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COMPUTATION METHOD OF THE LOAD CARRYING CAPACITY OF CONICAL SHELLS WITH SIMPLY SUPPORTED LOWER EDGE

METODA VÝPOČTU ÚNOSNOSTI KUŽELOVÝCH SKOŘEPIN S PROSTĚ PODEPŘENÝM SPODNÍM OKRAJEM

Abstract

The aim of this paper is to suggest an approximate analytical method that could allow an inexpensive and fast computational control of stability of conical shells with lower edge angle α_c=5°÷15° with simply supported lower edge. Standard stability computational methods according to European Recommendation ECCS [1] are not applicable due to the geometry and boundary condition of the examined shells (see [4,7]). Approximate method is based on the results of a set of numerical analyses of load carrying capacity of examined structures. Numerical analyses are performed by FEM computer program COSMOS/M [5].

Abstrakt

Cílem tohoto článku je představení navržené přibližné analytické metody, která by mohla umožnit levnou a rychlou kontrolu stability prostě podepřených kuželových skořepin s okrajovým úhlem z rozsahu α_c=5°÷15°. Standardní metody výpočtu stability jsou uvedeny v evropském doporučení pro výpočet tenkostěnných skořepinových konstrukcí ECCS European Recommendation [1]. Vzhledem ke geometrii a zvolené okrajové podmínce zkoumaných konstrukcí není možné tyto postupy použít (viz [4,7]). Přibližná metoda výpočtu je odvozena na základě výsledků souboru numerických analýz. Numerické analýzy jsou prováděny v programu založeném na metodě konečných prvků (MKP) COSMOS/M [5].

Keywords

Loss of stability, conical shell, FEM, ECCS

1 INTRODUCTION

Conical shells have practical applications in the field of process and power industry (conical roofs of small vertical storage tanks). Unfortunately, they are considerably sensitive to loss of stability. Available standards and recommendations [1], [2] provide engineers useful procedures to solve the stability of the cones with the edge angle higher than 25 degrees. Furthermore, the rules should be used only for cones with boundary conditions BC1r (nomenclature by ECCS; clamped edges) and BC2f (edge with stiff ring). These rules do not cover shells with free edges (BC3, see Fig. 1) or with edges restrained using a light flexible ring.

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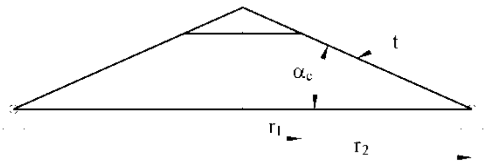

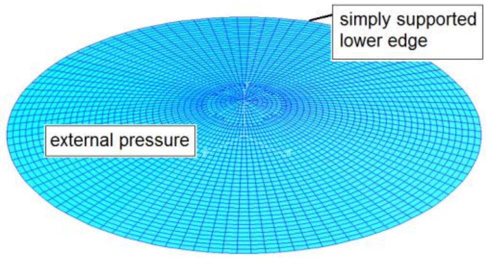

Fig. 1 Scheme of the simply supported conical shell (boundary condition BC3 according to ECCS)

Fig. 2 Numerical model of the simply supported conical shell subjected to an external pressure – shell elements SHELL 4T

This article deals with the already mentioned case of simply supported conical shells with lower edge angle ranging between 5 and 15 degrees. This problem is very difficult to describe. It is not possible to use procedures described in standards because of the nonlinearity of the solving problem. Standard methods are based on linear theory of shells. The procedure is based on sets of numerical analyses of load carrying capacity of conical shells under external pressure. The validity of the method must be verified experimentally.

Numerical analysis taking into account geometrical nonlinearity is performed in computer program COSMOS/M [5] based on the finite element method. Investigated structures may collapse in asymmetrical shape. For this reason it is not possible to use planar elements with rotational symmetry. The numerical model consists of four-node shell elements SHELL4T (see Fig. 2).

2 THEORETICAL BACKGROUND

The loss of stability is one of the limit states which can occur in an excessively loaded thin-wall structure. It is proved in the shell theory that the thin-walled structures can collapse in various shapes depending on geometrical parameters, boundary conditions, loading conditions, material characteristics and initial imperfections. The stability collapse is induced by minimum load corresponding to a particular form of deformation. The membrane strain energy is converted to both the membrane and bending strain energy. As the membrane stiffness of the shell structures is several orders higher than the bending stiffness, the loss of stability is attended by large displacements of a wave character often visible to the naked eye. Examples of loss of stability of excessively loaded thin-walled structures are shown in Fig. 3.

