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MATHEMATICAL SOLUTION OF A LINEAR HYDRAULIC SERVOMECHANISM

MATEMATICKÉ ŘEŠENÍ LINEÁRNÍHO HYDRAULICKÉHO SERVOPOHONU

Abstract

In this paper the mathematical model of linear hydraulic servomechanism for lifting heavy loads which was used to creation a mathematical model in Matlab Simulink and also Matlab SimHydraulics is described. Calculated results of piston rod velocity for various operating conditions are compared to each other in the section of results evaluation.

Abstrakt

Ve článku je popsán matematický model lineárního hydraulického servomechanismu zvedacího zařízení, který byl dále využit k tvorbě matematického modelu v prostředí Matlab Simulink a také v prostředí Matlab Simhydraulics. Získané výsledky simulace rychlosti pohybu pístnice za různých pracovních podmínek jsou vzájemně porovnány v kapitole týkající se vyhodnocení výsledků.

1 INTRODUCTION

Design of the hydraulic linear servomechanism for lifting heavy loads in steady state is not always enough, it is necessary to add the dynamic analysis of the system. This method is most often performed by using methods of mathematical modelling and computer simulation of dynamic behaviour of the investigated system now.

The mathematical model of the linear hydraulic servomechanism describes dynamic behaviour of the linear hydraulic servomechanism of the hydraulic circuit lifting device with the load on a rope (Fig. 1), and transfer a mathematical model onto the simulation model in Matlab - Simulink and realization computer simulation of selected operating conditions. The same circuit was built using new superstructures in Matlab, i.e. the use of problem-oriented toolboxes like Simscape - SimHydraulics. For practical purposes entering parameters of the circuit is easier and faster in this variant, but it is necessary to evaluate the quality of the results obtained by both approaches.

Lifting and lowering of the load is carried out by using the rope which is led over the block and tackle with 5:1 gear ratio to the hydraulic cylinder rod **HM**. Hydraulic motor is connected to the flow source, which is the pump **HG**. Velocity of extension and retraction of cylinder rod is controlled by changing speed of the pump **HG** by asynchronous electromotor with frequency converter. The object of modelling there is only a hydraulic drive with load.

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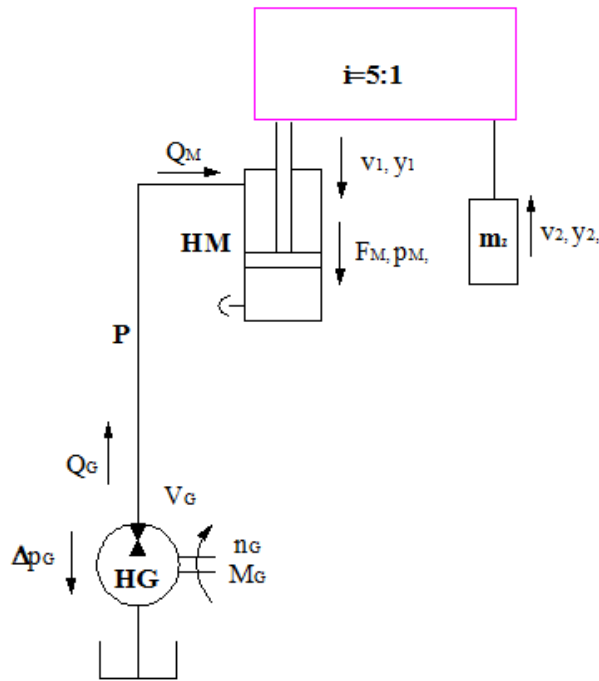


Fig. 1 Scheme of the hydraulic circuit.

