \right\}$, $N_1 = N_2 = \frac{E_1 A_1 \left[1 - (1 + \alpha_{t1} \Delta_t) \cos \alpha^*\right]}{\cos \alpha^*}$, $N_3 = E_3 A_3 \left(\frac{L_1}{L_3} \tan \alpha^* - \alpha_{t3} \Delta_t \right)$, $\Delta_1 = \Delta_2 = \frac{N_1 L_1}{E_1 A_1} + \alpha_{t1} \Delta_t L_1$
$v_A = \Delta_3 = \left(\frac{N_3}{E_3 A_3} + \alpha_{t3} \Delta_t \right) L_3$, $\sigma_1 = \sigma_2 = \frac{N_1}{A_1}$, $\sigma_3 = \frac{N_3}{A_3}$.	

Note that the same results as written in Tab. 2.1 can be derived in many other ways. One of these ways is based on the minimum of total potential energy Π [J] of the truss (i.e. on equation $\frac{\partial \Pi}{\partial \alpha^*} = 0$).

Another example (i.e. statically determinate truss) is presented in reference [3].

3 SIMPLE NUMERICAL SOLUTION

For the theory of 2nd order, a reasonably good initial estimate of angle α^* (i.e. α_0^*) can be derived by simplification via a Maclaurin series where $tan\,\alpha^* \approx \alpha_0^*$ and $cos\,\alpha^* \approx 1 - \frac{\alpha_0^{*2}}{2}$. Hence, trigonometric eq. (2.4) can be simplified into polynomial equation

$$\alpha_0^{*3} + \frac{E_3 A_3 L_1 - 2E_1 A_1 \alpha_{t1} \Delta_t L_3}{E_1 A_1 L_3 (1 + \alpha_{t1}\Delta_t)} \alpha_0^* - \frac{F + \alpha_{t3}\Delta_t E_3 A_3}{E_1 A_1 (1 + \alpha_{t1}\Delta_t)} = 0 \,. \tag{3.1}$$

Cubic eq. (3.1) can be solved via Cardano's formula; see [8]. Because the discriminant

$$\overline{D} = \left(\frac{F + \alpha_{t3}\Delta_t E_3 A_3}{2E_1 A_1 (1 + \alpha_{t1}\Delta_t)} \right)^2 + \left(\frac{E_3 A_3 L_1 - 2E_1 A_1 \alpha_{t1} \Delta_t L_3}{3E_1 A_1 L_3 (1 + \alpha_{t1}\Delta_t)} \right)^3 > 0 \,, \tag{3.2}$$

the eq. (3.1) has only one real root

$$\alpha_0^* = \sqrt[3]{\frac{F + \alpha_{t3}\Delta_t E_3 A_3}{2E_1 A_1 (1 + \alpha_{t1}\Delta_t)} + \sqrt{\overline{D}}} + \sqrt[3]{\frac{F + \alpha_{t3}\Delta_t E_3 A_3}{2E_1 A_1 (1 + \alpha_{t1}\Delta_t)} - \sqrt{\overline{D}}} \,. \tag{3.3}$$

From eq. (2.4) (by isolating trigonometric function $tan\,\alpha^*$) follows $tan\,\alpha^* = \frac{F + \alpha_{t3}\Delta_t E_3 A_3}{2E_1 A_1 \left[1 - (1 + \alpha_{t1}\Delta_t) cos\,\alpha^*\right] + \frac{E_3 A_3 L_1}{L_3}}$ and then can be derived

$$\alpha^* = atan\left(\frac{F + \alpha_{t3}\Delta_t E_3 A_3}{2E_1 A_1 \left[1 - (1 + \alpha_{t1}\Delta_t) cos\,\alpha^*\right] + \frac{E_3 A_3 L_1}{L_3}}\right) .$$

Thus, the iterative scheme with recursive relation (i.e. the application of the Fixed Point Iteration Method) can be derived as

$$\left.\begin{array}{l} \alpha_{j+1}^* = atan\left(\dfrac{F + \alpha_{t3}\Delta_t E_3 A_3}{2E_1 A_1 \left[1 - (1 + \alpha_{t1}\Delta_t) cos\,\alpha_j^*\right] + \dfrac{E_3 A_3 L_1}{L_3}}\right), \\ \text{for } j = 0, 1, 2, \ldots \,. \end{array}\right\} \tag{3.4}$$

Hence, with small and acceptable error (for small deformations, according to the 2nd order theory), a good solution can be written as $\alpha^* \cong \alpha_1^*$, i.e.

