

**Dan PILBAUER\*, Jaroslav BUŠEK\*, Vladimír KUČERA\*, Tomáš VYHLÍDAL\*\***

LABORATORY SET-UP DESIGN FOR TESTING VIBRATION SUPPRESSION ALGORITHMS WITH TIME DELAYS

NÁVRH LABORATORNÍ SOUSTAVY PRO TESTOVÁNÍ ALGORITMŮ TLUMENÍ VIBRACÍ S DOPRAVNÍM ZPOŽDĚNÍM

**Abstract**

Vibrations occur in wide field of application in mechanical engineering hence developing new and advanced algorithms is useful. For practical verification of theoretical control algorithms it is necessary not only to simulate them, but also to test them in laboratory conditions at least. This article describes design of multipurpose laboratory set-up for testing active vibration control algorithms which consists of series of carts connected to each other on linear guides. Carts are equipped with accelerometers and position sensors for feedback control. Linear electric motors on the carts are used for suppressing vibration. The laboratory set-up is controlled by Matlab/Simulink via DAQ card connected to PC. Next to the set-up itself, two control algorithms intended for testing are presented in this article. First, principle of algorithm of resonator absorber with delayed acceleration feedback is described. Furthermore algorithm using input shaping for pre-compensating the oscillatory modes of the system is presented. By adding time delays into closed loop it becomes an infinite dimensional and in this case different approach than in conventional methods is needed. Finally some successful simulation results are presented as a part of practical experiment preparation.

**Abstrakt**

Vibrace, jako nežádoucí efekt, se vyskytují v celé oblasti strojírenství. Z tohoto důvodu je vývoj nových algoritmů k jejich potlačování velmi přínosný. Pro praktické ověření teoretických řídicích algoritmů je nezbytné nejen simulovat matematické modely, ale také podpořit teorii alespoň v laboratorních podmínkách. Tento článek popisuje návrh univerzální laboratorní soustavy pro testování algoritmů pro aktivní tlumení vibrací. Soustava je tvořena několika vozíky vzájemně propojenými na lineárním vedení. Vozíky jsou osazeny akcelerometry a snímači polohy jednotlivých vozíků. K potlačení vibrací jsou pak využity lineární elektromotory. Laboratorní soustava je řízena pomocí softwaru Matlab/Simulink a DAQ karty připojené k PC. Kromě samotné soustavy jsou v článku popsány dva algoritmy, které budou na soustavě testovány. Nejprve je popsán princip absorberu vibrací s akcelerační zpožděnou zpětnou vazbou. Dále pak je popsán algoritmus pro tvarování vstupního signálu tak, aby kompenzoval kmitavé módy v systému. Přidáním dopravního zpoždění do soustavy vznikne soustava nekonečného řádu a z tohoto důvodu je nutné systémy takového charakteru zkoumat jinými než běžnými metodami. Na závěr jsou zobrazeny výsledky ze simulací, sloužící k přípravě praktických experimentů.

**Keywords**

Vibration control, delayed resonator, signal shapers, experimental design

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\* Ing, Department of Instrumentation and Control Engineering, Faculty of Mechanical Engineering, Czech Technical University in Prague, Technická 4, Praha, tel. (+420) 224 352 563, e-mail dan.pilbauer@fs.cvut.com

\*\* Prof, Department of Instrumentation and Control Engineering, Faculty of Mechanical Engineering, Czech Technical University in Prague, Technická 4, Praha, tel. (+420) 224 352 563, e-mail tomas.vyhliidal@fs.cvut.cz

# 1 INTRODUCTION

As the main contribution, we present a laboratory set-up that has been designed to test various algorithms for the active vibration suppressions. Particularly, we focus on two types of algorithms that utilize time delays in the algorithm structure. The first algorithm is the delayed resonator and the second is the input shaper. Next, we consider a communication time delay in the control loop.