HM – hydraulic cylinder, m_z – mass of load, **HG** – pump, **P** – pipeline, V_G – geometric volume of the pump, n_G – speed of the pump, M_G – torque, Q_G – pump flow, Q_M – cylinder flow, F_M – cylinder force, p_M – pressure at the cylinder inlet, y_1, v_1 – piston position and piston velocity, y_2, v_2 – load position and velocity, i – gear ratio

2 HYDRAULIC DRIVE DESIGN

Using known mass of load $m_z = 400$ kg the force on the piston rod of hydraulic cylinder F_M was calculated. For this value of the mass, was calculated the reduced mass m_R from equal of kinetic energies, eq. (1), (2), and based on the selected operating pressure has been designed with dimensions from hydraulic cylinder with dimensions $\phi D/\phi d$ – stroke $h = \phi 63/\phi 45$ – 600 mm.

$$F_M = i \cdot m_z \cdot g = 5 \cdot 400 \cdot 9,81 = 19620 \text{ N} \quad (1)$$

$$m_R = m_z \cdot \frac{v_z^2}{v_p^2} = 400 \cdot \frac{1^2}{0,2^2} = 10000 \text{ kg} \quad (2)$$

where:

m_z – mass of load [kg],

v_p – max. velocity of the piston rod HM, [m . s⁻¹]

v_z – max. velocity of the load, [m . s⁻¹].

From the specified piston rod velocity v_p has been calculated the pump flow $Q_{G,S} = 4,386 \cdot 10^{-4} \text{ m}^3 \cdot \text{s}^{-1}$, the pump HG was designed with geometric volume $V_G = 16 \text{ cm}^3$ and computed required pump speed n_G :

$$n_G = \frac{Q_{G,S}}{V_g \cdot \eta_{Q,G}} = \frac{4,286 \cdot 10^{-4}}{1,6 \cdot 10^{-5} \cdot 0,96} = 27,9 s^{-1} \quad (3)$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K \cdot S_M}{m_R \cdot y}} = \frac{1}{2\pi} \sqrt{\frac{1 \cdot 10^9 \cdot 2,1 \cdot 10^{-3}}{10000 \cdot 1,5}} = 1,88 Hz \quad (4)$$

where:

m_R – reduced mass of load, [kg],

K – bulk modulus [Pa],

S_M – piston area,

y – piston stroke [m].

2.1 Linear mathematical model

The liner model is composed from an ideal hydraulic components, i.e. resistance to motion of the liquid R , acceleration resistance H and resistance to deformation D , or their reciprocal values, i.e. the permeability $G = 1 / R$ and capacitance $C = 1 / D$. Those resistance components are used for creating the model of pump, hydraulic pipeline, single acting hydraulic cylinder and the load of mass. Such created model of hydraulic circuit shows the dynamic behaviour. The model of pipe takes into account only the pipeline capacity C_p , model of hydraulic cylinder takes into account the internal resistance to motion R_{VM} , internal hydraulic permeability G_M , the weight of the load (resistance H_M) and the compressibility of the liquid in the cylinder (capacity C_M). The pump mathematical model consists only from the model of ideal pump and internal pump permeability G_G . Capacities of pipeline and hydraulic cylinder are added together.

Fig. 2 shows the resistance scheme of the hydraulic circuit, which was used for the mathematical description in the form of a set of algebraic and differential equations. By means of these equations the simulation model in Matlab – Simulink was created.

Runge – Kutta fifth degree method was used for numerical simulation. The parameters of the numerical calculation were following: the minimum of time step was chosen to $1 \cdot 10^{-5}$, the maximum of time step is $1 \cdot 10^{-2}$ s the accuracy of calculating has the value of $1 \cdot 10^{-6}$. Thanks to openness of Simulink software it is possible to create clear hierarchical structure of very complicated systems too. Thus prepared structure is relatively well tuned progressively from the lowest to higher subsystems. Blocks and their parameters can be easily modified.

The mathematical model of the hydraulic circuit is based on the following equations

