Transactions of the VŠB – Technical University of Ostrava, Mechanical Series
No. 1, 2012, vol. LVIII
article No. 1897

Martin IVAN* **Alessandra GROSSO****

DELIVERY RELIABILITY OPTIMALIZATION IN DISTRIBUTION SYSTEMS WITH BACKUP COVERAGE
OPTIMALIZACE SPOLEHLIVOSTI DODÁNÍ V DISTRIBUČNÍCH SYSTÉMECH SE ZÁLOŽNÍM POKRYTÍM

Abstract

The basic two requirements for distribution systems include the maximum economic efficiency and reliability. Higher reliability can be achieved by considering such criterion when optimising. While a number of approaches [1], [2] or [3] have been published in terms of economic efficiency of distribution systems, the literature does not include much about modelling their structure. In our article, higher reliability will be achieved through the so-called backup coverage of the customer. The purpose of this article is to familiarise the readers with the optimisation approach to handle this issue based on a mathematical model. The article includes a mathematical model and sample example verifying its functionality. The mathematical model solution has been performed using the Xpress-IVE optimisation software. Structure optimalization of distribution system with backup coverage can be applied in a wide range of supplies. This article aims to familiarize readers with the theoretical part, where it will be given a mathematical model. Following the example of a solution procedure using an optimalization system Xpress – IVE.

Abstrakt

Základními dvěma požadavky kladenými na distribuční systémy je co nejvyšší ekonomická efektivita a spolehlivost. Vyšší spolehlivosti lze dosáhnout tím, že bude takové kritérium při optimalizaci uvažováno. Zatímco v oblasti ekonomické efektivity distribučních systémů byla publikována celá řada přístupů [1], [2] nebo [3], v oblasti modelování jejich spolehlivosti literatura mnoho informací literatura neobsahuje. V našem článku bude vyšší spolehlivosti dosahováno prostřednictvím tzv. záložního pokrytí zákazníka. Cílem tohoto článku je seznámit čtenáře s optimalizačním přístupem pro řešení tohoto problému na bázi matematického modelu. Článek obsahuje matematický model a vzorový příklad, který ověřuje jeho funkčnost. Vlastní řešení matematického modelu bylo provedeno v optimalizačním software Xpress-IVE.

1 INTRODUTION

Apart from the distribution system costs optimisation, there is now more and more often a requirement in the distribution logistics that an emergency transport of goods is provided in case of failure of any of the sources or an obstacle in the transport way on which goods are transported. There are several different approaches to deal with this issue. An approach based on using optimisation models is one of them. The article discusses the creation of a mathematical model for

* Ing. Martin IVAN, VŠB – Technical University of Ostrava, Faculty of Mechanical Engineering, Institut of Transport, 17. listopadu, 708 33 Ostrava – Poruba, Tel.: +420774959280, E-mail: martin.ivan@vsb.cz

** Ing. Alessandra GROSSO, VŠB – Technical University of Ostrava, Faculty of Mechanical Engineering, Institut of Transport, 708 33 Ostrava – Poruba, Tel.: +420597325754, E-mail: agrosso15@yahoo.com

designing the structure of a distribution system with backup coverage. The optimality criterion includes the reliability of goods distribution.

2 BASIS FOR DESIGNING A MATHEMATICAL MODEL

To prepare a model, we need to define the issue, lay down the optimisation process goals and then process and evaluate the results obtained. Let us consider a graph where the peaks represent stocks of goods and individual customers requiring to be supplied with goods in time. The edges represent the minimum paths between stocks and customers. For each stock its capacity will be known and for each customer his requirement will be known. The timely shipment delivery reliability will be defined for each path between the stock and the customer. This is suggested by the operational conditions on the given way, e.g. there may be a situation when there is a significant probability of a failure to deliver a shipment in time on an economically advantageous path while there is a higher probability of delivering a shipment in time on a more expensive way. The aim of the model is to prepare an optimum method of supplying customers (searching for an optimum combination of the most reliable paths in the graph) with a necessity of backup coverage in case of failure of the stock primarily designed for supplying being considered.

The general procedure for preparing the mathematical model is given e.g. in [4].

Recapitulation of the input values for the task being solved:

I Set of stocks $[-]$
J Set of customers $[-]$
b_j Annual requirement of the end customer $j \in J$ $[pu \cdot year^{-1}]$
q_i Capacity of buffer stock if built in the location $i \in I$ $[pu]$
p_{ij} Probability of delivering a shipment from stock $i \in I$ to the customer in time $j \in J$ $[-]$

3 MATHEMATICAL MODEL

As stated above, it is necessary to include in the model variables that will define individual decisions. In such a case, it is needed to take a standard decision, i.e. which stock will be used for primary supplying and backup coverage of the end customer.

To maximise the timely shipment delivery reliability, we must include in the model another variable that will consider the reliability. For our needs, a variable representing the sum of reliabilities of supplying the customer from the designated stocks will be selected. However, it is necessary to point out that this cumulative value does not represent reliability in itself. Its value will be maximised to optimise it. The final variable is important in terms of the selected solution procedure. As the so-called cascade approach will be used to solve the task, e.g. [5] one more type of a non-negative variable, the meaning of which will further be specified, must be introduced in the task. However, let us first give some important pieces of information as to the usability and substance of the mentioned cascade approach. The cascade approach can best be used in situations where the minimum value of the objective function is maximised or where, on the contrary, the maximum value of the objective function is minimised. For the course of these tasks, it is common that the optimisation algorithm ends the calculation in situations where it is no longer possible to improve theminimum or maximum objective function value. This usually happens in one of the conditions while such improvement is possible for other restricted conditions. As to a distribution system with backup coverage, conditions with information on the achieved reliability level (the value of which is maximised) will be concerned. The essence of the cascade approach is that in interactions there is gradual relaxation of the restricted conditions, the meeting of which does not enable to improve the optimization criterion value. The key issue is the identification of conditions, the meeting of which may also be achieved in case of higher (in maximization) or lower (in minimization) objective function values. For this reason, an auxiliary variable must be added to each condition enabling objective function improvement. The purpose of such auxiliary variable is to signal possible objective function improvement through such condition.