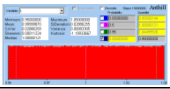
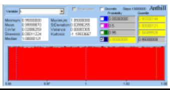
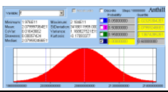
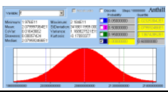
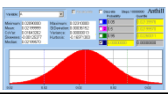
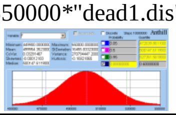
$$\alpha^{*} \cong atan\left(\frac{F + \alpha_{t3}\Delta_t E_3 A_3}{2E_1 A_1\left[1-\left(1+\alpha_{t1}\Delta_t\right)\cos\,\alpha_0^{*}\right]+\frac{E_3 A_3 L_1}{L_3}}\right). \tag{3.5}$$

Correctness of the derived results (i.e. their error) can be checked via Pythagoras' theorem too, i.e. $\left(L_1+\Delta_1\right)^2 = v_A^2 + L_1^2$; see Fig.2.3.

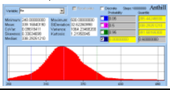
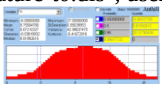
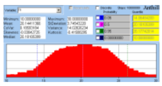
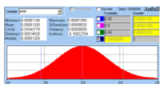
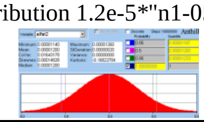
4 PROBABILISTIC INPUTS

For a solution using a stochastic approach, calculating the probability that plastic deformation will occur and performing a probabilistic reliability assessment, the probabilistic inputs must be defined; see Tab. 4.1 and 4.2. Anthill software (i.e. the SBRA Method, direct Monte Carlo approach) was applied in this stochastic modelling; see references [4], [5], [6], [7], [9], [10], [11] and [12].

Tab. 4.1 Stochastic inputs and their basic characteristics (simple pin-connected truss, statically indeterminate, Anthill 2.6 software)

Random inputs	Description	Histogram applied in Anthill software	Min.	Max.	Median	Mean
L_1 [m]	Length of members "1" and "2"	"Uniform" distribution 	0.95	1.05	1	1
L_3 [m]	Length of member "3"	"Uniform" distribution 	2.90	3.1	3	3
E_1 [Pa]	Modulus of elasticity of members "1" and "2"	Modified (truncated) normal distribution 2.08×10^{11}*"n1-05.dis" 	1.976×10^{11}	2.184×10^{11}	2.080×10^{11}	2.080×10^{11}
E_3 [Pa]	Modulus of elasticity of member "3"	Modified (truncated) normal distribution 2.08×10^{11}*"n1-05.dis" 	1.976×10^{11}	2.184×10^{11}	2.080×10^{11}	2.080×10^{11}
A_1 [m^2]	Area of cross-section of members "1" and "2"	Modified (truncated) normal distribution 0.022*"n1-05.dis" 	0.0209	0.0231	0.0220	0.0220
A_3 [m^2]	Area of cross-section of member "3"	Modified (truncated) normal distribution 2×10^{-3}*"n1-04.dis" 	0.002016	0.002184	0.0021	0.0021
F [N]	External vertical force acting in joint "A*"	Modified (truncated) dead distribution 550000*"dead1.dis" 	449900	550000	500147	499950

Tab. 4.2 Stochastic inputs and their basic characteristics (simple pin-connected truss, statically indeterminate, Anthill 2.6 software)

