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DESIGN AND ANALYSIS OF THE MEMBRANE STRUCTURE

NÁVRH A VÝPOČET MEMBRÁNOVÉ KONSTRUKCE

Abstract

The article deals with problems in designing and analyzing the membrane structure of a stadium roof. Attention is especially given to problems relating to the optimal design of the shape of the membrane (form finding) and the supporting cable system with respect to static behavior, functionality and architectural appearance. The work was carried out with respect to the companies Ing. Software Dlubal, s.r.o. and FEM consulting, s.r.o. intention to develop a new module of the RFEM program system capable of ascertaining the initial shape of a roof's membrane construction and also its analysis for both static and dynamic wind loads.

Abstrakt

Článek se zabývá návrhem a výpočtem membránové konstrukce zastřešení stadionu. Pozornost je věnována především problematice optimálního návrhu tvaru membrány (form finding) a nosného lanového systému z hlediska statického působení, funkčnosti i architektonického vzhledu. Práce byla zadána s ohledem na záměr firem Ing. Software Dlubal, s.r.o. a FEM consulting, s.r.o., které spolupracují na vývoji programového system pro výpočet konstrukcí RFEM, doplnit tento programový systém o modul pro hledání výchozích tvarů membránových konstrukcí a jejich výpočet pro statické zatížení, ev. pro dynamické zatížení větrem.

Keywords

Membrane structures, initial shape, form finding, minimal surface, isotropic stress field

1 INTRODUCTION TO FORM FINDING

The process of ascertaining an appropriate initial shape of membrane (form finding) is a key question in designing membrane structures. To achieve sufficient membrane bearing, stability and shape constancy, it is necessary to apply prestressing. The directions and magnitudes of prestressing are determined by the shape of the analyzed membrane [2].

2 DESIGN OF THE MEMBRANE STRUCTURE OF A STADIUM ROOF

The aim of the presented work was to test the RFEM program capability to design and analyze large membrane structures, particularly the membrane of a stadium roof. The project took into account not only the structure itself, but also its dispositional and esthetic aspects. Attention was given to the search for an appropriate membrane form with attention also given to the design of the supporting cable system which would bear the membrane.

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The cable system has been designed with respect to static requirements and an undisturbed spectator view from the stands. Therefore the columns, supporting the cable system, are located outside the stadium. The specific requirements stated above are met by means of appropriate curvatures of the cables and prestressing magnitudes. The scheme of the entire spatial structure can be seen in Fig. 1. The actual principle of the functioning cable construction is shown in Fig. 2, where it is analyzed in detail in cross section A-A, the position of which can be seen in Fig. 1. In Fig. 2 we can see both the initial equilibrium cable system and its behavior when applying applications of different load types. The stability of the structure is ensured by variations of cable tension caused by displacement of the applied loads. Both the geometrical stiffness and the size of the subsequent deformation, is a result primarily of the magnitude of initial prestressing.

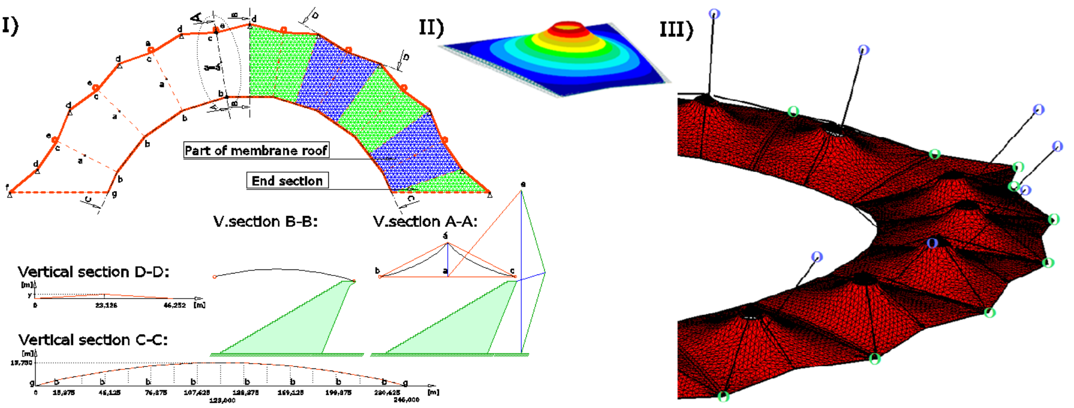

Fig. 1 I) Static schema of one half of roof stadium, II) one of sixteen parts of the same shape, III) part of 3D global model

