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MEASUREMENT OF THE LIGHTWEIGHT ROTOR EIGENFREQUENCIES AND TUNING OF ITS MODEL PARAMETERS

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Abstract

The common sizes and weights of rotors, which can be found e.g. in the energy production industry, allow to employ a standard methodology of an experimental modal analysis. However, certain applications with rotors of small weights lead to the usage of alternative measuring methods suitable for the identification of rotor eigenfrequencies. One of these methods, which is characterized by the measuring of noise, is introduced in this paper and the results for a particular rotor is presented. Moreover the tuning of the finite element rotor model on the basis of such measured values is shown.

Abstrakt

Standardní velikosti a hmotnosti rotorů, které se běžně vyskytují například v energetických aplikacích, umožňují využití standardních metod experimentální modální analýzy. Určité aplikace charakterizované nízkou hmotností rotorů však vedou na využití alternativních metod měření vhodných pro identifikaci vlastních frekvencí rotoru. Jedna z těchto metod, která je založen na měření hluku, je představena v tomto článku a jsou ukázány vybrané výsledky na konkrétním rotoru. Dále je popsán matematický model rotoru založený na metodě konečných prvků a ukázáno ladění parametrů tohoto modelu s využitím naměřených vlastních frekvencí.

1 INTRODUCTION

Rotor dynamics is an interesting and useful engineering area with many types of industrial applications. An experimental verification of rotor models is a necessary part of rotor analyses and various calculations. The knowledge of real-life rotors eigenfrequencies and modal shapes is a key to the successful verifications. Small and lightweight metal rotors have high eigenfrequencies – in the band up to 3200 Hz can usually be found but the first two eigenfrequencies. The conventional experimental modal analysis [1] can be difficult to perform in the frequency band above 3200 Hz because such a test can be affected by one or more of the following issues:

- the frequency spectrum of an excitation force is not constant enough in the frequency band in which the modal analysis is carrying out (the decrease between low and high end of the band is greater than 20 dB),
- the frequency range of used transducers is insufficient due to the low resonant frequency of the transducer (an uncertainty of amplitude and phase measurement caused by mechanical properties

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of the transducer can be ignored in the band up to ca. $0.3 \cdot f_r$, where f_r is the resonant frequency of the transducer),

- connections between the rotor and the ground, e.g. a structure suspending, transducer cables or a modal shaker arm, cannot be neglected because they have an influence on modal properties of the rotor (too stiff couplings can decrease the values of the eigenfrequencies by more than 10 % and unsuitably chosen positions of such couplings affect a form of some modal shapes).

The abovementioned problems can be eliminated by employing a testing method, which is not dependant on the quality of the excitation force, and by use of transducers, which are not mounted on the structure's surface, e.g. a microphone or a laser proximity sensor.

2 MEASUREMENT OF ACOUSTIC RESPONSE

An analysis employing the measurement using microphones cannot be considered to be a standard modal analysis because it does not provide any information regarding modal properties of the tested structure except values of certain eigenfrequencies. For this reason only structures of known mode shapes order, such as lightweight, thin rotors, can be tested. For these rotors the knowledge of the mode shapes order can be achieved by performing a computational modal analysis with the use of the finite element method.

The identification of eigenfrequencies from the acoustic response diminishes or completely eliminates the influence above mentioned issues. The analysis is not limited by the frequency range of a transducer, the lowest values of microphone's low pass are usually around 15 kHz. The number of connections between the tested structure and the ground always decreases – transducer cables are no longer needed and the number of suspensions can be often reduced. The setup for a measurement of a lightweight rotor acoustic response is shown in Fig. 1.