Random inputs	Description	Histogram applied in Anthill software	Min.	Max.	Median	Mean
R_p [MPa]	Yield limit for material of members "1", "2" and "3"	Measurement for A36-M steel (truncated user defined distribution) "a36-m-cont.dis" 	248	500	338.29	339.15
t_0 [°C]	Initial temperature of members "1", "2" and "3"	"temperature-t0.dis", user defined 	-8	27	9.820	9.755
t_1 [°C]	Initial temperature of members "1", "2" and "3"	"temperature-t1.dis", user defined 	10	30	20.181	20.144
α_{t1} [°C^{-1}]	Coefficient of thermal expansion of members "1" and "2"	Modified (truncated) normal distribution 1.2e-5*"n1-05.dis" 	1.14×10^{-5}	1.26×10^{-5}	1.2×10^{-5}	1.2×10^{-5}
α_{t2} [°C^{-1}]	Coefficient of thermal expansion of member "3"	Modified (truncated) normal distribution 1.2e-5*"n1-05.dis" 	1.14×10^{-5}	1.26×10^{-5}	1.2×10^{-5}	1.2×10^{-5}
$P_{ALLOWABLE} = 4\times10^{-4}$ = 0.04 % is the allowable working probability that plasticity will occur in members "1", "2" or "3"						

Thirteen chosen probabilistic inputs (i.e. mutually independent variables) of random type, and their notation via histograms, are shown in Tab. 4.1 and 4.2. These random variables cover real variabilities and fluctuations in technical practice for the truss presented here.

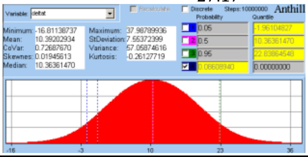
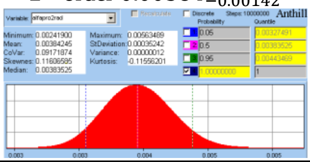
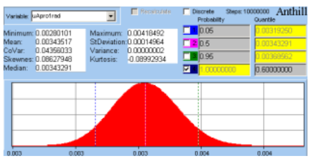
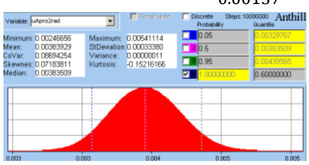
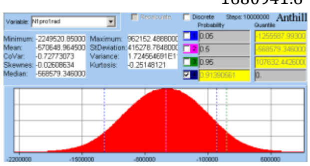
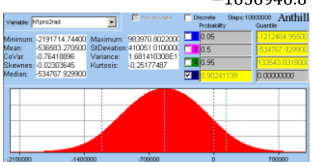
Tab. 4.1 and 4.2 presents all basic statistical information (i.e. minimum, maximum, median and mean values) and histograms. In Anthill software, the histogram "Uniform" means truncated uniform distribution, "n1-04.dis" means truncated normal distribution ±4%, "n1-05.dis" means truncated normal distribution ±5%, "dead1.dis" means dead load truncated distribution $^{+0\%}_{-18.9\%}$, "a36-m-cont.dis" means asymmetric yield stress truncated distribution for carbon steel A36, "temperature-t0.dis" means truncated and asymmetric user distribution $^{+174.95\%}_{-181.47\%}$ and "temperature-t1.dis" means truncated and asymmetric user distribution $^{+48.65\%}_{-50.45\%}$; see [5], [6] and [9].

Thus, the given stochastic inputs are used to calculate the stochastic outputs Δ_t, $\overset{*}{\alpha}$, v_A, $N_{1,2,3}$, $\sigma_{1,2,3}$, and $\Delta_{1,2,3}$ via histograms and distributed functions, as presented in Tab. 4.1 and 4.2. All calculations are performed and evaluated for $N_{TOTAL} = 10^7$ Monte Carlo random simulations.

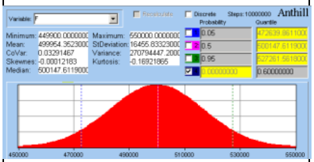
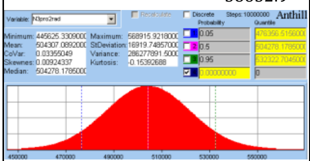
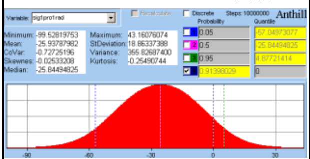
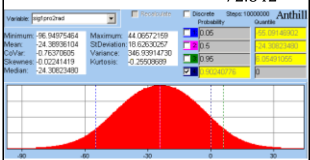
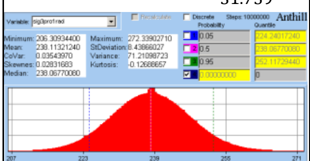
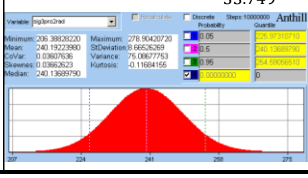
5 PROBABILISTIC OUTPUTS