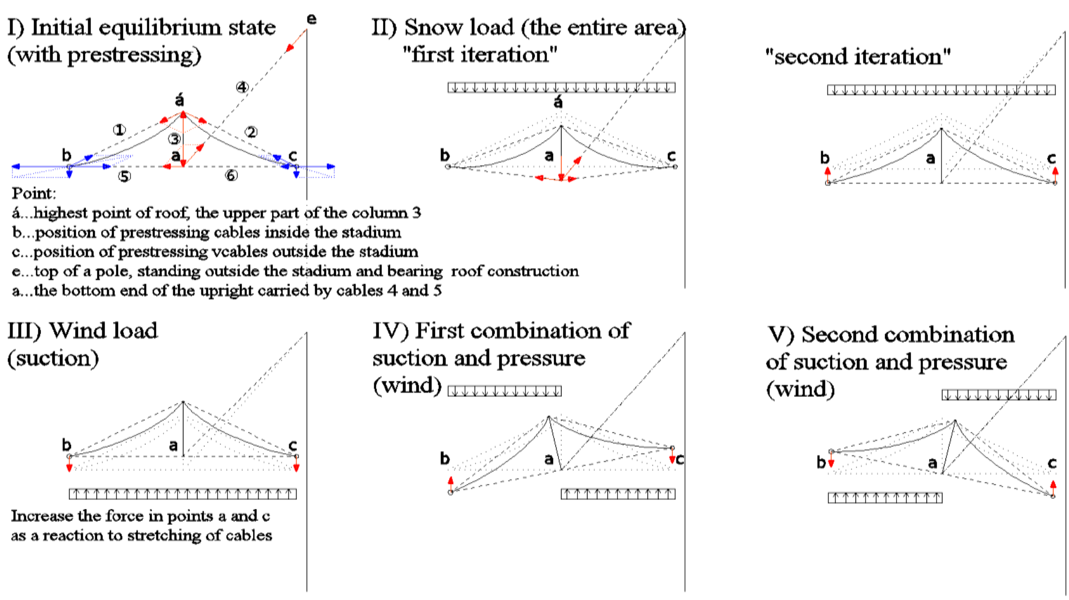

Fig. 2 Schema of the static system in cross section A-A

The above described cable system creates boundary conditions for the membrane itself. This membrane can be divided into sixteen parts of the same shape. One half of the roof's symmetric structure is presented in Fig. 1. The search for an appropriate initial shape was carried out on one of the sixteen sections (Fig. 1). This search is based on the theory of minimal surfaces [2]. This includes defining the membrane boundaries and a search for the minimum of all possible surfaces, which satisfy the boundary conditions. The process is approached numerically [1]. (An analytical solution of the problem can be found for example in a search for the minimum surface connecting the two given circles of the example. This created shape is called the catenoid.) After preloading the membrane which geometrically corresponds to the minimum surface between the specified boundaries, we can observe the isotropic stress field in the membrane (Fig. 3). This corresponds to the theory of soap bubbles, which established an equivalence between direct geometric access and access of mechanics. After ascertaining the appropriate initial shape of the membrane using the aforementioned section, the process of form finding was applied to the whole membrane structure.

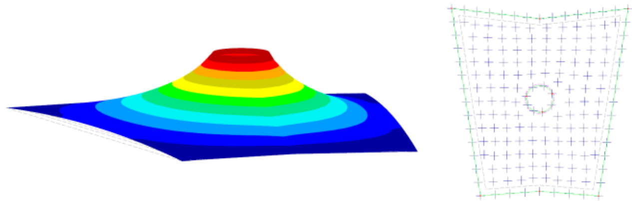

Fig. 3 The shape of the minimal surface and isotropic stress field

3 ANALYSIS OF THE MEMBRANE STRUCTURE OF A STADIUM ROOF

The global initial shape of the membrane was achieved by imposing vertical displacement on the supporting nodes of the original planar model and the application of contraction to the membrane. These loads are not real, but they are fictitious, and only serve in finding a suitable shape of the membrane structure. The calculation of the strain and stress caused by this load is irrelevant and has nothing to do with the real state of stress and strain. Therefore, these variables are not subject to limitations on the strength and ductility of the material used. The resulting shape is then used as an initial shape, i.e. the shape, with zero strain and stress, in calculating the proper response to actual membrane static or dynamic loads.