Fig. 3 Practical examples of the limit state of the loss of stability

2.1 Load carrying capacity of conical shells according to ECCS

Section of ECCS dedicated to non-reinforced conical shells is prepared by R. Greiner and C. Poggi [3]. Geometry of the conical shell is converted to an osculating cylinder (Figure 4). Then the load carrying capacity of a cylindrical shell with dimensions l_e and r_e (length and radius of the osculating cylinder) is solved.

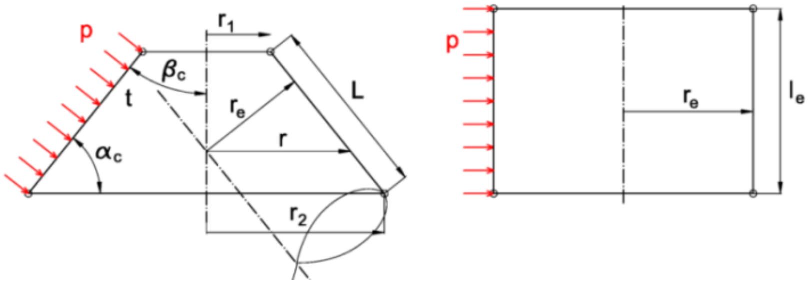

Fig. 4 Conversion of the geometry of the conical shell loaded by external pressure

Length of the osculating cylinder loaded by external pressure is expressed by the relation

$$l_e = \min\left[L; \left(\frac{r_2}{\sin\beta_c}\right)(0.53 + 0.125\beta_c)\right] \tag{1}$$

where the variables are shown in Fig. 4. If the length of the osculating cylinder is $l_e = \left(\frac{r_2}{\sin\beta_c}\right)(0{,}53 + 0{,}125\beta_c)$ then its radius is

$$r_e = 0.71 r_2 \frac{1-0.1\beta_c}{\cos\beta_c} \tag{2}$$

Critical stress of the cylinder loaded by external pressure is expressed by the well-known relation based on linear theory of shells

$$\sigma_{\theta kr} = 0.92E\frac{t}{l_e}\sqrt{\frac{t}{r_e}} \tag{3}$$

Critical stress (critical elastic buckling stress in the circumferential direction) express the relationship (see the European recommendation [1])

$$\sigma_{\theta Rcr} = 0.92E\,\frac{C_\theta}{\omega}\frac{t}{r_e} \tag{4}$$

where the influence of geometry and boundary conditions on the critical stress is taken into account through coefficient C_θ. Dimensionless parameter ω depends on the specific geometry of the cylindrical shell.

$$\omega = \frac{l_e}{r_e}\sqrt{\frac{r_e}{t}}$$

Value of critical stress $\sigma_{\theta Rcr}$ is further modified by further coefficients which take into account the influence of the nonlinear behavior of the material, the influence of initial imperfections, etc. The resulting design load in the circumferential direction depends on the external pressure according to the formula

$$\sigma_{\theta Ed} = p\left(\frac{r_e}{t}\right) \tag{5}$$

2.2 Conical shells with small camber

Conical shells with a smaller edge angle show a considerably nonlinear behavior (see [6]). Therefore it is *not possible to use the linear solution*, used in the ECCS recommendations. Nonlinearity is caused by

- ***the size of the edge angle*** $\boldsymbol{\alpha} < 25°$. The location of maximal bending meridian moment is shifted from the edge to the center of the cone during loading. When $\alpha_c \to 0$ the cone becomes a circular plate with the maximum bending moment in the middle of the plate. The task of the loss of stability is then changing into the task of strength. The radius of the osculating cylinder is dependent on the formula $1/\sin\alpha_c$. Radius grows beyond all bounds when edge angle approaching zero. Mentioned reasons may cause a significant distortion of the result of the critical load.