$$Q_{Gt} = Q_{Mt} + Q_{GZ} + Q_{MC} \quad (5)$$

$$Q_{MC} = Q_{Gt} - Q_{Mt} - Q_{GZ} \quad (6)$$

$$Q_{MC} = C_{M,p} \frac{d\Delta p_M}{dt} \quad (7)$$

From the equation (7) the pressure drop Δp_M was expressed

$$\Delta p_M = \frac{1}{C_M} \int Q_{MC} \cdot dt \quad (8)$$

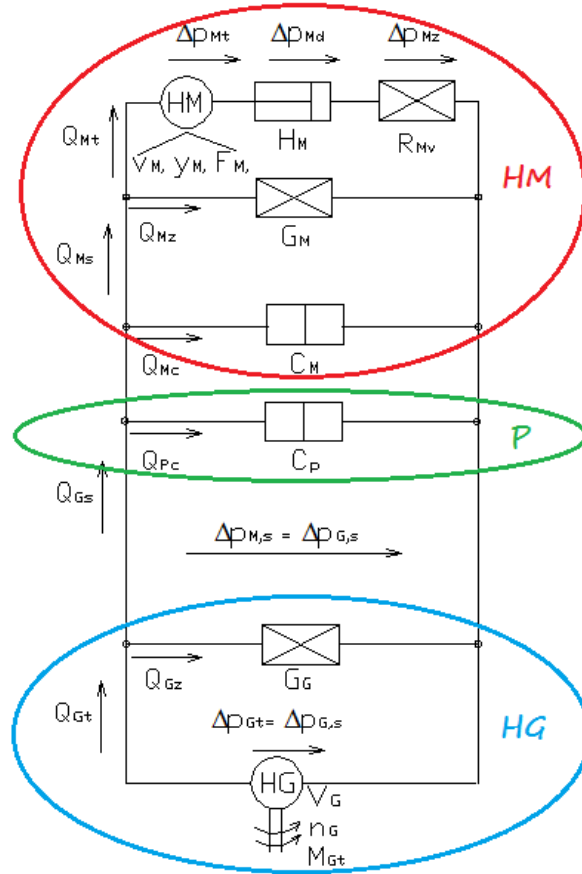


Fig. 2 The resistance scheme of the hydraulic circuit.

HM - ideal model of the hydraulic cylinder, **HG** – ideal model of the pump, **P** – pipeline model G_M – permeability of cylinder (leakage), G_G – permeability of pump (leakage), R_{VM} – resistance to motion (linear), n_G – pump speed, V_G – pump geometric volume, v_M – velocity of the piston rod, $y_M = y_l$ – piston stroke, F_M – force, Q_{Gt} – ideal pump flow, Q_{Mt} – ideal cylinder flow, $Q_{P,c}$, $Q_{M,c}$ – flow into capacities of pipeline and hydraulic cylinder, C_M , C_P – capacities of pipeline and hydraulic cylinder, Δp_M , Δp_G – pressure drop of the pump and the hydraulic cylinder, $\Delta p_{M,z}$, $\Delta p_{G,z}$ – the required pressure drop to overcome the internal frictional resistances at steady state, $\Delta p_{M,d}$ – pressure drop on cylinder resistance to acceleration (dynamic).

From the equation (8) flow loss Q_{MZ} was calculated

$$Q_{MZ} = \frac{\Delta p_M}{R_S} = G_M \cdot \Delta p_M \quad (9)$$

And ideal flow $Q_{M,t}$ was calculated from following equation

$$\Delta p_M = \Delta p_{M,t} + \Delta p_{M,d} + \Delta p_{M,z} \Rightarrow \Delta p_{M,d} = \Delta p_M - \Delta p_{M,t} - \Delta p_{M,z} \quad (10)$$

$$\Delta p_{M,d} = H_M \cdot \frac{dQ_{M,t}}{dt} \quad (11)$$

$$\Delta p_M - \frac{F_M}{S_M} - R_{V,M} \cdot Q_{M,t} \quad (12)$$

$$Q_{M,d} = \int \frac{\Delta p_{M,d}}{H_M} . dt \quad (13)$$

2.2 Simulation in Simulink (software)

Simulation model of the hydraulic circuit of the linear hydraulic servomechanism was created in Matlab – Simulink (software) according to the set of equation (5) – (13). The model was supplemented by the calculated values of hydraulic resistors that we consider linear. The input parameters are speed n_G and force F_M . Output variables can be monitored by adding "scope".

