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CALIBRATION OF CHABOCHE MODEL WITH A MEMORY SURFACE

KALIBRACE CHABOCHEOVA MODELU S PAMĚŤOVOU PLOCHOU

Abstract

This paper points out a sufficient description of the stress-strain behaviour of the Chaboche nonlinear kinematic hardening model only for materials with the Masing's behaviour, regardless of the number of backstress parts. Subsequently, there are presented two concepts of most widely used memory surfaces: Jiang-Sehitoglu concept (deviatoric plane) and Chaboche concept (strain-space). On the base of experimental data of steel ST52 is then shown the possibility of capturing hysteresis loops and cyclic strain curve simultaneously in the usual range for low cycle fatigue calculations. A new model for cyclic hardening/softening behaviour modeling has been also developed based on the Jiang-Sehitoglu memory surface concept. Finally, there are formulated some recommendations for the use of individual models and the direction of further research in conclusions.

Abstrakt

Příspěvek poukazuje na dostatečný popis napěťově-deformačního chování Chabocheova modelu nelineárního kinematického zpevnění pouze pro materiály, které vykazují Masingovo chování, a to nezávisle na počtu kinematických částí. Následně jsou prezentovány dva nejpoužívanější koncepty paměťových ploch – modely Jiang-Sehitoglu (v deviátorové rovině) a Chaboche (prostor poměrných deformací). Na oceli 11523 jsou pak ukázány možnosti zachycení hysterezních smyček v rozsahu obvyklém pro nízkocyklovou oblast. Byl také vyvinut nový přístup pro modelování cyklického zpevňování/změkčování na základě paměťové plochy Jiang-Sehitoglu. V závěru jsou také formulována doporučení pro použití jednotlivých modelů a popsán směr dalšího výzkumu.

Keywords

Cyclic plasticity, low cycle fatigue, memory surface, Chaboche model, FEM.

1 INTRODUCTION

Recently the most popular cyclic plasticity models are those of Armstrong-Frederick type. These models incorporate a nonlinear kinematic hardening rule, which ensures the capture of so called Bauschinger effect and it also makes possible accurate ratcheting prediction or modeling of cyclic hardening/softening behaviour of metallic materials. In spite of that such pure kinematic hardening models are not useable for materials with Non-Masing's behaviour.

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The basic concept of kinematic and isotropic hardening has been enhanced for example by memory surfaces introduction [1–2] and by evolution shape of yield locus surface shape evolution [3]. The first mentioned improvement corresponds to the fact that there is a non-hardening strain region in a material point in the case of variable amplitude loading as Ohno explained in [4]. In the second case, the aim of researchers is to capture real anisotropy induced by plastic deformation. Both approaches can lead to significant improvements of classical models of cyclic plasticity of Armstrong-Frederick type, but number of material parameters sometimes drastically increases.

There are compared two basic concepts of memory surface in this paper, one proposed in plastic strain space developed by Chaboche et al. [1] and the second introduced in stress space by Jiang and Sehitoglu [2]. Main aim is a description of the procedure for material parameters identification, especially those influencing modelling of Non-Masing behaviour. All numerical analysis (finite element analysis, FEA) in this paper has been carried out using software Ansys.

2 CONCEPT OF SINGLE YIELD SURFACE

There are many approaches in the incremental theory of plasticity suitable for metals, but the most popular one is the concept of single yield surface. Such cyclic plasticity models under assumption of rate-independent material's behaviour consist mostly of von Mises yield criterion

$$f = \sqrt{\frac{3}{2}(\mathbf{s}-\mathbf{a}):(\mathbf{s}-\mathbf{a})} - Y = 0\,,\ Y = \sigma_Y + R\,, \tag{1}$$

the associative flow rule

$$d\boldsymbol{\varepsilon}_p = d\lambda \frac{\partial f}{\partial \boldsymbol{\sigma}}\,, \tag{2}$$

the kinematic hardening rule

$$d\mathbf{a} = g\left(\boldsymbol{\sigma},\mathbf{a},\boldsymbol{\varepsilon}_p,d\boldsymbol{\sigma},d\boldsymbol{\varepsilon}_p,etc.\right), \tag{3}$$