- ***the possible displacement in the radial direction***. This displacement occurs during loading the simply supported conical shell and conical shells with thin stiffening rings. The membrane meridian force F_x occurs in the shell wall during the process of the loading. This force grows in dependence on the size of the edge angle α_c according to relation $1/\sin\alpha_c$ (see Fig. 5). Radial component of this force causes additional bending stress in the location of the support. When $\alpha_c \to 0$ then for meridian force applies theoretically $F_x \to \infty$. Meridian force causes both the vertical (of the peak of the cone) and horizontal (of the lower edge in radial direction) displacement. Horizontal displacement generates further increase of meridian force. This process can lead to breaking the shell into its inverse position.

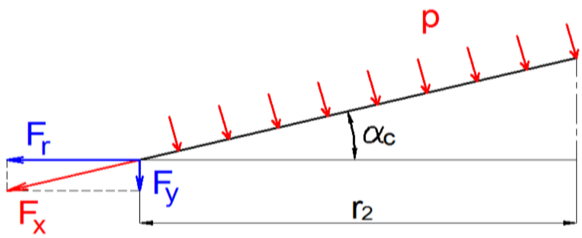

Fig. 5 Forces acting in the location of the support of the conical shell

The problem should be solved using numerical analysis taking into account geometrical and material nonlinearities. Furthermore, it is necessary to experimentally verify the results of the analyses. The main aim of the paper is to find a simple pseudo-analytical formula based on a critical stress equation (4)

$$\boldsymbol{p = K \cdot E \frac{1}{\omega}\left(\frac{t}{r_e}\right)^m} \quad (6)$$

Due to the significant complexity and breadth of the described problem would be very difficult and costly to investigate the stability of conical shells using only experiments. Using numerical analyzes it is possible to quickly and cheaply simulate a wide range of experiments. Numerical analyses are performed in computer programs based on the finite element method (FEM).

3 ACHIEVEMENTS

Dependence of limit pressure (results of geometrically nonlinear analyzes - GNA) of simply supported conical shells with edge angle $\alpha_c = 10°$ on the thickness parameter r_e/t is shown in Figure 6 (further graphs are listed in [8]). Young's modulus of elasticity $E = 2 \cdot 10^5 MPa$ and Poisson number $\mu = 0.3$ are considered during the analysis.

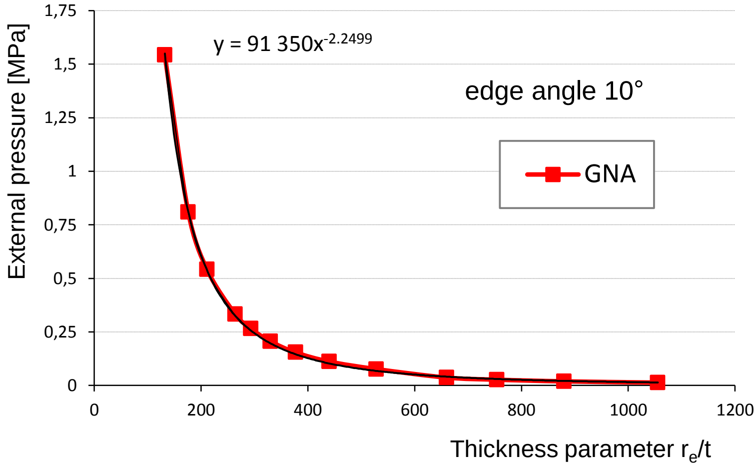

Fig. 6 Dependence of the limit pressure (results of numerical analyses GNA) on the thickness parameter - simply supported conical shell with edge angle $\alpha_c = 10°$; regression equation displayed

It is possible to assemble the regression curves in the form of power functions due to the course of those dependencies

$$\boldsymbol{p = K' \cdot \left(\frac{t}{r_e}\right)^m = K' \cdot \left(\frac{r_e}{t}\right)^{-m}} \tag{7}$$

Where K' is the coefficient of power curves which already includes the effect of the material and geometry of the shell by means of the modulus of elasticity E and the dimensionless parameter ω

$$K' = K \cdot E \frac{1}{\omega} \tag{8}$$

The values of coefficients K' and m of regression curves of simply supported conical shells are listed in Table 1.