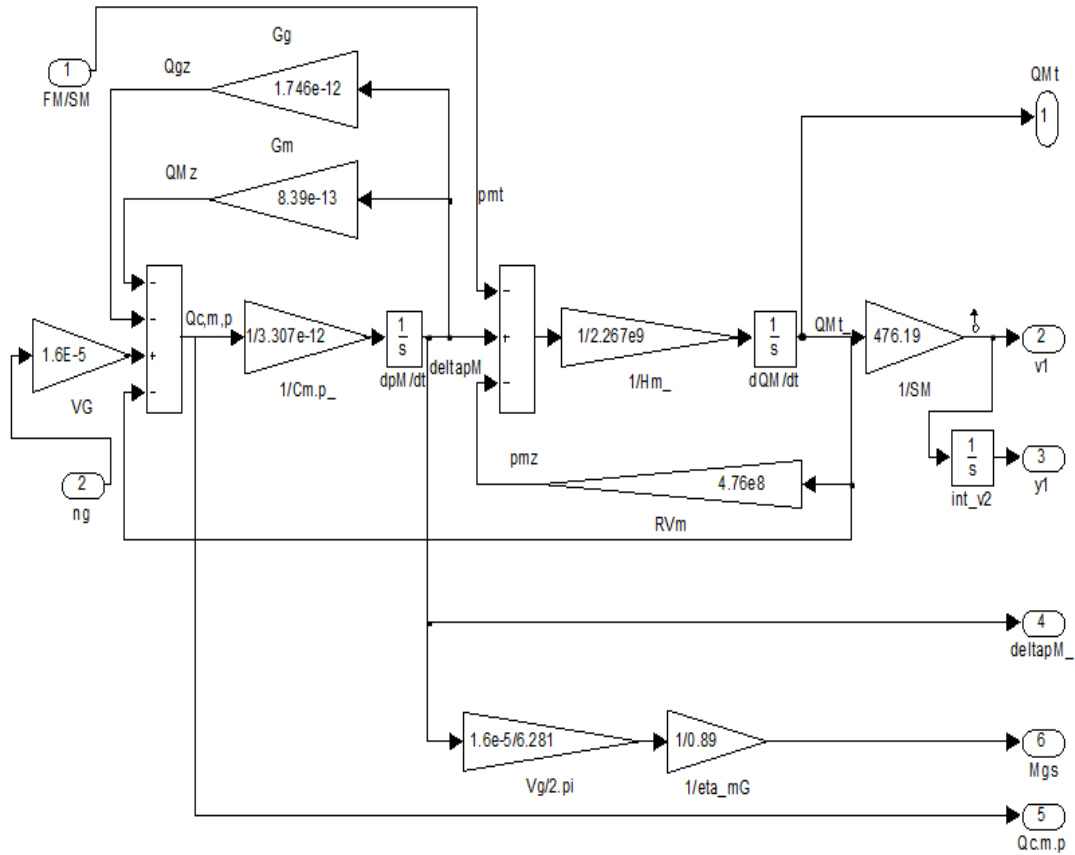


Fig. 3 Simulation scheme (simulation model) of the hydraulic circuit.

The simulation scheme of the hydraulic circuit with the load is shown in Fig. 3. This scheme describes the hydraulic circuit for lifting load. The adjusted course of speed n_g begins at time $t = 4$ s with value of $n_g = 0$ and its linearly increases for time $t_R = 5,5$ s to value $n_g = 27,9 \text{ s}^{-1}$. Loading force F_M is also increasing linearly since $t = 0$ till the time 4 s.

