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TEMPERATURE IDENTIFICATION IN THE STRUCTURAL ELEMENTS OF NON-CONTACTING FACE SEALS BY USING TREFFTZ FUNCTIONS

IDENTIFIKACE TEPLOTY V KONSTRUKČNÍCH PRVCÍCH BEZKONTAKTNÍCH TĚSNĚNÍ POMOCÍ TREFFTZ FUNKCÍ

Abstract

Phenomena of the heat transfer in non-contacting face seals was described by partial differential equation of the second order and boundary conditions. In this way, the mathematical model was developed for the sealing rings. The distributions of temperature in the structural elements was obtained by the Trefftz method. It is a simple method of solving direct and inverse problems described by a homogeneous or an inhomogeneous partial differential equation. The main idea of the method is to determine functions satisfying a given differential equation (Trefftz functions) and to fit the linear combination of them to the governing boundary conditions.

Abstrakt

Jev přenosu tepla v bezkontaktních mechanických těsněních byl popsán parciální diferenciální rovnicí druhého řádu s okrajovými podmínkami. Tímto způsobem, byl vyvinut pro těsnicí kroužky matematický model. Distribuce teploty v konstrukčních prvcích byla získána metodou Trefftz. Jedná se o jednoduchý způsob řešení přímých a inverzních problémů popsaných homogenní nebo nehomogenní parciální diferenciální rovnicí. Hlavní myšlenkou této metody je určit funkce, které by splňoval danou diferenciální rovnici (funkce Trefftz) tak, aby jejich lineární kombinace vyhověli okrajovým podmínkám.

Keywords

Heat transfer, non-contacting face seals, Trefftz functions, inverse problem, the Trefftz method

1 INTRODUCTION

In the paper an approximate solution of the inverse problem in the heat transfer is presented. The head transfer in the stator was described by partial differential equation of the second order. The distributions of temperature in the stator was obtained by the Trefftz method. In this method, the linear combination of the Trefftz functions is used. The concept of using the functions is that they satisfy a given differential equation. Additional information about the Trefftz functions can be found in [1],[8],[9],[15]. Many examples of using the Trefftz functions to solve invers problems were presented in [2], [3], [11], [5]. It is worth to notice that this method can be used for solving various partial differential equations. It was applied for the wave equation and thermoelasticity problems in [12], [10], [6], for the equation of a beam and plate vibration in [13], [14] and for the nonlinear heat condition problem in [7].

2 PROBLEM DESCRIPTION

Mechanical seals are crucial elements of most turbomachines, ensuring their high reliability. They are common in pumps, mixers and other industrial equipment. Designed primarily to reduce power loss, they are also responsible for preventing leakage of the fluid to the environment. This can be achieved by maintaining a proper fluid-filled micro-clearance between the mating sealing rings. Seals of this type are called non-contacting face seals.

The diagram of the non-contacting face seal used in the analysis is shown on Figure 1. The seal consists of two rings, one of which is the stator (3) and the other is the flexibly mounted rotor (4), rotating with the shaft

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(1) of a turbomachine. The main design assumption was to maintain a slight clearance gap between the mating rings during the operation of the seal, which is filled with a medium.

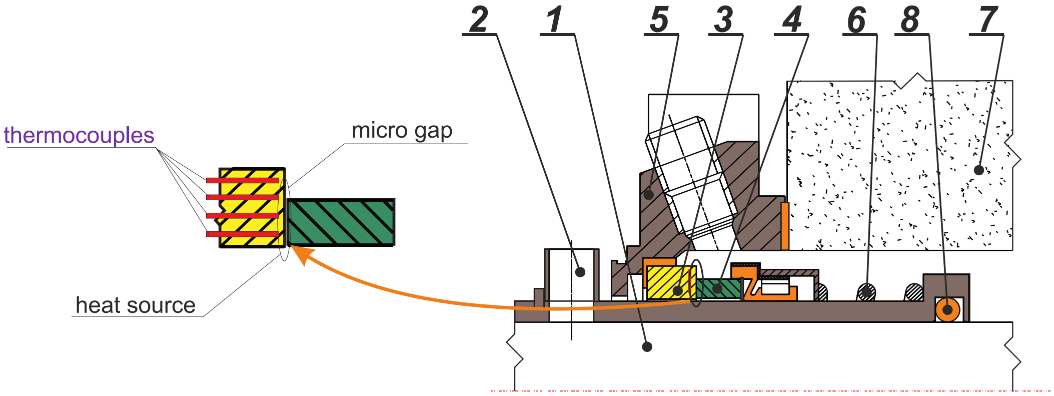

Fig. 1 Diagram of the non-contacting face seal: 1 – shaft, 2 – steady pin, 3 – stator, 4 – rotor, 5 – housing of mechanical seal, 6 – spring, 7 – device housing, 8 - 0-ring.