The stochastic (probabilistic) results (i.e. stochastic outputs), see Tab. 5.1 and 5.2, can be used for the probabilistic reliability assessment of the solved truss (Anthill software, SBRA Method; see [5], [6] and [9]).

Tab. 5.1 Stochastic outputs and their basic characteristics (simple pin-connected truss, statically indeterminate, Anthill 2.6 software, result of 10^7 Monte Carlo random simulations)

Stochastic outputs	Description		Min.	Max.	Median	Mean
Δ_t [°C]	Uniform temperature increasing 1st and 2nd order $10.36^{+27.63}_{-27.17}$ 		-16.81	37.99	10.36	10.39
α^* [rad]	Angle in deformed structure	1st "0"	0	0	0	0
		2nd order $0.00384^{+0.00179}_{-0.00142}$ 	0.00242	0.00563	0.00384	0.00384
v_A [m]	Displacement of point "A"	1st order $0.00343^{+0.00075}_{-0.00063}$ 	2.801	4.185	3.433	3.435
		2nd order $0.00384^{+0.00157}_{-0.00137}$ 	2.467	5.411	3.835	3.839
$N_{1,2}$ [N]	Normal forces in members "1" and "2"	1st order $-568579.3^{+1530731.8}_{-1680941.6}$ 	-2249521	962152	-568579	-570649
		2nd order $-534767.9^{+1518738.7}_{-1656946.8}$ 	-2191715	983971	-534768	-536583

Tab. 5.2 Stochastic outputs and their basic characteristics (simple pin-connected truss, statically indeterminate, Anthill 2.6 software, result of 10^7 Monte Carlo random simulations)

Stochastic outputs	**Description**		**Min.**	**Max.**	**Median**	**Mean**
N_3 [N]	Normal force in member "3"	1st order $5\times10^5 \pm 50000$ 	449900	550000	500147	499950
		2nd order $504278.2^{+64637.7}_{-58652.9}$ 	445625	568916	504278	504307
$\sigma_{1,2}$ [MPa]	Stresses in members "1" and "2"	1st order $-25.845^{+69.006}_{-73.683}$ 	-99.53	43.16	-25.85	-25.94
		2nd order $-24.308^{+68.374}_{-72.642}$ 	-96.95	44.07	-24.31	-24.39
σ_3 [MPa]	Stress in member "3"	1st order $238.068^{+34.271}_{-31.759}$ 	206.31	272.34	238.07	238.11
		2nd order $240.137^{+38.767}_{-33.749}$ 	206.39	278.90	240.14	240.19

Negative values of $N_1 = N_2$ and $\sigma_1 = \sigma_2$ mean compression state of loading. However, in this case, there can be tensile or compression states in members "1" and "2"; see Tab. 5.1 and 5.2.

In this case, the reliability functions $R_{F\,i}$ [MPa] can be defined as

$$R_{F1} = R_{\mathrm{P}} - |\sigma_1|\ , \quad R_{F3} = R_{\mathrm{P}} - \sigma_2\,. \tag{5.1}$$