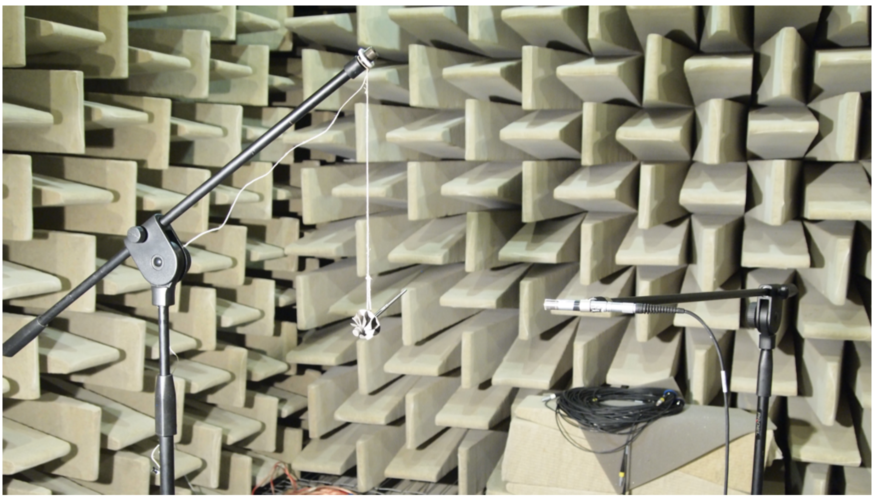

Fig. 1 The measurement of acoustic response in an unechoic chamber. Note that only one link with a stand is needed

Furthermore, the analysis does not depend on the quality of an excitation force if a response is measured instead of a frequency response function (FRF) generally given by

$$H(f) = \frac{X(f)}{F(f)} \tag{1}$$

where $X(f)$ and $F(f)$ are Fourier spectra of a response $x(t)$ and an excitation $f(t)$, f is a frequency.

Although knowledge of real and imaginary parts of FRFs allows a fast and accurate identification of eigenfrequencies using standard procedures and commercial applications, the acoustic response can be also used. The response has to be converted from the time domain to the frequency domain and peaks (tones) have to be found in the resulting spectrum.

Such a task can be performed either manually or programmatically. The eigenfrequencies related to the bending modes are doubled for symmetric rotors, because such rotors have the same bending stiffness in both horizontal and vertical planes in theory. However, values of bending stiffness of real-life rotors differ slightly for each plane due to manufacturing inaccuracy, material nonhomogeneity etc. The difference causes the separation of the doubled eigenfrequencies into two entities. The separation can be easily observed in the frequency domain of the response – there are two or more close peaks instead of one peak, which would be present if the bending stiffness in both aforementioned planes was equal. This problem is illustrated in Fig. 2.

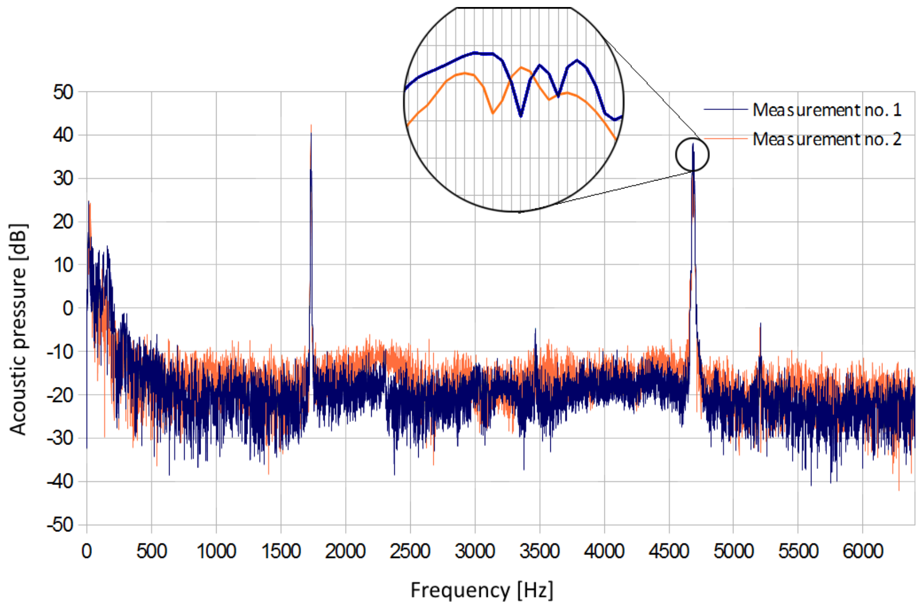

Fig. 2 Frequency spectrum of a real-life rotor acoustic response with the divided peak in the detail

