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APPLICATION OF SBRA METHOD IN MECHANICS OF CONTINETAL PLATES

APLIKACE METODY SBRA V GEOMECHANICE KONTINENTÁLNÍCH DESEK

Abstract

This paper shows the probabilistic SBRA Method application to the model of the behaviour of the lithosphere of the Earth. The method extends our initial work where we created the geomechanical model of the lithosphere. The basic idea was about the generation of thermoelastic waves due to thermal expansion of the rock mass and the ratcheting mechanisms.

Abstrakt

Cílem práce je ukázat možnost použití pravděpodobnostní metody SBRA při modelování chování litosféry Země. Metoda rozšiřuje naše úvodní práce na vytváření geomechanického modelu litosféry. Základní myšlenka je postavena na teorii termoelastické vlny, jejíž chování je závislé především na teplotní roztažnosti masívu a západkovém mechanismu.

1 INTRODUCTION

Our work extends the works of M. Hvozdara et al. [3] and J. Berger [7], who detected and described the behavior of the thermoelastic wave, and the work of J.Croll [8], who described the ratcheting mechanism.

In the first step, we created the geomechanical model of the lithosphere, where we tested the directions of the relative expansion of the lithosphere plate in two places. This model assumes that the main part of the deformation depends on the solar irradiation. We used the simplified mathematical model which consists of linear differential equation focused to the Eurasian continental plate. The probabilistic SBRA method showed the possibility to simulate some physical quantities and the limits of the linear model where the non-linear behavior and the ratcheting begin.

2 NUMERICAL MODEL, PROCESS OF CALCULATION OF STRAIN ON BOUNDARIES OF THE CONTINENTAL PLATE

The temperature profile in continental rocks was calculated in one-day steps and in one-year cycles for each latitude. The same temperature profile was calculated in 30-minutes steps in one-day cycles. Both cycles were superimposed and the relative temperature development was calculated in 30-minutes steps during one year.

Than the relative strains ε_{xx} , ε_{yy} and ε_{zz} /1/ in the far field under a surface can be evaluated as an integral (or they are directly proportional to this integral) of temperature (depth) profile multiplied by linear thermal expansion coefficient α /K^{-1}/. Because the attenuation of thermal wave with the depth is high (due to low thermal expansion of rock), the far field is supposed to be in order of one kilometre outside the expanded block of lithosphere. We are able to evaluate the equivalent

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(normalised) relative strains of each block and to evaluate the principal component of stress tensor and its relative development in time. It depends mainly on the geometry and geographical position of the continents. We calculated the relative values of the principal component of the stress tensor.

The maximum and minimum strain in diurnal period was evaluated and the annual strain development was calculated for the points on the border of continents. Japan (140.625E, 50N) and Italy (16.47E, 40N) were chosen as examples. The principal component of relative strain changes its direction in time in both cases. At the end of March the direction is towards the continent. It is the result of the contraction of the Eurasian lithosphere plate after winter. In the case of Italy, it is in the direction of the NE and, in the case of Japan, it is in the direction of the NW. The opposite direction can be seen in September. This is the result of the expansion of the Eurasian lithosphere plate after summer. The results are in accordance with the field GPS measurement of continental deformations at both places.

Cyclic variations of temperature $\vartheta_{(h,t)}$ /K/ close to Earth‘s surface can be evaluated for homogeneous isotropic environment by equation:

$$\vartheta_{(h,t)} = \vartheta_0 e^{-h\sqrt{\omega/2a}} \cos\left(\omega t - h\sqrt{\frac{\omega}{2a}}\right), \tag{2.1}$$

where ϑ_0 /K/ is an amplitude of temperature variations on the Earth’s surface as a function of time t /s/, h /m/ is depth, $\omega = 2\pi/\tau$ /s^{-1}/ (τ /s/ is period), $a \in < 7.10^{-7};\ 22.10^{-7} >$ /m^{2}s^{-1}/ is temperature conductance coefficient.

The character of the stress tensor behavior of rock is entirely dependent on temperature variations in layers close to the surface due to the small velocity of the temperature penetration into depth and its large attenuation. The equivalent (normalised) relative strain at point is given:

$$\varepsilon \approx \int_0^H \alpha\ \Delta\vartheta_{(t,h)}\ dh\ , \tag{2.2}$$

H /m/ is depth, which the heat penetrates during five periods with $a\ =\ 13.10^{-7} m^2 s^{-1}$ (mean value). To estimate the ratio between the lowest and the highest strain and their direction at the border of the continent it will be sufficient to simplify the function $\alpha\ \Delta\vartheta_{(t,h)}$ inside the integral (2.2) in this way:

$$\Delta\bar{\vartheta}_{(t,h)} = \frac{T_{(t,h)} - T_s}{T_s}\ e^{-h\sqrt{\omega/2a}}, \tag{2.3}$$