## 1.1 Delayed resonator

The concept of delayed resonator proposed by N. Olgac and co-workers, see [1] and references therein, is depicted in Fig. 1. It consists of an absorber

$$m_a \ddot{x}_a(t) + c_a \dot{x}_a(t) + k_a x_a(t) = u(t) \quad (1)$$

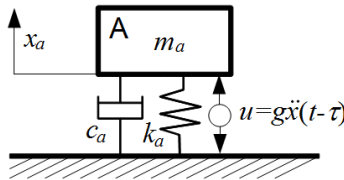
where  $m_a, c_a, k_a$  are the mass, the damping and the stiffness parameters of the passive absorber and  $x_a(t)$  is its displacement and of an active, delayed acceleration feedback force

$$u(t) = g \ddot{x}_a(t - \tau) \quad (2)$$

where  $g$  is the feedback gain and  $\tau > 0$  is the delay. The characteristic equation of this linear time-invariant (LTI) system (1) and (2) is a quasipolynomial equation

$$R_a(s) = m_a s^2 + c_a s + k_a - g s^2 e^{-\tau s} = 0 \quad (3)$$

which has infinitely many roots.



**Fig. 1** Delayed resonator absorber

Following the approach in **Chyba! Nenalezen zdroj odkazů.**, the ideal absorber could be achieved if it behaved as a resonator, that is, with dominant characteristic roots placed at  $s_{1,2} = \pm j\omega$ . This condition yields the feedback formation

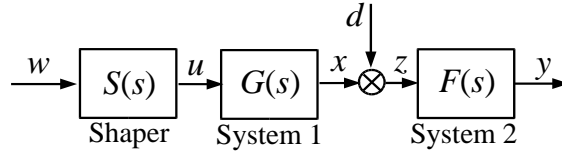
$$g = \frac{1}{\omega^2} \sqrt{(c_a \omega)^2 + (m_a \omega^2 - k_a)^2} \quad (4)$$

$$\tau = \frac{1}{\omega} \left( \text{atan} \left( \frac{c_a \omega}{m_a \omega^2 - k_a} \right) + 2(l - 1)\pi \right) \text{ for } l=1, 2, \dots, \quad (5)$$

where  $l$  denotes a counter associated with phase wrap-around and  $\omega$  is the frequency of the vibrations that need to be suppressed, see [6, 10] for detailed analysis of the absorber dynamics and design recommendations.

## 1.2 Signal Shaper

Signal shapers are mostly used in applications as reference command filters for positioning of the system with flexible or oscillatory modes. As demonstrated in Fig. 2, the reference command  $w(t)$  of the system  $G(s)$  is shaped by the shaper  $S(s)$  in order to target the oscillatory mode of the flexible part of the system  $F(s)$  so that it is not excited. As the basic concept of signal shaping, O. J. Smith Posicast [3] published in 1950's can be considered. Nowadays, these types of shaper are known from the work of Singer and Seering [4, 5], in the 1990's. They developed idea of zero vibration shaper (ZV) and alternatives that lead to more robust suppression over the target mode, such as zero-vibration-derivative (ZVD) and extra insensitive (EI) shapers.



**Fig 2** Signal shaper basic concept

For the compensation of  $F(s)$  oscillatory mode given by the complex conjugate poles  $s_{1,2} = \alpha + \beta j$ , we can use the ZV shaper in a form of equation as follows,

$$u(t) = Aw(t) + (1 - A)w(t - \tau) \quad (6)$$

where  $w$  and  $u$  are the shapers input and output. The parameters of the shaper are the gain  $A$  and the time delay  $\tau \in \mathbf{R}^+$ . The zeros of the shaper, given as the roots of the equation

$$S_{zv} = A + (1 - A)e^{-s\tau} = 0 \quad (7)$$

are given as follows

$$r_{2k+1, 2k+2} = \frac{1}{\tau} \ln \frac{A}{1-A} \pm j \frac{\pi}{\tau} (2k + 1), k = 0, 1, \dots, \infty \quad (8)$$

Placing the dominant zeros  $r_{1,2}$  of the shaper (5) at the position of modes  $s_{1,2}$  of the flexible system with the objective to compensate it, provide

$$A = \frac{e^{\frac{\beta}{\Omega}\pi}}{1 + e^{\frac{\beta}{\Omega}\pi}}, \tau = \frac{\pi}{\Omega} \quad (9)$$

Next to the above described classical ZV shaper, new concepts of signal shaper with a distributed delay [7, 8] will be tested on the laboratory set-up.