The manual analysis of the real-life rotor acoustic response can be difficult because of ambiguous positions of individual peaks and thus the programmatic approach can yield more accurate results.

Peaks can be detected by looking for downward zero-crossings in the smoothed first derivative and peak amplitudes that exceed predefined threshold values, and the position, height, and approximate width of each peak can be determined by least-squares curve-fitting the top part of the peak. The exact code and the mathematical background is beyond this article reach and further details can be found in [2], where MATLAB script, which is able to locate and measure the positive peaks in a noisy data sets, is introduced.

In order to achieve as accurate results as possible, the rotor – or another tested structure – should be sequentially excited in several reference points and the acoustic response should be recorded and converted to the frequency domain. Each of the reference points should be excited at least twice.

The estimates of the first few rotor eigenfrequencies are the arithmetic means of found peaks with similar frequencies. The uncertainty σ^j of jth estimate can be determined as standard deviation with applied Bessel's correction given by

$$\sigma^j = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(f_i^j - \bar{f}_i^j\right)^2}, \tag{2}$$

where n is the number of analyzed acoustic responses, f_i^j is the frequency of jth peak in ith frequency spectrum, $\bar{f}^j$ is the arithmetic mean of jth peak frequency.

The standard deviation is one of the basic parameters of probability distributions. It shows how much variation exists from the mean value.

The central limit theorem [3] states that the distribution of a mean value of many independent, identically distributed random variables tends towards the normal distribution. Assuming $\bar{f}^j$ is the expected value of the normal distribution and σ^j equals its standard deviation, a probability density function can be reconstructed and the upper deviation $\bar{\sigma}^j$ of the measurement can be calculated. $\bar{\sigma}^j$ is usually chosen so as the interval $\left\langle \bar{f}^j - \bar{\sigma}^j,\ \bar{f}^j + \bar{\sigma}^j \right\rangle$ covers 90 % or 95 % possible values.

The first few rotor eigenfrequencies can be determined with the error lower than 1 %.

3 MATHEMATICAL MODEL OF A ROTOR

The measured eigenfrequencies are significant mainly for sake of the parameter identification of mathematical models or their verification and therefore the chosen modelling approach will be briefly described in this section. It will be introduced with important effects of rotation with respect to a further model usage.

The rotating shaft can be considered as a one-dimensional continuum with undeformable cross-section and can be discretized by shaft finite elements (Fig. 3). The circular cross-section according to Fig. 3 is considered in this paper, the derivation of the more general mathematical model for a nonsymmetrical cross-section was presented e.g. in [5].

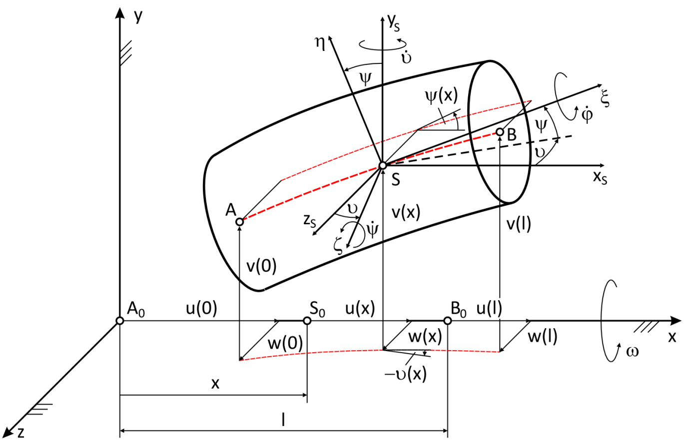

Fig. 3 Simple scheme of a shaft finite element, red curve represents deformed centerline