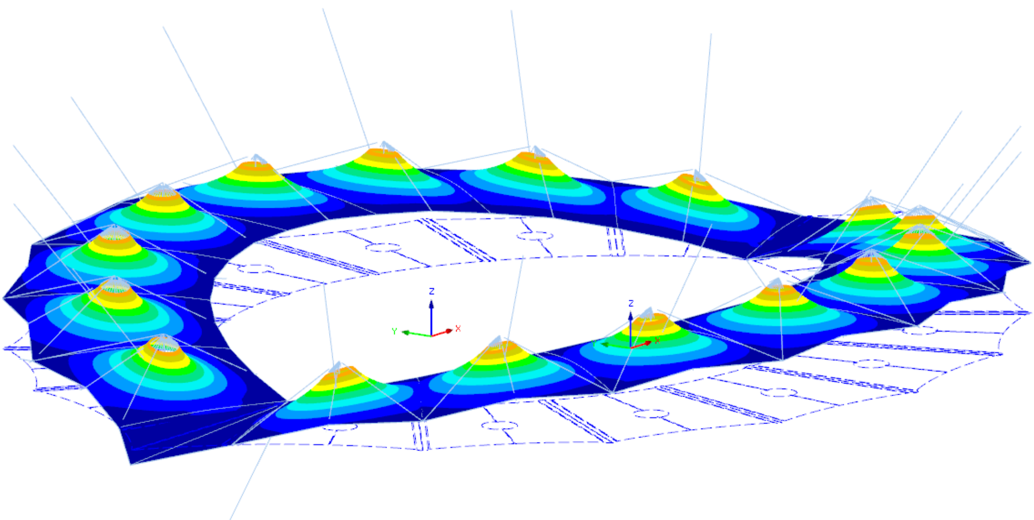

Fig. 4 The geometry of global model construction

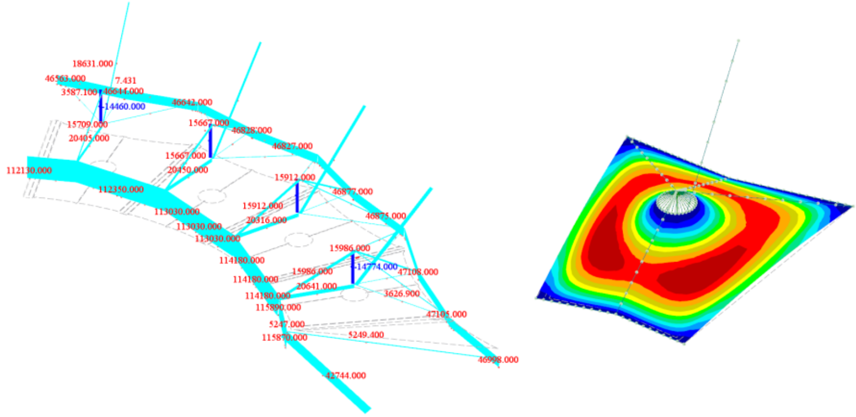

Fig. 5 The normal forces in the cables system as a result of design load values (left) and membrane deformation from characteristic load values (right)

3 CONCLUSIONS

The initial membrane shape was founded during the search process, which corresponds with theory of minimal surfaces. The real prestressing and loading values were further applied to the shape. On the loaded construction, calculations of the stress and deformation values were performed. For illustrative purposes, Fig. 5 presents various outputs. On the left for better clarification in Fig 5 the values of the design axial forces in the cable system with struts are demonstrated using one-quarter of the global model. On the right is presented one of the sixteenth parts of the deformation of the membrane shape resulting from characteristic load values.

Analysis of membrane structure, described above, was first result of this work. The second result is an understanding of the problem of minimal surfaces generation. This is one of the famous methods for design of shape for membrane structures.

This work is the beginning of cooperation on creation of module for design of membrane structures shapes in the RFEM program system.

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