## 1.3 Positioning with communication delay

Next to considering time delays in the control algorithms, communication delay will be considered in the feedback loops. The delay will arise by placing the controller to a remote PC and connecting it with the set-up using the internet. The overall communication delay  $\tau$  will be varying, depending on routing the signal through TCP/IP communication connection. The controlled algorithms then will need to be designed robust against this type of uncertain delay parameter.

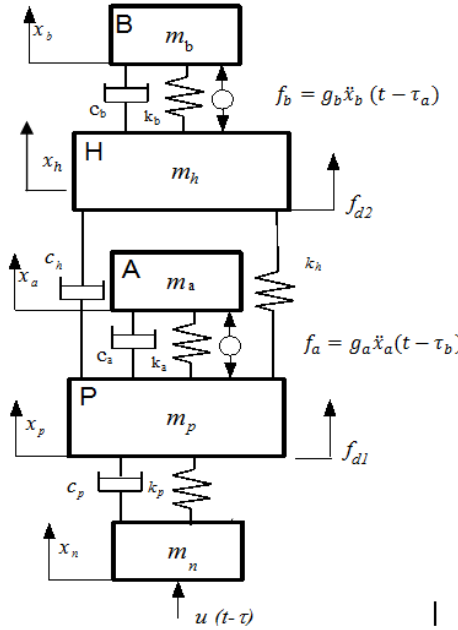
## 2 MODEL, DESIGN AND SIMULATIONS

The basic design framework for the laboratory set-up is multi-degree of freedom structure with multiple resonators, see its scheme in Fig. 3. In the set-up, we consider one or two periodical external harmonic forces  $f_{d1}(t)$ ,  $f_{d2}(t)$ , characterized by the frequencies  $\omega_1, \omega_2$  that excite vibrations of the

masses ( $m_p, m_h$ ). The masses are together joined by the  $k_h, k_p$  springs and  $c_h, c_p$  dampers. The absorbent masses ( $m_a, m_b$ ) are also connected with the main structure by the springs ( $k_a, k_b$ ) and the dampers ( $c_a, c_b$ ). The resonators are controlled by the delayed feedback from the acceleration sensors as explained in section 1.1. In an ideal case, the absorbers acts such that the deflections  $x_p, x_h$  of the masses ( $m_p, m_h$ ) are equal to zero despite the excitation forces. The masses  $m_p, m_h$  are positioned through the input  $u(t - \tau)$ , where the time delay  $\tau$  is caused by intended long distance communication between remote controller and local control devices.

The equations for the set-up are derived from standard force equilibrium equations combined with accelerated delayed feedback (2) as follows.

$$\begin{aligned}
 m_a \ddot{x}_a(t) + c_a \dot{x}_a(t) + k_a x_a(t) - g_a \ddot{x}_a(t - \tau_1) &= c_a \dot{x}_p + k_a x_p(t) \\
 m_b \ddot{x}_b(t) + c_b \dot{x}_b(t) + k_b x_b(t) - g_b \ddot{x}_b(t - \tau_2) &= c_b \dot{x}_h + k_b x_h(t) \\
 m_p \ddot{x}_p(t) + (c_p + c_h + c_a) \dot{x}_p(t) + (k_p + k_h + k_a) x_p(t) &= \\
 \{c_a \dot{x}_a(t) + k_a x_a(t) - g_a \ddot{x}_a(t - \tau_1)\} + c_h \dot{x}_h(t) + k_h x_h + f_{d1}(t) &= \quad (10) \\
 m_h \ddot{x}_h(t) + (c_h + c_b) \dot{x}_h(t) + (k_h + k_b) x_h(t) &= \\
 \{c_b \dot{x}_b(t) + k_b x_b(t) - g_b \ddot{x}_b(t - \tau_2)\} + c_h \dot{x}_p(t) + k_h x_p + f_{d2}(t) &= \\
 m_n \ddot{x}_n + c_p \dot{x}_n + k_p x_p &= c_p \dot{x}_p + k_p x_p + u(t - \tau) - f_{d1}
 \end{aligned}$$



**Fig. 3** Model of laboratory set-up for vibration testing and positioning

## 2.2 Design requirements

All the mechanical components need to be designed with respect to functionality in the achievable frequency ranges of the primary actuators - the absorbers that are to be implemented using voice coils (magnetic shakers). Springs are designed to allow deflections within the voice coil ranges and the maximal force. Dampers are not explicitly included but they are included in springs themselves and in rolling carts on the rails.