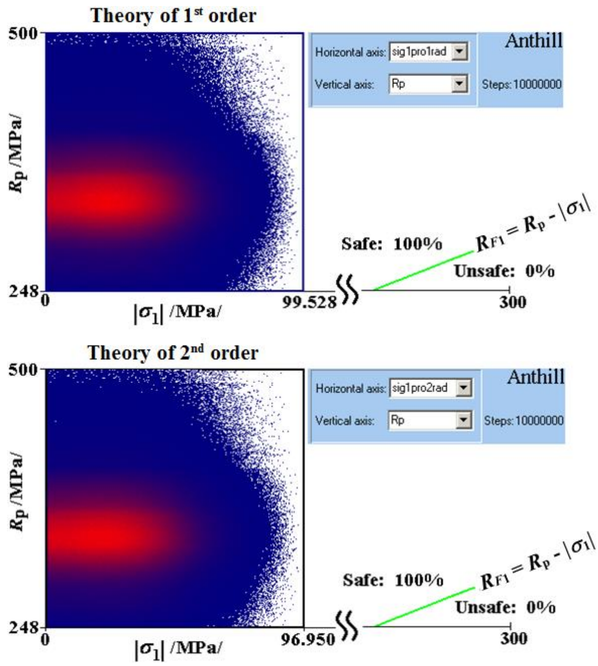


Fig. 5.1 Probabilistic reliability assessment for members "1" and "2" (SBRA Method, simple pin-connected truss, statically indeterminate, Anthill 2.6 software, result of 10^7 Monte Carlo random simulations)

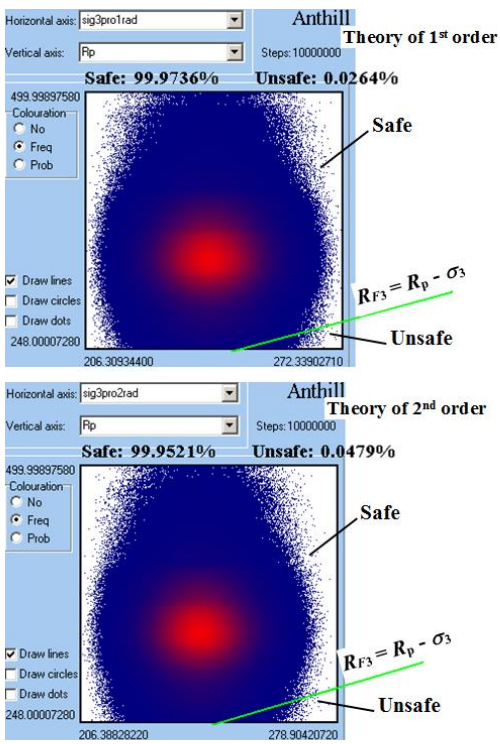


Fig. 5.2 Probabilistic reliability assessment for member "3" (SBRA Method, simple pin-connected truss, statically indeterminate, Anthill 2.6 software, result of 10^7 Monte Carlo random simulations)

The reliability functions (i.e. 2D histograms $|\sigma_1|$ vs. R_p and σ_3 vs. R_p) are presented in Fig. 5.1 and 5.2. Hence, it is evident that if $R_{F\,i} > 0$ (i.e. yield limit R_p is greater than positive value of normal stress σ_i), the stress is below the yield limit (safe loading, no plasticity occurs). Otherwise, if $R_{F\,i} \leq 0$, then plasticity occurs (i.e. an unsafe and undesirable situation); see Fig. 5.3.

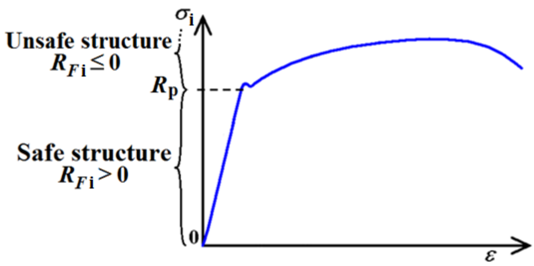


Fig. 5.3 Stress-strain diagram of material – definition of safe and unsafe structure

6 PROBABILISTIC RELIABILITY ASSESSMENT

The probability P_f of an unsafe situation (i.e. a situation when $R_{F\,i} \leq 0$) is calculated in Anthill software by the expression

$$P_f = \max(P_{f\,i}),\ \text{where}\ P_{f\,i} = P_{\left(R_{F\,i} \leq 0\right)} = \frac{N_{f\,i}}{N_{TOTAL}} \tag{6.1}$$

and where $N_{f\,i}$ is the number of unfavorable states (i.e. states when $R_{Fi} \leq 0$) and, in our case, $N_{TOTAL} = 10^7$ Monte Carlo random simulations.