2.3 Simulation in Matlab – SimHydraulics (software)

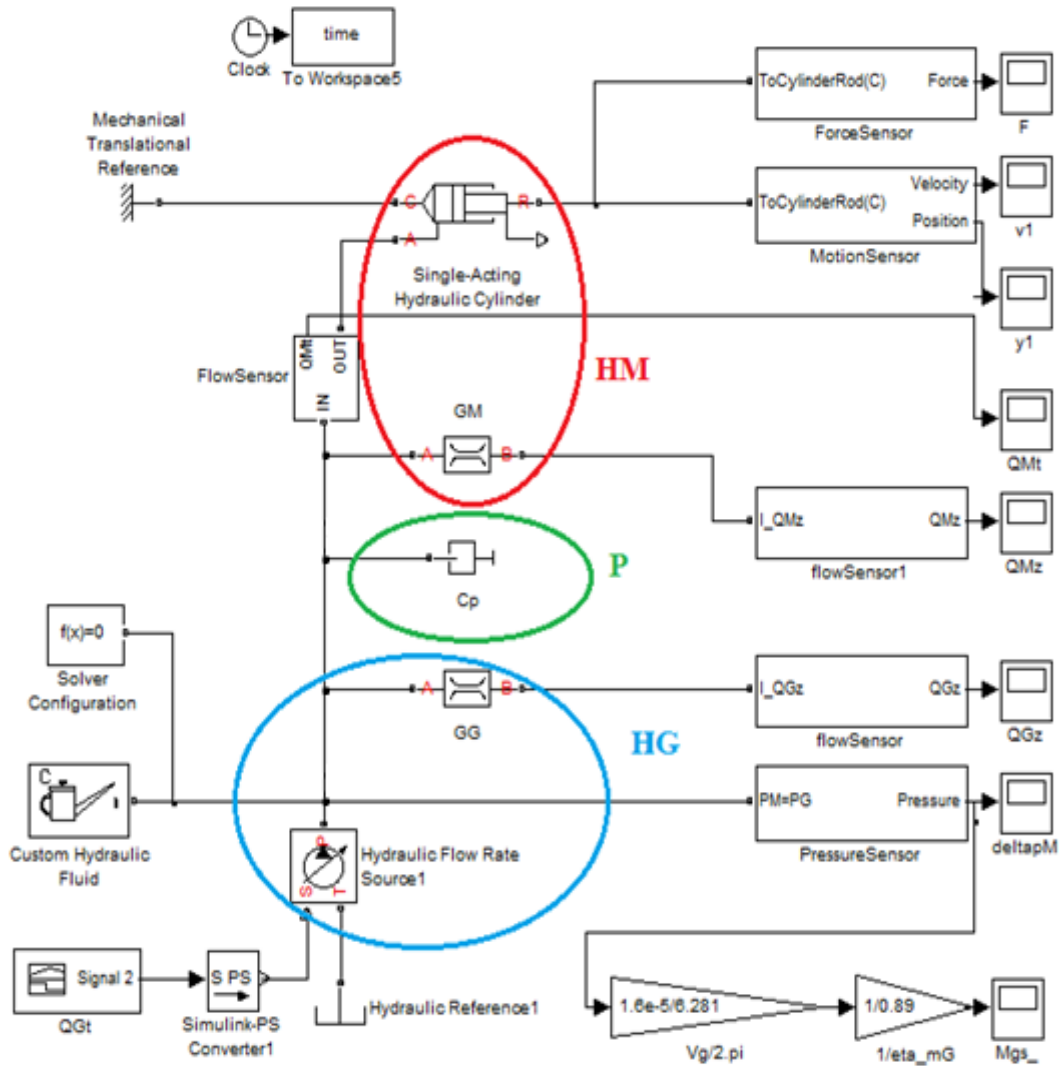


Fig. 4 Simulation scheme of hydraulic servomechanism in SimHydraulics.

The scheme of the SimHydraulics model is shown in the Fig. 4. The pipeline is replaced only by the capacity C_p block in order to comparison. The capacity block is given only by pipeline volume, and the bulk modulus is given in the block Custom Hydraulic Fluid. The pump is defined as ideal Flow Rate Source with permeability G_G . The Single Acting Cylinder block is described by the variable capacity which depends on piston stroke and piston area, and the equation (14). Hydraulic parameters are entered directly in this block, resistance to movement is defined by damping b in the block "damper" and piston area S , and resistance to acceleration is based on the specified load. Both resistances are in subsystem "forcesensor" also force F_M .