The range of considered frequencies also depends on the available control units and their sampling. Laboratory set-up includes lots of electronic parts such as a servo drive, accelerometers, position sensors etc., which are discussed in the following section.

Balancing all the constraints, the design parameters for the set-up have been selected as given in Tables 1. and 2. Parameters are based on simulation of the models described in the introduction and a complete stability analysis done in [6].

**Tab. 1** Parameters of the proposed set-up

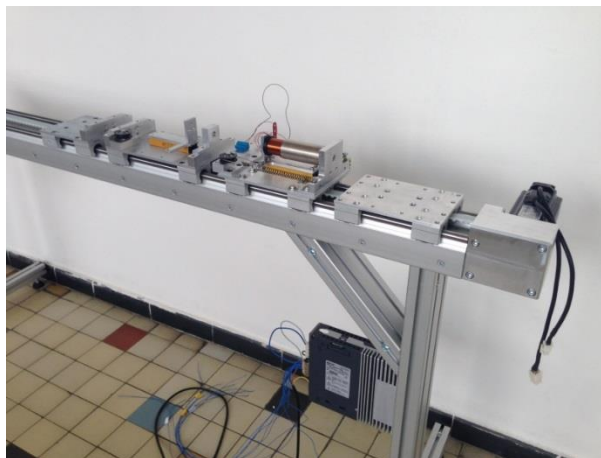
Parameter	value	Units
Range of frequencies	5-15	Hz
Deflection of absorbers	$\pm 20$	mm
Moving mass weight, $m_p, m_h$	$>1$	kg
Amplitude of the excitation Force	$\pm 5$	N

**Tab. 2** Parameters of the absorbers

Parameter	Value	Units
Resonator weight, $m_a, m_b$	0.2	kg
Spring $k_a k_b$	280	$\text{Nm}^{-1}$
Dampers $c_a c_b$	1.4	$\text{Nsm}^{-1}$

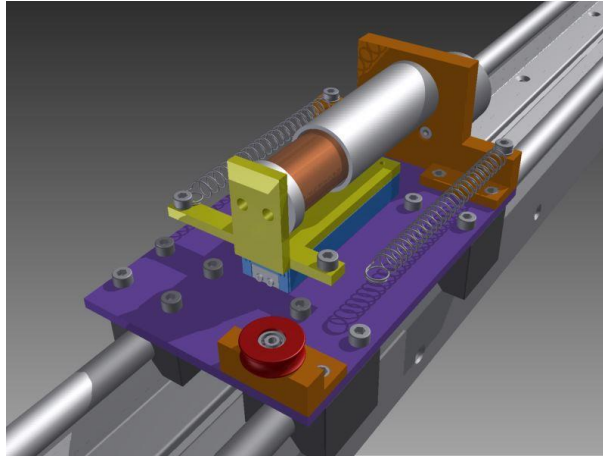
Laboratory set-up has also been designed in order to allow wide range of modularity, which allows a simple scaling of system parameters and assembling various device configurations.

Basic part of the laboratory set-up (see Fig. 4) is a frame built of modular aluminium profiles allowing easy reconfigurability and quick attachment of any electronics and other mechanical parts. A linear ball-bearing rails are used for cart assembling in order to minimize their movement friction. Length of the rails is 1.5 m which is enough for placing up to five carts on them. Positioning of one cart is provided by a toothed belt placed along the rails.



**Fig. 4** Laboratory set-up with attached carts

The cart in Fig. 5, main mass, consists of few easy replaceable parts. There are four ball bearing units at the bottom of the cart for precise movement with low-friction. The resonator absorber is a coil of the linear voice coil placed on a small linear ball-bearing guide. Magnetic housing of the voice coil is fixed to the cart. Two springs are connected to the coil - one through a plastic pulley with ball bearing and the other spring is connected directly.