and the isotropic hardening rule

$$dR = h\left(R,\mathrm{dp},\boldsymbol{\sigma},\mathbf{a},\boldsymbol{\varepsilon}_p,\boldsymbol{\varepsilon}_p,etc.\right), \tag{4}$$

where

s - the deviatoric part of stress tensor $\boldsymbol{\sigma}$,

a - the deviatoric part of back-stress $\boldsymbol{\alpha}$,

Y – corresponds to the current size of the yield surface,

R - the isotropic variable,

σ_Y - the initial size of the yield surface,

$\boldsymbol{\varepsilon}_p$ - the plastic strain tensor and

$d\lambda$ - the plastic multiplier, which corresponds to the equivalent plastic strain increment dp.

The symbol: denotes the inner product between tensors ($\mathbf{x}:\mathbf{y} = x_{ij}y_{ij}$).

3 POSSIBILITIES OF CLASSICAL CYCLIC PLASTICITY MODELS

The most important for cyclic plasticity models is kinematic hardening rule. Chaboche model which is often implemented in FE codes is based on the superposition of particular backstress parts

$$\boldsymbol{a} = \sum_{i=1}^{M} \boldsymbol{a}_i \, , \;\; d\boldsymbol{a}_i = \frac{2}{3} C_i d\boldsymbol{\varepsilon}_p - \gamma_i \boldsymbol{a}_i dp \, , \tag{5}$$

where

C_i - material parameter,

γ_i - material parameter.

In the case of cyclic plasticity modeling, parameters can be identified using a large uniaxial hysteresis loop or cyclic strain curve of the investigated material. Unfortunately, accuracy of the Chaboche model is poor in the case of a material which exhibits Non-Masing's behaviour. This fact is presented by predictions of the Chaboche model with three backstress parts (M = 3) for the ST52 steel considering both ways of model calibration in the Fig. 1. In the case of calibration using large hysteresis loop the cyclic strain curve is not correctly captured and vice versa [5]. The material parameters used in these simulations are stated in the Tab. 1. Similar results can be obtained also for a multilinear material model with kinematic hardening, for example the model of Besseling (MKIN in Ansys), so the mentioned conclusions are valid for this class of classical hardening models too [6].

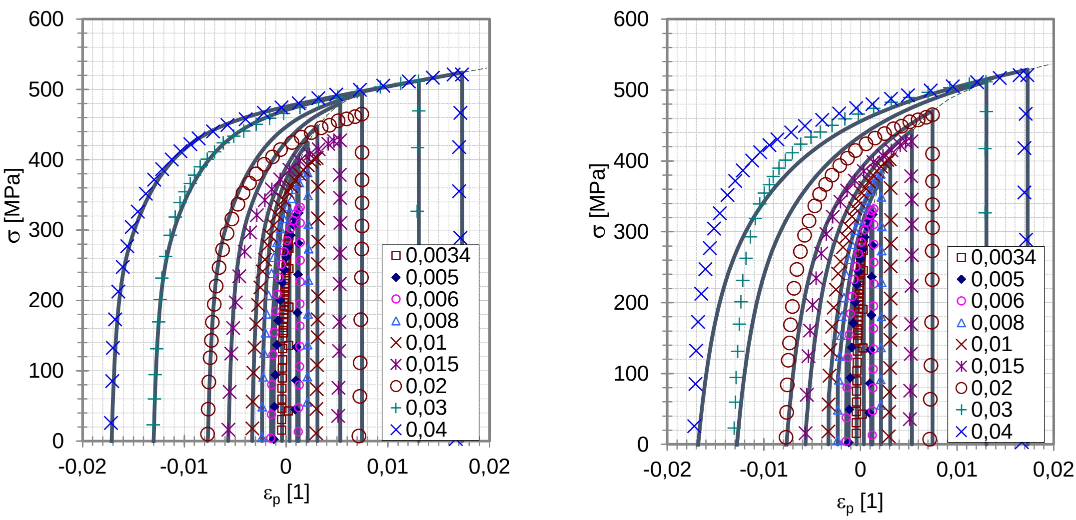

Fig. 1 Simulation of uniaxial hysteresis loops with various strain ranges by Chaboche model calibrated from: a) hysteresis loop, b) cyclic strain curve