T_s /K/ is the mean temperature on the surface, $T_{(t,h)}$ /K/ is the relative temperature curve on the surface:

$$T_{(t,h)} = T_s \cos\left(\omega t - h\sqrt{\frac{\omega}{2a}}\right), \tag{2.4}$$

the term $-h\sqrt{\omega/2a}$ represents the delay of the temperature variations at the depth h with the respect to the variations on the surface. When we substitute (2.4) into the relationship (2.3), we obtain:

$$\Delta\bar{\vartheta}_{(t,h)} = e^{-h\sqrt{\omega/2a}} \cos\left(\omega t - h\sqrt{\frac{\omega}{2a}}\right). \tag{2.5}$$

When we substitute (2.5) into the integral (2.2), we obtain:

$$\varepsilon \approx \int_0^H e^{-h\sqrt{\omega/2a}} \cos\left(\omega t - h\sqrt{\frac{\omega}{2a}}\right) dh\ . \tag{2.6}$$

An primitive (antiderivative) function of the integral (2.6) is:

$$F(x) = \frac{e^{-h\sqrt{\omega/2a}}}{2\sqrt{\omega/2a}} \left(\sin(h\sqrt{\omega/2a} - \omega t) - \cos(h\sqrt{\omega/2a} - \omega t)\right) + C\ . \tag{2.7}$$

Due to a fast attenuation the enumeration was only done for five annual periods, which means $H = 113{,}48799\ \ m$.

3 THE PROBABILISTIC SBRA METHOD APPLICATION (ANTHILL SOFTWARE)

The first approximate calculation was modified in this way. We added a random variable, which simulated a temperature conductance coefficient a /m^2s^{-1}/ (see figure 3.1). The relationship (2.6) describes enumeration of the variable ε - the equivalent (normalised) relative strain**.** For this very reason, it is necessary to determine an initial condition – the actual time. The temperature field close to the surface is dependent on the temperature variations at the surface. It may be assumed that in the case of constant surface temperature, the strain and the evaluated value ε will tend towards zero. We made a simulation of the behaviour of this variable for the diurnal period. For the relationship (2.7), the random variable *t* was used with uniform distribution in the range <0; 86 400> /s/. Therefore, according to the relationship (2.7) we obtain:

$$\varepsilon_{vyp} = F(H) - F(0) , \qquad (3.1)$$

where H /m/ is the depth, to which the heat penetrates during five periods. The value of the parameter H depends on the temperature conductance coefficient a; the example of generated values are in Fig. 3.2. and Fig. 3.3 shows the relationships of the calculation of the reliability (SW AntHill).

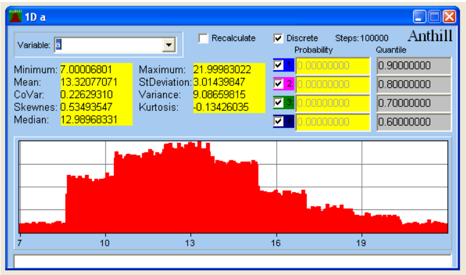

Fig. 3.1 The temperature conductance coefficient a

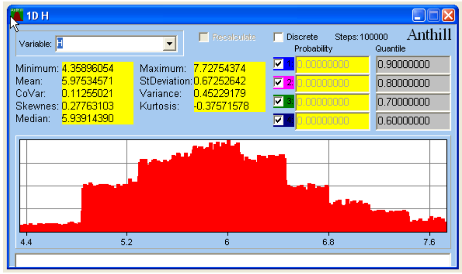

Fig. 3.2 Values of H - depth of temperature field

The behaviour of the variable ε_{vyp} is shown in figure 3.4, which shows that the strain inside the rocks varies during the diurnal period. Although the relative values are shown they respect the dynamics of the model well (maximum and minimum). We added the new variable “epsvyp0“ to show the behaviour of strain, which corresponds to the relationship (3.1).

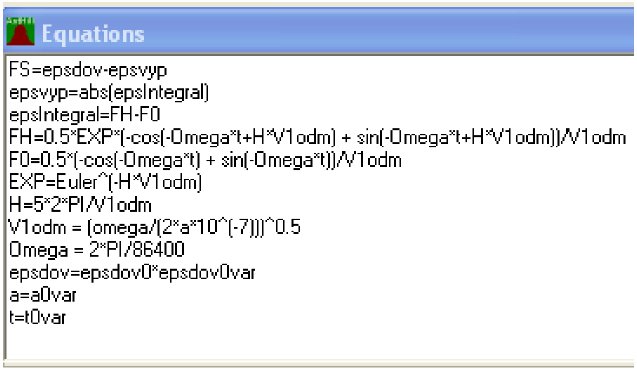

Fig. 3.3 Calculation description.