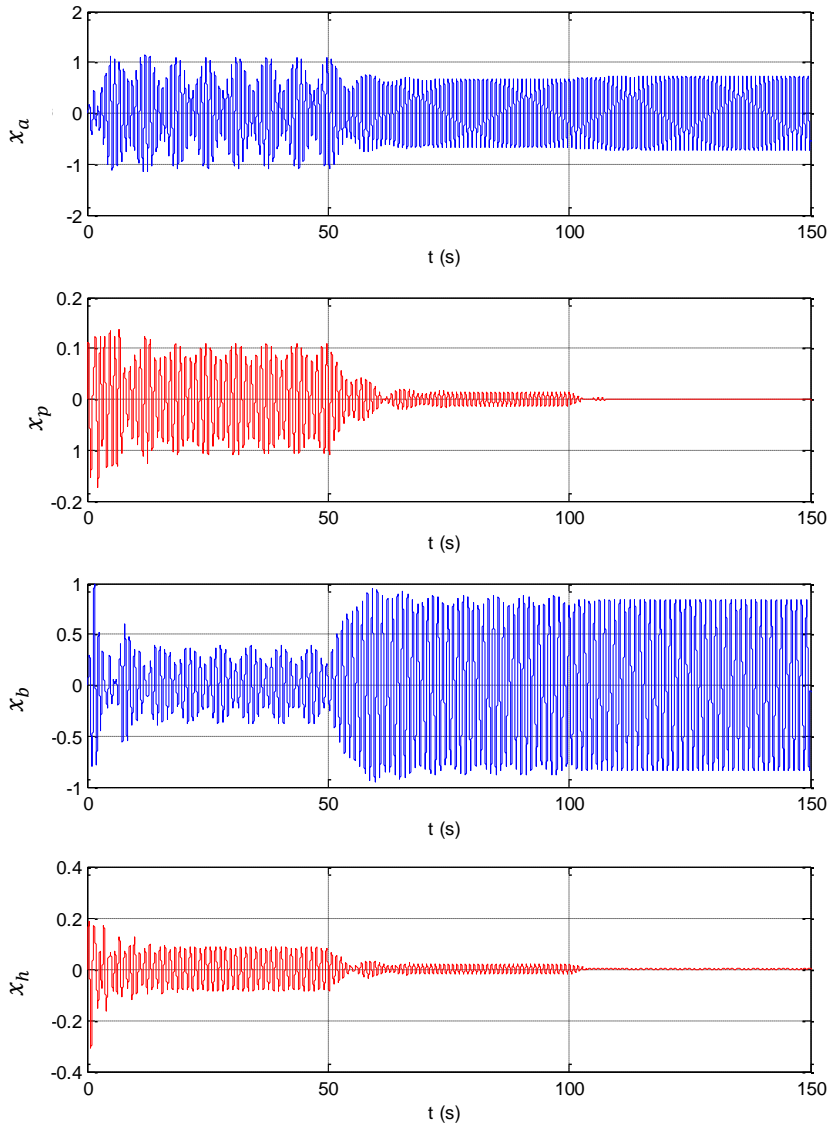
**Fig. 5** 3D design of the cart

### 2.3 Simulation results

To demonstrate the functionality of the set-up design, we provide the following simulation example. Consider the primary structure coupled with the delayed resonators as described in Fig. 3. with  $m_p = 10kg$ ,  $c_p = 20kgs^{-1}$ ,  $k_p = 500Nm^{-1}$ , and  $m_h = 10kg$ ,  $c_h = 20kgs^{-1}$ ,  $k_h = 500Nm^{-1}$ , connected together. The  $m_h$  mass in this example is fixed as stationary part of the system.

The objective is to suppress the external periodical force with frequency at  $\omega_1 = 7 \text{ rad.s}^{-1}$  and  $\omega_2 = 6 \text{ rad.s}^{-1}$  exciting the structure at  $m_h$  and  $m_p$ . Let us consider the absorber masses are given as  $m_a = 2kg$  and  $m_b = 2kg$ . Consider  $j_c = 1$ , we obtain the feedback parameters  $\tau_a = 0.4007s$ ,  $g_a = 0.0432kg$  for frequency  $\omega_1 = 7 \text{ rad.s}^{-1}$  and  $\tau_b = 0.5200s$ ,  $g_b = 0.778kg$  for frequency  $\omega_1 = 7 \text{ rad.s}^{-1}$ .