Table 1 Material parameters of Chaboche model

Calibration from	*parameters*
hysteresis loop	$\sigma_Y = 250 MPa, C_{1-3} = 250000, 34860, 2670 \; MPa$ $\gamma_{1-3} = 2500, 273, 0$
cyclic strain curve	$\sigma_Y = 235 MPa, C_{1-3} = 67800, 20763, 2670 \; MPa$ $\gamma_{1-3} = 694, 136, 0$

4 MEMORY SURFACE CONCEPTS

As noted in introduction, there are basically two concepts of memory surfaces. The pioneer model, which states a memory surface in plastic strain space, is the model of Chaboche [1].

New internal variables $\boldsymbol{\zeta}$ and ρ define the position and the radius of the cyclic non-hardening region introduced in the plastic strain space

$$F = \frac{2}{3}\sqrt{\frac{3}{2}\left(\boldsymbol{\varepsilon_p} - \boldsymbol{\zeta}\right) : \left(\boldsymbol{\varepsilon_p} - \boldsymbol{\zeta}\right)} - \rho \le 0 \ . \tag{6}$$

If the point corresponding to the current plastic strain state is inside the region, then its position and its size are fixed. The evolution of the cyclic non-hardening region occurs only in the case of F=0 and $\frac{\partial F}{\partial \boldsymbol{\varepsilon_p}} : d\boldsymbol{\varepsilon_p} > 0$.

The works of Chaboche [1] and Ohno [4] lead to the generalization of evolution rules for the new variables, thus

$$d\rho = \eta \ H\ (F)\ \langle \boldsymbol{n} : \boldsymbol{n}^* \rangle dp \ , \tag{7}$$

$$d\boldsymbol{\zeta} = \frac{\sqrt{3}}{2}(1 - \eta)\ H\ (F)\ \langle \boldsymbol{n} : \boldsymbol{n}^* \rangle \boldsymbol{n}^* dp \ , \tag{8}$$

where the unit normal vectors are defined as follows

$$\boldsymbol{n} = \frac{\partial f}{\partial \boldsymbol{\sigma}} = \sqrt{\frac{2}{3}}\ \frac{d\boldsymbol{\varepsilon_p}}{dp} = \sqrt{\frac{2}{3}}\ \frac{\boldsymbol{s} - \boldsymbol{a}}{Y - R} \ , \tag{9}$$

$$\boldsymbol{n}^* = \frac{\partial F}{\partial \boldsymbol{\varepsilon_p}} = \sqrt{\frac{2}{3}}\frac{\boldsymbol{\varepsilon_p} - \boldsymbol{\zeta}}{\rho} \boldsymbol{.} \tag{10}$$

Finally, the new variable ρ influences the limit value of isotropic variable R in the nonlinear isotropic hardening rule

$$dR = b(Q(\rho) - R)dp \ , \tag{11}$$

$$Q(\rho) = Q_M - (Q_M - Q_0)e^{-2\beta\rho} \ , \tag{12}$$

where

Q_0 and Q_M are initial, and limit values of Q, respectively,

β is an evolution parameter.

One of the representatives of the memory surfaces established in stress space is the concept of Jiang and Sehitoglu [2], which introduces a scalar function to represent the memory surface in the deviatoric stress space

$$\boldsymbol{g} = \|\boldsymbol{\alpha}\| - R_M \le 0 \ , \tag{13}$$

where

$\|\boldsymbol{\alpha}\|$ is the magnitude of the total backstress $\|\boldsymbol{\alpha}\|$, which is defined as $\|\boldsymbol{\alpha}\| = \sqrt{\boldsymbol{\alpha} : \boldsymbol{\alpha}}$.

The evolution for the radius of memory surface R_M is

$$\boldsymbol{d}R_M = H(g)\ \langle \frac{\boldsymbol{\alpha}}{\|\boldsymbol{\alpha}\|} : d\boldsymbol{\alpha} \rangle - c_M\ \langle 1 - \frac{|\boldsymbol{\alpha}|}{R_M} \rangle\ dp \ , \tag{14}$$

where

c_M is a parameter influencing rate of memory surface contraction.