In the case of 1st order theory, from the presented results it is calculated that $P_f = P_{f\,2} = P_{f\,3} = 2.6444\times10^{-4}$ (i.e. approx. 0.0264% of all possible random simulations cause plastic deformations).

In the case of 2nd order theory, from the presented results it is calculated that $P_f = P_{f\,2} = P_{f\,3} = 4.7938\times10^{-4}$ (i.e. approx. 0.0479% of all possible random simulations cause plastic deformations).

Finally, the probabilistic reliability assessment can be performed by checking the inequation

$$P_f \leq P_{ALLOWABLE}. \tag{6.2}$$

In the case of 1st order theory, the inequation (6.2) is fulfilled (i.e. $2.6444\times10^{-4} < 4\times10^{-4}$); the solved truss therefore satisfies the probabilistic reliability condition.

However, in the case of 2nd order theory, the inequation (6.2) is not fulfilled (i.e. $4.7938\times10^{-4} > 4\times10^{-4}$); the solved truss therefore does not satisfy the probabilistic reliability condition.

There is possible to calculate percentage error of calculations $\Delta_{\%}$ [%], for example, by comparing median values of the results from the theory of 2nd and 1st order (Tab. 5.1 and 5.2). Thus

$$\Delta_{\%} = 100\,\frac{value_{\text{2nd order}} - value_{\text{1st order}}}{value_{\text{2nd order}}}\,. \tag{6.3}$$

From calculated results is obvious the legitimacy application of the theory of 2nd order which gives results close to the reality. **Some errors of the theory of 1st order are alarming**; see Tab. 6.1.

Tab. 6.1 Errors of the theory of 1st order in comparing with the theory of 2nd order (simple pin-connected truss, statically indeterminate)

Description	$\Delta_{\%}$ [%]
Error of median values for angle α^* according to the theory of 1st order	**100**
Error of median values for displacement v_A according to the theory of 1st order	**10.48**
Error of median values for normal forces $N_1 = N_2$ according to the theory of 1st order	**-6.32**
Error of median values for normal force N_3 according to the theory of 1st order	**0.82**
Error of median values for stresses $\sigma_1 = \sigma_2$ according to the theory of 1st order	**-6.32**
Error of median values for stress σ_3 according to the theory of 1st order	**0.86**
Error of median values for probability of unsafe state P_{f1} according to the theory of 1st order	**0**
Error of median values for probability of unsafe state $P_f = P_{f2} = P_{f3}$ according to the theory of 1st order	**44.84**

3 CONCLUSIONS

It is a fact that, the planar truss structures appear to be the easiest ways of introducing, explaining and solving geometrical and material nonlinearities (in this case, a simple pin-connected and statically indeterminate truss of three members). The focus is on the understanding, step-by-step derivation, applications, possible simplifications, programming and solution of nonlinear problems which are widely applied mostly by civil and mechanical engineers. The solutions according to the 2nd order theory always lead to a set of nonlinear equations. However, there are possibilities to solve such a task directly via iterative approaches, or to linearize and simplify it (via a Maclaurin series in this case) and then to solve it easily with only small errors. Simplifying a relatively complicated nonlinear set of equations usually enables a relatively easy application. The stochastic approach (direct Monte Carlo Method, Simulation-Based Reliability Assessment (SBRA) Method, probabilistic reliability assessment) is a modern, quite popular trend in mechanics. Hence, the SBRA Method (i.e. stochastic inputs and outputs) was applied in order to determine the probability that plastic deformations will occur in the structure. Finally, a probabilistic reliability assessment was performed by checking the inequation $P_f \le P_{ALLOWABLE}$ (i.e. the probability that plastic deformation will occur).

In the case of 1st order theory (i.e. linear solution), the solved truss satisfies the probabilistic reliability condition. However, in the case of 2nd order theory (i.e. nonlinear solution), the solved truss does not satisfy the probabilistic reliability condition). Hence, from calculated results is obvious the legitimacy application of the theory of 2nd order which gives results close to the reality. Some mentioned errors of the theory of 1st order are alarming.

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