3 MATHEMATICAL MODEL

Generally, a model of heat transfer for a non-contacting face seal is a complex system of cross-correlated differential equations taking into consideration a number of boundary conditions necessary for the correct solution.

The heat transfer in the stator can be describe by the Laplace equation in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}+\frac{\partial^2 T}{\partial z^2}=0 \tag{1}$$

The Laplace equation has been completed by boundary conditions:

$$\frac{\partial T\left(r,-0.01\right)}{\partial z}=0, \tag{2}$$

$$\frac{\partial T\left(0.04,z\right)}{\partial r}=0, \tag{3}$$

$$-\lambda\frac{\partial T\left(0.045,z\right)}{\partial r}=\alpha\left(T\left(0.045,z\right)-T_0\right), \tag{4}$$

where:

T_0 – the temperature of the surrounding fluid $[{}^\circ C]$,

λ – the thermal conductivity $\left[\frac{\mathrm{W}}{mK}\right]$,

α – the convection coefficient $\left[\frac{\mathrm{W}}{m^2 K}\right]$,

Additionally, we assume that the temperature for $z=-0.002$ is known. The exact solution to this problem is given by the following formula:

$$T=T_0+\frac{1}{2}\frac{\mu\omega^2 r^2}{\lambda^f}+\sum_{n=1}^{\infty}B_n\cosh\left(s_n z\right)\left[J_0\left(s_n r\right)-\frac{J_1\left(s_n r_i\right)}{Y_1\left(s_n^r r_i\right)}Y_0\left(s_n r\right)\right] \tag{5}$$

4 THE TREFFTZ METHOD

The Trefftz method is an approximate method of solving direct and inverse problems which are described by a partial differential equation. The unknown solution T of Laplace equation with suitable boundary conditions was approximated by the linear combination of the Trefftz functions

$$T(r,z) \approx \bar{T}(r,z) = \sum_{n=1}^{N} c_n V_n \tag{6}$$

where:

c_n – unknown coefficient of the linear combination,

V_n – the Trefftz functions satisfying the Laplace equation.

The Trefftz functions for the Laplace equation in cylindrical coordinates were presented in [4]. Before numerical calculations, the Trefftz functions have to be generated with the recursive formula:

$$V_0(r,z) = 1 \tag{7}$$

$$V_1(r,z) = z \cdot V_0(r,z) = z \tag{8}$$

$$V_{(k+1)}(r,z) = \frac{((2k+1)\cdot z \cdot V_k(r,z) - (r^2+z^2)\cdot V_{(k-1)}(r,z))}{(k+1)^2} \quad \text{for } k = 1,2,\ldots \tag{9}$$

Tab. 1 The Tefftz functions from degree 0 to 5.

Degree	The Trefftz function
0	1
1	z
2	$\frac{1}{2}z^2 - \frac{1}{4}r^2$
3	$\frac{1}{6}z^3 - \frac{1}{4}zr^2$
4	$\frac{1}{24}z^4 - \frac{1}{8}z^2r^2 + \frac{1}{64}r^4$
5	$\frac{1}{120}z^5 - \frac{1}{24}z^3r^2 + \frac{1}{64}zr^4$

The coefficients c_n of the combination are determined in such a way to minimize the functional describing the fitting of an approximation to known boundary conditions. The functional for this problem has the following form:

$$I = \int_{-0.01}^{0} \left(\frac{\partial \overline{T}(0.04,z)}{\partial r} \right)^2 dz + \int_{-0.01}^{0} \left(\lambda \left(\frac{\partial \overline{T}(0.045,z)}{\partial r} \right) + \alpha \left(\overline{T}(0.045,z) - T_0 \right) \right)^2 dz + \\ + \int_{0.04}^{0.045} \left(\frac{\partial \overline{T}(r,-0.01)}{\partial z} \right)^2 dr + \int_{0.04}^{0.045} \left(\overline{T}(r,-0.002) - W \right)^2 dr \tag{10}$$

where:

T_0 – the temperature of the surrounding fluid $[{}^{\circ}C]$,

λ – the thermal conductivity $\left[\frac{\mathrm{W}}{m\,K}\right]$,

α – the convection coefficient $\left[\frac{\mathrm{W}}{m^2 K}\right]$,

W – the interpolation polynomial for a given set of data points (thermocouples),

The necessary condition to minimize functional has a form:

$$\frac{\partial I}{\partial c_1} = \frac{\partial I}{\partial c_2} = \ldots = \frac{\partial I}{\partial c_N} = 0 \tag{11}$$

Hence the linear system of equations nice to be solved.

5 RESULTS

Figure 2 presents the exact solution and its approximation. It depicts the approximation for the 40, 42, 51 and 62 the Trefftz functions.