In this paper we incorporate the new variable R_M directly into the nonlinear isotropic hardening rule in the same way as for model with Chaboche memory surface

$$dR = b(Q(R_M) - R)dp \, , \tag{15}$$

$$Q(R_M) = \sigma_Y a_k e^{c_k R_M} \, . \tag{16}$$

Firstly, the parameters σ_Y, C_i, γ_i are identified from a large hysteresis loop.

To determine parameters of equations (12) and (16) we have to calculate data for fitting from cyclic strain curve:

$$Q(\varepsilon_{ap}) = \sigma_a(\varepsilon_{ap}) - \sum_{i=1}^{M} \frac{C_i}{\gamma_i} \tanh(\gamma_i \varepsilon_{ap}) \, . \tag{17}$$

Therefore main parameters are determined for example by a nonlinear least-squares method using approximation function (12) for Chaboche memory surface and equation (16) for Jiang-Sehitoglu memory surface, which can be improved for uniaxial symmetric strain loading

$$Q(\varepsilon_{ap}) = \sigma_Y a_k e^{c_k \sum_{i=1}^{M} \frac{C_i}{\gamma_i} \tanh(\gamma_i \varepsilon_{ap})} \, . \tag{18}$$

Described way of calibration leads to sufficient description of upper parts of hysteresis loops and cyclic strain curve as can be seen in the Fig. 2. It is clear, that Chaboche model can fit experimental data better, because of three parameters in comparison with Jiang-Sehitoglu model having two parameters only. Results of both approximations are presented graphically at the Fig. 3.

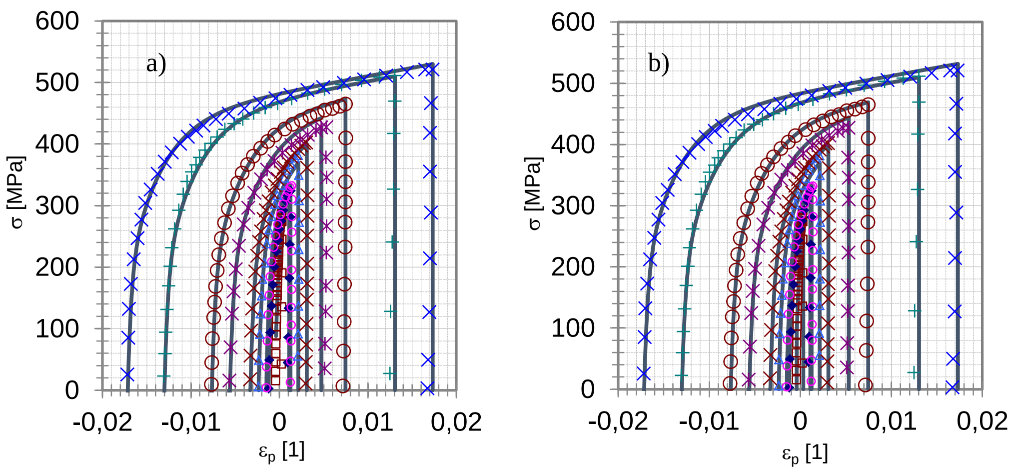

Fig. 2 Simulation of uniaxial hysteresis loops with various strain ranges by Chaboche model with (symbols-experiment):
a) Chaboche memory surface, b) Jiang-Sehitoglu memory surface

