

Dušan TEICHMANN*, Markéta HLAVSOVÁ**, Erika KONUPČÍKOVÁ***

SET COVERING PROBLEM IN TRANSPORT NETS EXPANDING MODELS

ROZŠIŘUJÍCÍ MODEL Y ÚLOHY O POKRYTÍ VRCHOLŮ DOPRAVNÍ SÍŤE

Abstract

The article deals with the problem of optimal coverage of utility centres in transport network. We can often encounter similar types of tasks in a real life. A lot of cases relate to the public utility systems that mean systems intended for providing basic public services to all population in a certain territory. These services can include services of Integrated Rescue System. The tasks of this category are called D_{\max} tiling tasks in a scientific literature. The classic version of D_{\max} cover task is known for quite a long time but its shape does not always cover real traffic conditions. The aim of the article is to show other modifications of the basic variation of the tasks. The authors expect the proposed application will enable mathematical apparatus to be used to solve problems of this type to the greater extent.

Abstrakt

Článek se zabývá problémem optimálního pokrytí dopravní sítě obslužnými středisky. S podobnými typy úloh se můžeme setkat v reálném životě velice často. Velké množství případů se vztahuje k tzv. veřejným obslužným systémům, tj. systémům, které jsou určeny k zajištění základních veřejných služeb poskytovaných všemu obyvatelstvu na určitém území. Takovými službami mohou být např. služby integrovaného záchranného systému. Úlohy této kategorie se v odborné literatuře označují jako D_{\max} pokrývací úlohy. Klasická varianta D_{\max} pokrývací úlohy je známa poměrně dlouhou dobu, svým tvarem však vždy nevyhovuje podmínkám reálného provozu. Článek si klade za cíl ukázat další vybrané modifikace základní varianty úlohy. Autoři očekávají, že navržené aplikace umožní využívat matematický aparát k řešení tohoto typu úloh ve větší míře.

1 INTRODUCTION

Availability to the general public is a standard of efficiency for services provided by the state. In this context a fact is discussed that public services (education, health, transport and many other services that are guaranteed by state) must be available in the required time. These and similar tasks are possible to divide in the point of view of modelling into several categories. Further in the text the authors will deal with security service sites in the transport network - the coverage tasks. Some types of coverage tasks are formulated in publications [2], [4], [5] of this article [3]. In the article [3] there

* Ing. Dušan TEICHMANN, Ph.D., VŠB – Technical University of Ostrava, Fakulta strojní, Institut dopravy, VŠB - TU Ostrava, 17. listopadu 15, 708 33 Ostrava - Poruba, tel. (+420) 597 324 575, e-mail: dusan.teichmann@vsb.cz

** Ing. et Ing. Markéta HLAVSOVÁ, Vysoká škola chemicko-technologická, Fakulta chemicko-inženýrská, Technická 5, 166 28 Praha 6 - Dejvice (+420) 581 259 144, e-mail: marketa.hlavsova@vslg.cz

*** Mgr. Erika KONUPČÍKOVÁ, VŠB – Technical University of Ostrava, Fakulta metalurgie a materiálového inženýrství, 17. listopadu 15, 708 33 Ostrava - Poruba, tel. (+420) 581 259 135, e-mail: erika.konupcikova@vslg.cz

is an important application shown - the use of D_{max} coverage task in draft of designing emergency services in Slovakia.

Let us now formulate the basic type of D_{max} coverage task.

2 FORMULATION OF THE BASIC TYPES OF TASKS

A transport network is given in which there are two sets of vertices (they can or cannot be disjoint). The set of vertices I which you can realize the operator from (here in after referred to "locations") and set of vertices J that require operation (here in after referred to as "customers"). We know the distance d_{ij} (or other information characterizing availability) from each location $i \in I$ to every customer $j \in J$. It is also defined value D_{max} that represents the maximum limit of availability. If $d_{ij} \leq D_{max}$ is true we say that the customer $j \in J$ is available from the location $i \in I$. If $d_{ij} > D_{max}$ is true we say that the customer $j \in J$ is not available from the location $i \in I$. Assuming at this phase and even in other variations to the model that each customer can be covered by at least one location; the researchers expect to determine the locations in which service centres should be provided. The requirements are that each customer is covered by a minimum of one centre and the total number of operation centres is minimal.

If decisions are expected to be formulated by the authors; a decision bivalent variable y_i has to be implemented into the task

Therefore together with a common used convention the following can be applied: if $y_i = 1$, then the operation centre will be provided in the locality $i \in I$, if $y_i = 0$, then the operation centre will not be in the locality $i \in I$. Because we implement bivalent variable into each locality, the number of the decisive variables is the same as the number of the localities. Thy symbol I_j describes a set of localities where the customer $j \in J$ is available. Now, we can formulate the model of optimization. Mathematical formulation for a basic type of optimization model D_{max} of the tilling task is the following:

$$\min f(y) = \sum_{i \in I} y_i \tag{1}$$

subject to

$$\sum_{i \in I_j} y_i \geq 1 \text{ for } j \in J \tag{2}$$

$$y_i \in \{0;1\} \text{ for } i \in I \tag{3}$$

The expression (1) represents an objective function – total number of operating utility centres in localities. The group of the constraints (2) ensures that each customer will be covered from at least one locality, where the utility centre will operate. The number of constraints in the group represents the number of customers. The group of constraints (3) declares domains of variables.