Deformations of the shaft finite element in dependance on the longitudinal coordinate x are described by the longitudinal displacement $u(x)$, transverse displacements $v(x)$ and $w(x)$ in two perpendicular directions, angular displacements of the cross-section $\vartheta(x)$ and $\psi(x)$ and torsional angular displacement $\varphi(x)$. The cross-section is supposed to remain perpendicular to the deformed shaft axis and the shaft is rotating with the constant angular velocity ω.

A common approach for the rotating shaft model derivation is the expression of the kinetic energy and the strain energy of the shaft finite element and their usage in the Lagrange formalism to the formulation of mass $\boldsymbol{M}$, gyroscopic $\boldsymbol{G}$ and stiffness $\boldsymbol{K}$ matrices. These finite element matrices are then used for the creation of the equations of motion of a discretized rotating shaft externally loaded by vector $\boldsymbol{f}(t)$ in the form

$$\boldsymbol{M}\,\ddot{\boldsymbol{q}}(t) + \omega\,\boldsymbol{G}\,\dot{\boldsymbol{q}}(t) + \boldsymbol{K}\,\boldsymbol{q}(t) = \boldsymbol{f}(t), \tag{3}$$

where $\boldsymbol{q}(t)$ is a vector of shaft generalized coordinates. The model can be extended by effects of damping [5], by models of rigid disks or similar bodies and by a model of various bearings [4].

4 TUNING OF MODEL PARAMETERS

Equation (3) can be simplified for $\omega_s = 0$ and for zero excitation with respect to the conditions of the measurement. The modal analysis can be then performed and rotor eigenfrequencies and corresponding eigenvectors can be calculated. The goal of the tuning of model parameters is the identification of such parameters in vector $\boldsymbol{p}$ to obtain a sufficient agreement of measured and model eigenfrequencies. The real rotor mathematical model (Fig. 1, discretized into 12 finite elements) is implemented in the MATLAB system. The suitable model parameters are diameters of particular shaft finite elements and inertia properties of coupled rigid disks (together 17 parameters). The parameter tuning is performed as an optimization process. Initial model parameter values are defined by estimated values and relative optimization parameters are used for a better numerical performance, so the initial parameter values are equal to 1.

The objective function is chosen in the form

$$\psi(\boldsymbol{p}) = \sum_{i=1}^{2} \left(1 - \frac{f_i(\boldsymbol{p})}{\hat{f}_i}\right)^2, \tag{4}$$

where $f_i(\boldsymbol{p})$ are actual values of eigenfrequencies corresponding to the first two eigenmodes with dominant bending deformation and $\hat{f}_i$ are measured eigenfrequencies (1735.35 Hz and 4687.37 Hz) corresponding to these eigenmodes.

Tab. 1 Goal, initial and final values of the tuned eigenfrequencies for particular parameter constraints

	Goal	Initial values	Final values
f_1 [Hz]	1735.35	1663.27	1735.36
f_2 [Hz]	4687.37	4824.39	4687.36