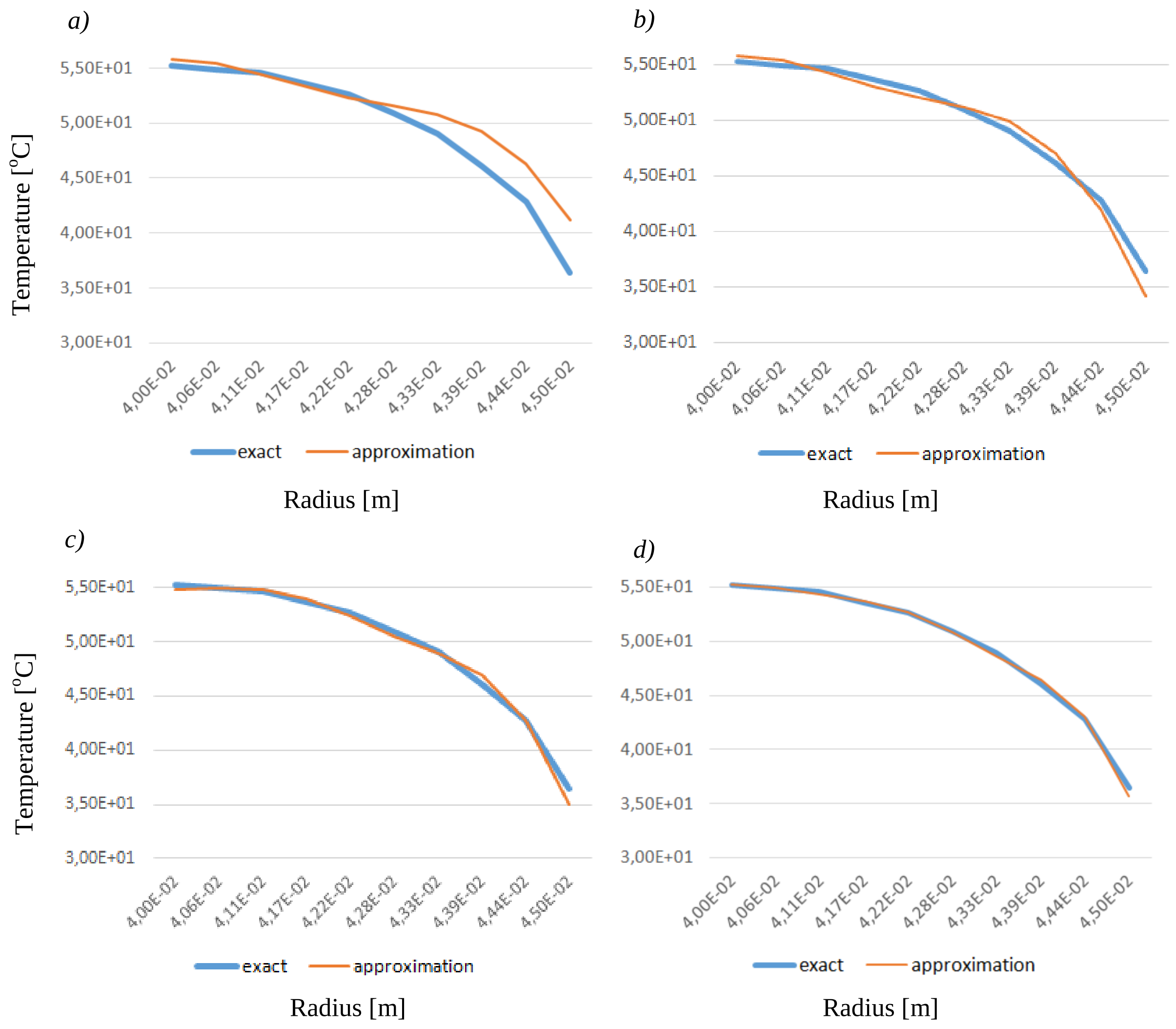

Fig. 2 The temperature distribution on the surface of the stator for A – 40, B – 44, C – 51 and D – 62 the Trefftz functions

On the basis of the presented results, it can be observed that the greater the number of Trefftz functions the better the approximation. The obtained approximations are satisfactory. This results show a remarkable efficiency of the Trefftz method for solving these types of problems.

6 CONCLUSIONS

The presented method is suitable for solving boundary inverse problems. The approximation of the exact solution is highly satisfactory. We can observe that when the number of the Trefftz functions was increased the results were improved. Moreover, the main advantage of the Trefftz method is its mathematical simplicity. The approximate solution is a linear combination of the functions satisfying identically the Laplace equation. Then the coefficients of the linear combination are determined by solving a linear system of equations.

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