To avoid too complicated entry of the model into the optimization software, it is more suitable to introduce incidence matrix A . In case $a_{ij} = 1$ the customer $j \in J$ is available from the locality $i \in I$. In case $a_{ij} = 0$, then the customer $j \in J$ is not available from the locality $i \in I$.

The group of restricted conditions (2) will consequently come to the format:

$$\sum_{i \in I} a_{ij} y_i \geq 1 \quad \text{for } j \in J \quad (4)$$

The following chapter will deal with some modifications of the basic format to the task.

3 MODIFICATIONS OF THE BASIC TASKS

The content of the chapter 3 includes three modifications of the original task. The aim is to demonstrate further usability of this type of tasks for practical solutions.

Option 1 – limited amount of operating costs

Let us suppose we use the entry formulated in chapter 2. The difference to the entry is a presumption of the fact that operating the centre in locality $i \in I$ will raise a certain amount of operating costs f_i . Total amount of costs available for operating the utility centres is F . Additional data input initializes creating a restriction which ensures the total amount of operational costs in utility centres will not exceed the limited amount of F .

Mathematical formulation of D_{\max} optimization model covering tasks with a limited operating expense for the centre is the following:

$$\min f(y) = \sum_{i \in I} y_i \quad (5)$$

subject to:

$$\sum_{i \in I} a_{ij} y_i \geq 1 \quad \text{for } j \in J \quad (6)$$

$$\sum_{i \in I} f_i y_i \leq F \quad (7)$$

$$y_i \in \{0,1\} \quad \text{for } i \in I \quad (8)$$

Meanings of the objective function and groups restricting conditions (6) and (8) are the same as in the basic variant. Constraint (7) is the only one and ensures compliance of the additional condition, that means ensures that the costs for operating centres do not exceed the available limited amount.

The authors consider correct to point out that the variant 1 of the model has one significant drawback. As the available amount intended to operate the service centres may not be sufficient to ensure that each customer will be covered from at least one operated centre. In that case the solving algorithm evaluates the situation in the way the set of acceptable solutions is empty and the calculation is finished, without any proposed suitable solution. If we want to prepare for the situation, ensure solvability and at the same time also determine the minimal amount to be used to ensure basic performance of the solved system we need to modify the model. A new variable must be introduced, let us mark it z . The new variable signals the amount which the limited amount F must be higher of, to ensure performance of the solved system.

The constraint (7) comes to the form of (9)

$$\sum_{i \in I} f_i y_i \leq F + z \quad (9)$$

and the original objective function comes to the form of scalar multi-objective function (10)

$$\min f(y, z) = T \sum_{i \in I} y_i + z \quad (10)$$

Objective function defined that way causes the value of the variant z will be stated as a minimum necessary to fulfil the restricted condition (9). If all customers are successfully covered within the available limited amount F , the objective function (10) defines that $z = 0$. If we do not add the variable z , the value of the objective function would be independent to the variable z and the solved algorithm would not be motivated to state its value as a necessary minimum (the amount necessary to fulfil the condition might be distorted).

In order to emphasize the importance of the first part of the objective function, the element was multiplied by suitable so-called prohibiting constant (justification of the step is provided further in the text).

Missing obligatory condition for the variable z must of course be added, that is:

$$z \geq 0 \quad (11)$$

It is important to realize, the value of the objective function (10) does not have any unambiguous economic importance as values in the objective function included are economically inconsistent (the expression $\sum_{i \in I} y_i$ represents the number of centres operated and the expression

z financial amount needed to spend for necessary coverage of all customers). As a result we need to analyse the value of the objective function according to particular units and consequently interpret them individually.

Option 2 – minimizing total operating costs in the centres

Let us suppose we use the entry formulated in chapter 2. The difference to the entry is a presumption of the fact that operating the centre in locality $i \in I$ will rise operational costs valued f_i . Our task is to decide which localities should be used to operate the utility centres in order to cover each customer by at least one centre and the total cost related to the services are minimized.

The difference between the option 1 and option 2 is the fact that there is no need to add any more variants nor constraints in 2. But we need to change the objective function as it should not express the number of operation centres but the total costs related to the operation.

Mathematical formulation of optimizing model D_{\max} covering tasks ensuring minimal total costs needed to operate the centres is as followed:

$$\min f(y) = \sum_{i \in I} f_i y_i \quad (12)$$

subject to

$$\sum_{i \in I} a_{ij} y_i \geq 1 \text{ for } j \in J \quad (13)$$

$$y_i \in \{0;1\} \text{ for } i \in I \quad (14)$$

Expression (12) represents a new objective function – total operating costs for centres, meanings of groups of condition restricting (13) and (14) are the same as the meanings of groups of condition restricting (3) and (4) in the basic option of the model.