Results in Fig. 6 show two delayed resonator operation. The first resonator attached to primary mass  $m_p$  starts operating at  $t = 50s$  and the resonator attached to the mass  $m_h$  starts operating at the time  $t = 100s$ . The first excited frequency is removed after the first resonator starts operating and the second is removed when the second resonator starts working, as shown in Fig. 6.



**Fig. 6** Simulation results of the vibration suppression by two delayed resonators

#### 4 CONTROL, ELECTRONICS AND SENSORS

Local PC control system is equipped with data acquisition card AD 622. The DAQ card contains 8 channel fast 14 bit A/D converter with simultaneous sample/hold circuit and 8 independent 14 bit D/A converters, which are used for system control. Sensor outputs and control boards of actuators are connected to the card using TB620 I/O terminal. Alternatively, embedded system Compact RIO-9068 developed by National Instruments is prepared to be connected and work with higher performance. The system is equipped with 667 MHz Dual-Core Controller, Artix-7 FPGA and 8 I/O expandable modules.

The main positioning movement of the movable mass elements on a linear sliding guide provides a servomotor actuator with toothed belt. Servo drive ProNet-04A is controlled by analog voltage signal 0 – 10 V in torque or speed control mode.

Secondary movements between mass elements are realized by voice coils, which are controlled by voice coil control unit board. The control unit works on the principle of current feedback. The voice coil winding (i.e. coil) provides the motive force to the ferromagnetic coil core by the reaction of a magnetic field to the current passing through it. The control unit brings appropriate accurate excitation current to winding of the coils and so the desired force of the linear actuator is exerted. The control unit is equipped with processor ST 32F100 which is 32 bit ARM processor. Sampling frequency of the current measurement is 20 kHz with 12-bit resolution. Power supply voltage range is between 9 and 48 VDC and current range is from 50mA to 10 A. Supported communication protocols are RS-232, RS-485, Profibus, CAN and Ethernet. In our case, the control unit is directly controlled by analogue output of AD 622 DAQ card.

Conditioning amplifier Brüel&Kjær NEXUS 2692 with accelerometer type 4375 was used for initial experiments. The accelerometer is a single-axis precise piezoelectric accelerometer with full scale range of 5000 g. But simple use and precise calibration of output with signal conditioning (bandwidth control, gain control etc.) are of course also expensive and therefore cheaper alternative was chosen. Polysilicon surface micromachined sensor ADXL325/ADXL326 is a small, low power and low cost, complete 3-axis accelerometer with signal conditioned voltage analog outputs - full-scale range of  $\pm 5$  g or  $\pm 16$  g. Bandwidths of the sensor can be selected to suit the application with a range of 0.5 Hz to 1600 Hz for X and Y axes and a range of 0.5 Hz to 550 Hz for the Z axis, which are sufficient parameters for basic measurement with the device. Small SMD package of the sensor (4 mm  $\times$  4 mm  $\times$  1.45 mm) and few necessary PCB components in practical circuit allow to make a small plate with all the components that can be easily stick anywhere on the small flat surface on the device.

## 5 CONCLUSIONS

The paper focuses on design of a laboratory set-up for testing various active vibration control laws with time delays. Next, we discussed problems of delayed resonator, signal shapers and positioning of the multi degree of freedom structure that will be tested on this set-up. Simulation results in chapter 2.3 show vibration suppression by two resonators attached on main structure which is excited by external periodical forces.

Some parts of the system have been already tested and recorded on video available on the project web page<sup>1</sup>. One cart is excited with external force provided through smaller magnetic shaker. Larger magnetic shaker then represents delayed resonator which suppresses the vibration, see also Fig. 1. Furthermore, the functionality of the signal shaper can be also seen in the video<sup>1</sup>.

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<sup>1</sup><http://www.cak.fs.cvut.cz/projects/resonator>



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