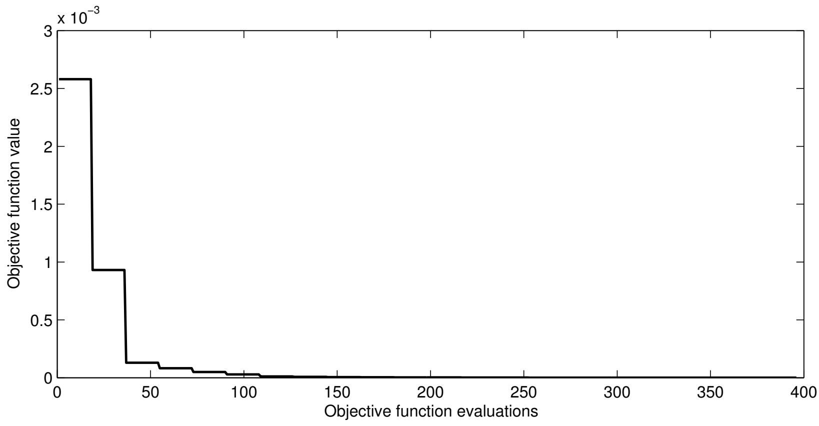

Fig. 4 Evolution of the objective function value during the optimization process

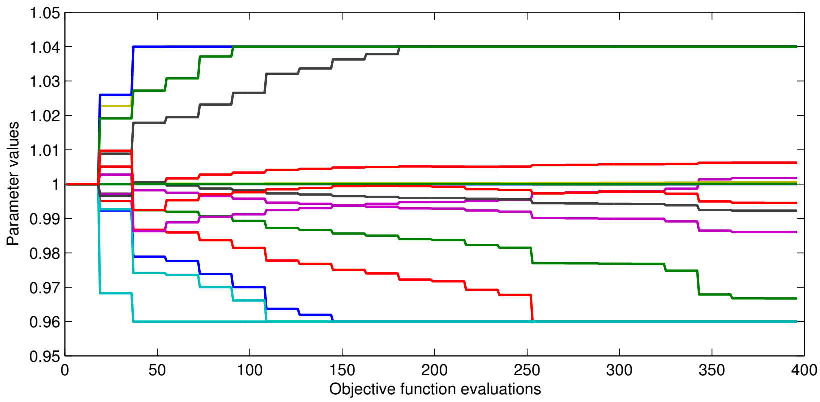

Fig. 5 Evolution of the model parameters during the optimization process

The parameters were constrained by lower and upper bounds and the optimization process was performed using gradient based algorithms [6]. Chosen results of the tuning are shown in Tab. 1. The dependencies of the objective function value and values of the optimization parameters in the course of the optimization process are shown in Figs 4 and 5.

5 CONCLUSIONS

The suitable approach for the experimental modal analysis of lightweight rotors was presented in this paper together with the succesive usage of the measurement results for the model parameter identification. The common rotating beam finite element was used for the modelling of rotors. However the introduced parameter tuning procedure can be generalized for other types of rotor mathematical models such as solid finite elements etc.

The measurement can be also employed to increase the precision of an already executed conventional experimental modal analysis.

ACKNOWLEDGEMENT

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REFERENCES

[1] IRRETIER, H. Mathematical foundations of experimental modal analysis in rotor dynamics. *Mechanical Systems and Signal Processing*. 1999, XXIII. Nr. 2., pp. 183-191.

[2] O'HAVER, T. C. *Peak Finding and Measurement* [online]. College Park (MD), 2013 [cit. 2013-03-17]. Available at: http://terpconnect.umd.edu/˜toh/spectrum/PeakFindingandMeasurement.htm.

[3] Wikipedia: the free encyclopedia. *Central limit theorem* [online]. San Francisco (CA): Wikimedia Foundation, 2001- [cit. 2013-03-17]. Available at: http://en.wikipedia.org/wiki/Central_limit_theorem

[4] BYRTUS, M. & HAJŽMAN, M. & ZEMAN V. *Dynamika rotujících soustav*. 1st ed. Plzeň : Západočeská univerzita, 2010. ISBN 978-80-7043-953-1.

[5] DUPAL, J. Stability of non-symetrical rotating 1D continuum with non-isotropic supports. In *Proceedings of UWB*, Vol 2/1998. Plzeň : Vydavatelství ZČU, 1998.

[6] NOCEDAL, J. & WRIGHT, S.J. *Numerical optimization*. New York : Springer, 1999. ISBN-10: 0387303030