Tab. 1 Values of coefficients of regression curves

Boundary condition	Edge angle $\boldsymbol{\alpha_c}$ [°]	Range $\boldsymbol{r_e/t}$	Coefficients	
			$\boldsymbol{K'}$	$\boldsymbol{m}$
SIMPLY SUPPORTED	5	260÷2080	104300	2.2175
	10	130÷1050	91350	2.2499
	15	90÷890	91858	2.28

Coefficients of regression curves are valid only for *specific values of examined edge angles*. The value of limit pressure the size of the edge angle between values 5°, 10° and 15° could be linearly interpolated. Figure 7 shows an example of linear interpolation of limit pressure of conical shell with edge angle $\alpha_c = 7.5°$ (geometry of the sample is shown in Table 2). A detailed calculation of the limit pressure is given in [8].

Tab. 2 Geometry of the example conical shell

$\boldsymbol{\alpha_c}$[°]	$\boldsymbol{t}$ [$\boldsymbol{mm}$]	$\boldsymbol{r_2}$[$\boldsymbol{mm}$]	$\boldsymbol{r_e}$[$\boldsymbol{mm}$]	$\boldsymbol{r_e/t}$[−]
7.5	12	2100	9778.2	814.85

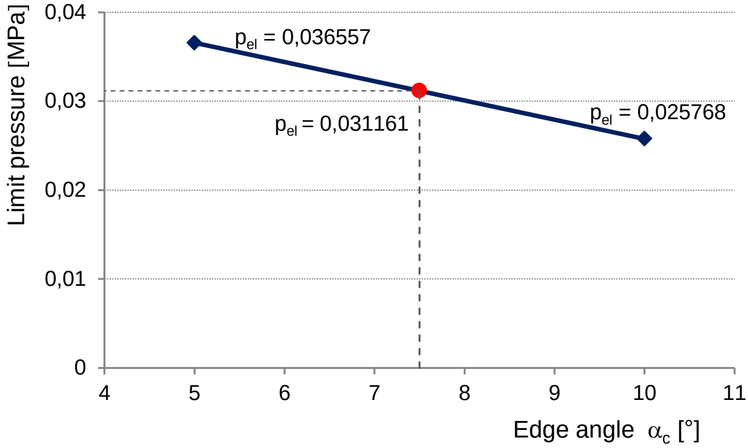

Fig. 7 Linear interpolation of limit pressure of conical shell with edge angle $\alpha_c = 7.5°$.

Results of numerical analysis GNA of the example cone and the value of elastic limit pressure calculated using presented method are compared in Table 3. Relative error of those results is listed in the last row of the table (specified to the result of numerical analysis).

$$\delta = \frac{p_{GNA} - p_{el}}{p_{GNA}}$$

Tab. 3 Comparison of results of calculated limit pressures of sample cone with edge angle $\alpha_c = 7.5°$

$p_{GNA}[MPa]$	$p_{el}[MPa]$
0,03215	0,031161
δ = 3.07%	

4 CONCLUSIONS

Method of solving the elastic limit pressure of simply supported conical shells with edge angle of the range $\alpha_c = 5 \div 15°$ is presented in this paper. This method is based on the formula of critical stress of osculating cylinder (4). Formula for the critical pressure (7) is supplemented by new coefficients (K, m) which reflected conical shell geometry (edge angle α_c).

$$p_{el} = K \cdot E \frac{1}{\omega} \left(\frac{t}{r_e}\right)^m, \text{ resp. } p_{el} = K' \cdot \left(\frac{t}{r_e}\right)^m$$

Coefficients K', m are determined from regression curves of limit pressure dependence on the thickness parameter (see Figure 6). After substituting the coefficients into the equation of limit pressure is calculated elastic limit pressure. Using similarity criteria (dimensionless thickness parameter r_e/t) can be calculated limit pressure conical shells with arbitrary dimensions within the studied range.

The results achieved in the paper could be a useful tool in the hands of ordinary designer. At present, must be conical shell construction with a small camber solved numerically. The possibility of using normative relations for the design comes much cheaper. The proposed method is quite simple and has the same physical basis as the calculation of load capacity specified in the standards and recommendations. Calculation of load capacity is based on the well-known formula of the critical stress of cylindrical shell

Numerical results are verified by first set of experiments (results and comparison are given in [8]). A test device for experiments of stability of shells of revolution is available at University of Pardubice. Results of the experiments are not included in this article because this article mainly deals with elastic limit pressure of ideal shells. Material nonlinearity and initial imperfection have an influence on experimental results. Influence of these effects is significant part of further research.

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