No leakage, internal or external, is taken into account therefore it is entered in parallel connected resistance G_M . So we can say that cylinder block HM corresponds to the description of the hydraulic resistance diagram in Fig. 2.

$$\Delta p_{Mt} = F \cdot S \quad (14)$$

$$\Delta p_{Md} = H \cdot Q_{Mt} = \frac{m}{s^2} \cdot Q_{Mt} \quad (15)$$

$$\Delta p_{Mz} = R_{VM} \cdot Q_{Mt} = \frac{b}{s^2} \cdot Q_{Mt} \quad (16)$$

where:

b – damping coefficient HM [$N \cdot m^{-1} \cdot s$],

R_{VM} – resistance to movement HM [$N \cdot m^{-5} \cdot s$],

H – resistance to acceleration [$kg \cdot m^{-4}$],

Q_{Mt} – hydraulic cylinder flow on the piston rod side [$m^3 \cdot s^{-1}$],

3 RESULTS EVALUATION

The graph in Fig. 5 shows that after putting the same input parameters into models in Simulink and SimHydraulics the small deviations in results are observed. We can compare velocity and other variables (e.g. pressure, flow, position) gained from monitors with negligible deviations. Model in SimHydraulics can be relatively easy improved by substitution block of pipe capacity for the "Segmented Pipeline" and the block hydraulic flow rate for the block "fixed displacement pump", which after entering the pump catalogue values considers the flow loss and does not consider the friction in the pump and the pump rotor inertia. Flow and total losses are given by efficiency. Use of SimHydraulics modelling is more appropriate in case of technical applications.

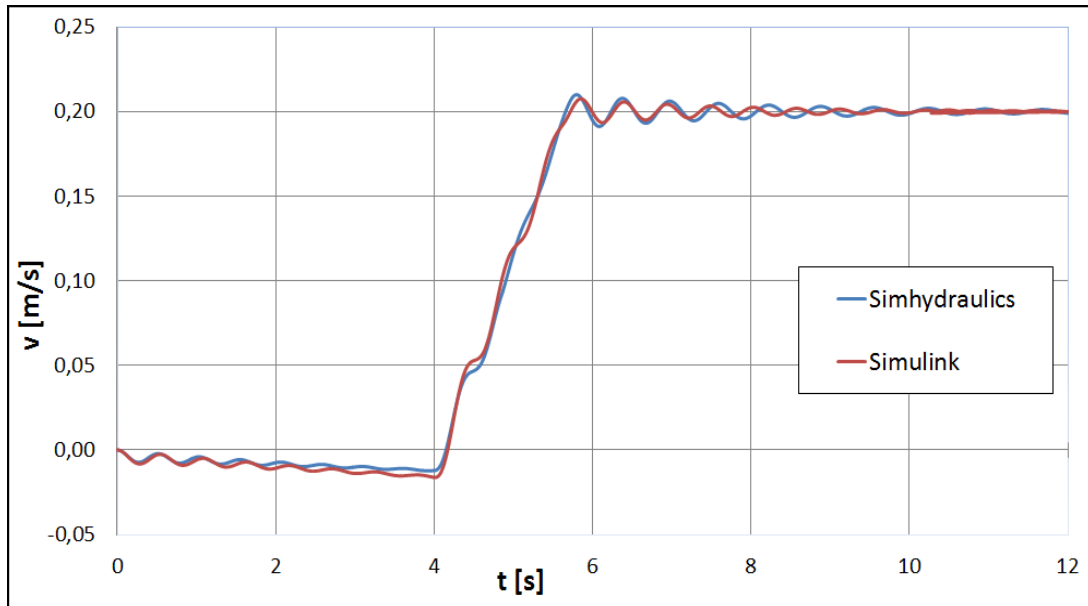


Fig. 5 Comparison of calculated piston rod velocity in Matlab-Simulink and SimHydraulics.

4 CONCLUSION

In this paper the mathematical model of the dynamic behavior of a hydraulic circuit of linear hydraulic servomechanism with load of the rope was described. This mathematical model was used for simulation of two models in (software) Simuling and SimHydraulics.

The simulation results show that we can use both ways to modelling of simple hydraulic circuits. In terms of practicality and simplicity it is better to use SimHydraulics with predefined elements, where it is not necessary to build a model based on differential equations and convert the parameters of liquid and hydraulic components into the values of R , H , D resistances. Some special phenomena, such as stick-slip of hydraulic cylinders, are difficult for modelling in both methods.

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