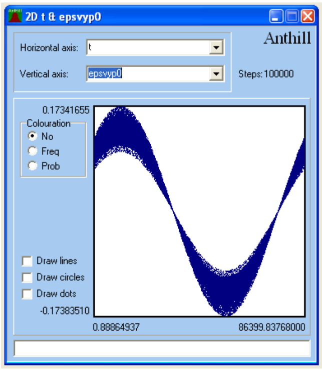

Fig. 3.4 Progression ε_{vyp} during diurnal period.

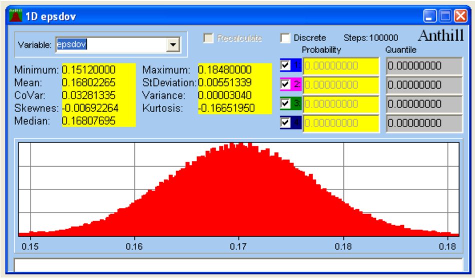

Fig. 3.5 Distribution of critical values ε_{dov} .

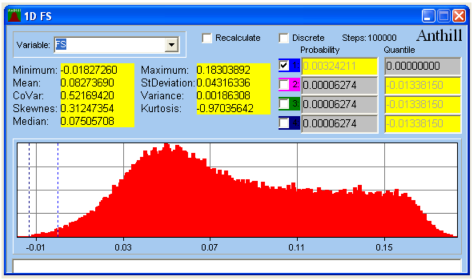

Fig. 3.6 Reliability function F_S.

There are two extremes in Fig.3.5. Both of them show the situation, when the model is approaching its limits when the linear law ceases to apply and the non-linear behaviour starts. One extreme describes the situation when the stress is approaching the ultimate compressive strength of the massif and the second describes the situation when the stress is approaching the ultimate tensile strength. The first of them, in a long perspective, leads to the creep or seismic events. The second extreme leads to the opening of the micro-cracks, cracks or faults, when ratcheting can occur.

To get a more accurate calculation, it will be necessary to take into account the dynamics of the strain growth and the relaxation of rocks for example a massif non-linearity, hysteresis or ratcheting. This is the subject of further work.

The distribution of frequencies corresponds to the chosen equation for computation of the temperature field close to the Earth's surface. To simulate a surface temperature, the model of the ideal periodic function was used. While, in this paper the ideal temperature function was used (defined by goniometric functions), the real temperature curves must be used in the real approach. These curves have asymmetric forms when the minimum of the diurnal temperature can be determined at the moment of the sunrise and the maximum can be determined approximately one hour after noon or we can use the real temperature development curves. This means that the heating of the Earth's surface is considerably faster then the cooling.

We used the numeric integration method for the evaluation of the temperature. If the surface temperature is constant then the integral will tend towards zero. Therefore, the computation was modified and the new variable $|\varepsilon_{vyp}|$ was set up.

Figure 3.5 shows the distribution of variable ε_{dov} minus critical value of the variable ε_{vyp} .

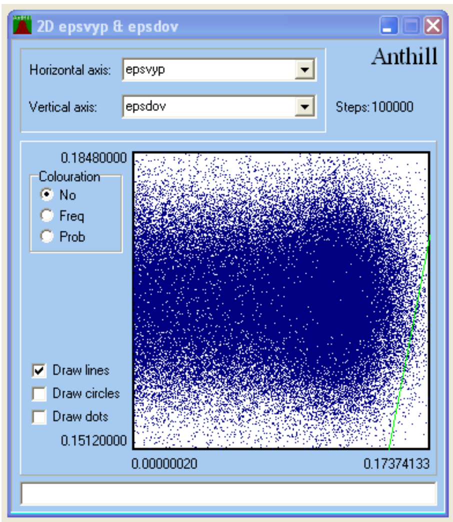

Fig. 3.7 Distribution admissible values ε_{dov} and critical values ε_{vyp}.

The value ε_{dov} was estimated by probabilistic approach (see figure 3.5). In this way the reliability function F_S could be estimated (see figure 3.6):

$$F_S = \varepsilon_{dov} - |\varepsilon_{vyp}| \, . \tag{3.2}$$

We also verified the methodology of the probabilistic SBRA method in the real geological situation and the number of simulations of 10^5 gave a fair result.

The probability of micro-movement P_f (i.e. the probability of the irreversible effect when $F_S < 0$) was evaluated as 0.00324 , i.e. 0.324%. The figures 3.6 and 3.7 show the principles of determining this value.

4 CONCLUSIONS

The aim of this paper was to test the possibility of using probabilistic calculations for modeling the behavior of the lithosphere of the Earth (the primary methodology for setting the limit values, which leads to the irreversible movement of the lithospheric plates - creep). We can say that the method can be used for the modeling of the massif, when its parameters do not enter as mean values, but by using Monte Carlo method. The method could be used directly inside the geomechanical model.

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