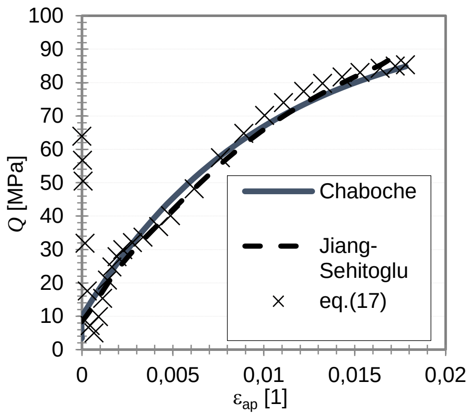

Fig. 3 Resulting approximations

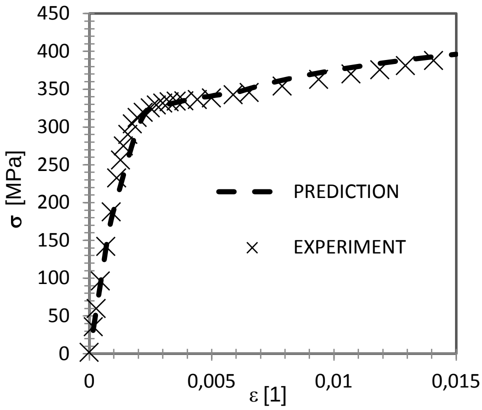

Fig. 4 Monotonic curve prediction

5 MODELLING OF CYCLIC HARDENING/SOFTENING BEHAVIOUR

Mainly stabilized behaviour is important for technical practice, but for more accurate fatigue analysis is necessary to calculate damage cycle by cycle considering transient effects in initial cycles. The investigated steel shows kinematic (cyclic softening/hardening) and isotropic hardening (Non-Masing's behaviour), so the memory parameter should appear also in evolution equation for kinematic hardening variables.

The presented model is based on the Jiang and Sehitoglu memory surface. We consider evolution rule of Marquis, thus

$$\mathbf{a} = \sum_{i=1}^{M=3} \mathbf{a}_i \, , \; d\mathbf{a}_i = \frac{2}{3} C_i d\boldsymbol{\varepsilon}_p - \varphi(p)\gamma_i \mathbf{a}_i dp \, , \tag{19}$$

$$\varphi(p) = 1 - (1 - \varphi_0(R_M)) \cdot e^{-\omega p} \, , \tag{20}$$

where ω is an evolution parameter and we define these suitable functions

$$\varphi_0(R_M) = \varphi_A \quad \text{for} \quad R_M \le R_M^L \quad , \tag{21}$$

$$\varphi_0(R_M) = 1.5 - 0.0017 \cdot R_M \quad \text{for} \quad R_M \ge R_M^P \quad , \tag{22}$$

$$\varphi_0(R_M) = \varphi_A + \varphi_B R_M + \varphi_C R_M^2 + \varphi_D R_M^3 \quad \text{otherwise.} \tag{23}$$

The saturated value of $\varphi(p)$ is 1 and dependency of φ_0 on the memory surface makes possible good description of the monotonic strain curve, see Fig. 4. Parameters obtained by Levenberg-Marquardt algorithm based on the nonlinear least-squares method for both models are shown in the Table 3.

The parameter γ_3 can be chosen higher than zero to predict ratcheting or mean stress relaxation behaviour of the material. The parameter *b* influencing the cyclic hardening rate at the equation (11) was estimated using a standard technique reported elsewhere [7].

Table 2 Material parameters of both models with memory

concept of memory	*parameters*
Chaboche	σ_Y=170MPa, C_1=2.5×10^{5}MPa, γ_1=2500, C_2=34860MPa, γ_2=273, C_3=1500MPa, γ_3=0, b=10, Q_0=3.6MPa , Q_M=99.3MPa,β=57
Jiang-Sehitoglu	σ_Y=170MPa, C_1=2.5×10^{5}MPa, γ_1=2500, C_2=34860MPa, γ_2=273, C_3=1500MPa, γ_3=0, ω=20, b=10, a_k=0.03, c_k=0.007918, φ_A=0.135, φ_B=0.000513, φ_C=-4.28×10^{-5}, φ_D=4.31×10^{-7}, $R_M^L=85$, $R_M^P=175$

6 CONCLUSIONS

The two memory surface concepts have been applied to the stress-strain description of ST52 steel to show its calibration. The Chaboche model including a superposition of three backstress parts makes possible very good representation of the cyclic strain curve or the shape of hysteresis loop but not simultaneously. As has been presented, the Chaboche model combined with a nonlinear isotropic hardening rule with the limit value of isotropic variable dependent on the size of a memory surface leads to the excellent description of both. Moreover, the actual cyclic hardening/softening behaviour can be modeled properly even under sequential loading. The Jiang-Sehitoglu memory surface has at least one advantage in comparison with the Chaboche model. The parameter R_M can be considered for ratcheting prediction improvement, because it can be associated with the current amplitude of loading.

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