Yet it remains to be noted that in the option 2 there is no risk, as it is in the option 1, of not being able to cover all the customers within the available limited amount of money. The option 2 works with the amount stated at minimal value.

Option 3 - multiple criteria D_{\max} tiling task

All above mentioned options of D_{\max} tiling task (with an exception including the z variant) were drawn up as single criterion. Single criterion option is not always considered to be the optimal

for a practical usage. The reason is usually because the practical usage requires multiple criteria to be taken into account. Our statement is declared in the following easy example.

There are two localities suitable for operating service centres and two customers, requiring the service we have to provide. In accordance to the specifications of the basic task we also have matrix of time availability of the customers from localities D .

$$D = \begin{pmatrix} 16 & 20 \\ 5 & 12 \end{pmatrix}$$

and value $D_{\max} = 20$.

At defined value $D_{\max} = 20$ it is immediately clear that in the basic option there are two optimal solutions. The first of them is a solution with the service centre to be placed to the L_1 locality, the second centre is placed in L_2 locality. For the objective function value we can apply

$\sum_{i=1}^2 y_i = 1$ in both cases. Both of the solutions seem to be of the same quality for optimizing

algorithm with conceived criterion in the form of $\sum_{i=1}^2 y_i$.

But it is obvious that better solution seems to be the one with the service centre located in L_2 locality as the sum of the time availability values of both serviced customers from the locality is better in both cases.

In case we want to take into account minimization of time availabilities when solving the task (of finding suitable localities), we also have to take into account the second objective function. The result of the thoughts will be two-criteria optimizing task.

In optimization tasks with more criteria, each of the criterion should be assigned a certain weight. Firstly it should be pointed out, that the weight to each criterion should be decided by a contractor. The weight of the criterion expresses the contractor's priority in solving the task. For this case we decide to give higher priority to the criterion meaning the number of operating centres.

The basic part of general formulation in option 3 task is not so far from formulation of the basic type task. The difference is in time availabilities which will appear explicitly, not through the incidence matrix A (as so far).

In our model we need to ensure that each customer is supplied from at least one service centre, further the allocation of a customer to the locality initializes operation of the centre in the locality. Formulation of the mentioned restrictions reminds of unlimited capacity location task, as for example in [1] publication. Apart from variants y_i modelling decisions about the service centres we also introduce another group of variants into the model - variants x_{ij} . Variant x_{ij} will be modelling decision about allocating (or not allocating) the customer $j \in J$ to the locality $i \in I$.

Mathematical model of optimization task will then be formulated in the following form:

$$\min f(y) = T \sum_{i \in I} y_i + \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \quad (15)$$

Subject to

$$\sum_{i \in I} a_{ij} x_{ij} \geq 1 \text{ for } j \in J \quad (16)$$

$$x_{ij} \leq y_i \text{ for } i \in I \ j \in J \quad (17)$$

$$x_{ij} \in \{0,1\} \text{ for } i \in I \ j \in J \quad (18)$$

$$y_i \in \{0,1\} \text{ for } i \in I \quad (19)$$

Expression (15) represents associated cost function. The element $\sum_{i \in I} y_i$, whose weight is emphasised by prohibited constant, again represents number of operated centres; the element $\sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij}$ represents aspects of time availability. The meaning of a group of restricting conditions (16) is the same as the one of the group of restricting conditions (4). The group of restricting conditions (17) ensures the fact that if we allocate a customer $j \in J$ to the locality $i \in I$, a centre will operate the locality $i \in I$. If there is no centre in the locality $i \in I$ no customer will be allocated there. The group of restricted conditions (18) and (19) defines the variable domains.

4 COMPUTATIONAL EXPERIMENTS WITH PROPOSED MODELS

Performance of all proposed model options was proved at a number of experiments. Numerical experiments were performed in optimization software Xpress-IVE and the difference was in the number of localities, and number of served customers – so called dimension of task. We used a demo version of the software, which is available free for academic purposes (Xpress). In the wider range tasks, exceeding the capacity of the demo version, we used the dynamic array function for the purpose of reducing the number of variants and constraints.

In numerical experiments with optimizing models we also have to observe their effectiveness that means what time is needed for reaching the optimal solution. Effectiveness is interesting for wider range tasks above all. Minimal range of task used for testing the calculation was 40 x 50; it means 40 locations and 50 customers. The time of calculation did not exceed 0.1 sec. for any of the proposed model. So we can suppose that all the options of the models are fast enough to the extent of the 40 x 50 range.

5 CONCLUSION

The article deals with issues of coverage tasks. It states the basic formulation of coverage tasks and three modifications to the basic model. Modifications include costs restrictions and costs criteria. One of the modifications consists of multi-criteria type of task. The case takes into account not only the basic criterion (number of operating centres) but also minimizing time availabilities.

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