The following variables are therefore included in the model:

x_{ij} Bivalent variable modelling a decision whether ($x_{ij}=1$) or not ($x_{ij}=0$) the end customer $j \in J$ is assigned
to the stock $i \in I$;

d Variable representing the minimum sum of values of the timely shipment delivery reliability;

e_j Auxiliary variable signalling the possibility of an increase in the reliability of supplying the customer $j \in J$ in time.

The input data with the defined variables will be generally used to define the objective function and set of restricted conditions. In the model, the restricted conditions must:

- Provide the defined level of coverage of the end customer $j \in J$,
- Provide that the sum of requirements of assigned customers does not exceed the capacity of stocks $i \in I$,
- Provide correct link of values of variables modelling individual decisions in the task,
- Specify the domains of individual variables.

Direct supplying by exactly one type of a vehicle will be considered in the proposed mathematical model option.

The mathematical model of the distribution system will have the following form:

$$\max f(d, e) = d + \sum_{j \in J} e_j \tag{1}$$

Under the following conditions

$$\sum_{i \in I} x_{ij} = 2 \qquad \text{for } j \in J \tag{2}$$

$$\sum_{j \in J} b_j \cdot x_{ij} \le q_i \qquad \text{for } i \in I \tag{3}$$

$$\sum_{j \in J} \sum_{i \in I} p_{ij} \cdot x_{ij} \ge d + e_j \qquad \text{for } j \in J \tag{4}$$

$$e_j \le \varepsilon \qquad \text{for } j \in J \tag{5}$$

$$x_{ij} \in \{0,1\} \qquad \text{for } i \in I; j \in J \tag{6}$$

$$e_j \ge 0 \qquad \text{for } j \in J \tag{7}$$

$$d \ge 0 \tag{8}$$

The function (1) represents the objective function – achieved value of the minimum timely shipment delivery reliability (term including the variable d). The group of restricted conditions (2), the number of which corresponds to the number of the end customers, will make sure each end customer is supplied by one stock and covered by another in case of emergency. The group of restricted conditions (3), the number of which in the model corresponds to the number of stocks, will make sure that the capacity of each stock is not even exceeded in a situation when the stock is designated for backup coverage. The group of restricted conditions (4) creates links between the assignment variables, minimum level of reliability and variables that signal the possibilities to increase the reliability level. The number of conditions corresponds to the number of the end customers $j \in J$. The group of restricted conditions (5) makes sure the values of auxiliary variables do not exceed the suitably selected constant ε. Such constant must be selected in a way it does not significantly affect the achieved optimum value (it is only an auxiliary variable that is to signal the possibility to increase the objective function value, not to significantly change its value). The groups of restricted conditions (6), (7) a (8) specify the domains of variables.

4 COMPUTING EXPERIMENTS

After preparing the mathematical model, it is necessary to check its functionality and obtain information on the calculation efficiency. To solve the issue above, a mathematical model was prepared using the Xpress – IVE optimization software.

The input data of the test task are given in tab. 1.

Tab. 1 – Input data of the proposed example.

	Timely shipment delivery probability			Customer requirements
	Stock 1	Stock 2	Stock 3	
Customer 1	0.9	0.75	0.4	100
Customer 2	0.7	0.45	0.6	200
Customer 3	0.6	0.85	0.7	300
Customer 4	0.6	0.75	0.9	400
Stock capacity	1,500	700	300	

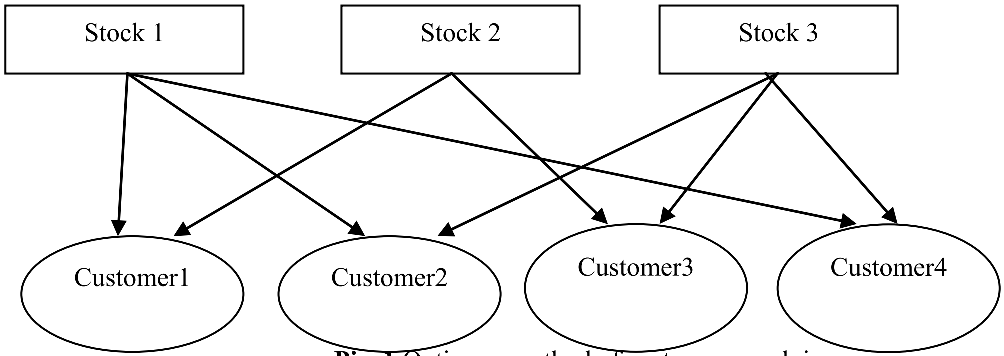
Based on the input conditions above, a mathematical model will be prepared, with the determination of the optimum method of supplying the end customers with possible backup coverage being the aim. Apart from the solution itself, the optimization software also enables to define types of outputs. The program text was defined in a way that after starting the algorithm the solver obtains not only information on the objective function value, summary of end customers assigned to individual buffer stocks but also information where the supplying reliability may be increased. To make things clear, there is a table of the resulting values after iterations (tab. 2).

Tab. 2 Solution genesis after iterations.

1st iteration	2nd iteration	3rd iteration
OF:1.3003	OF:1.5002	OF:1.65
x(1,1)=1	x(1,1)=1	x(1,1)=1
x(1,2)=1	x(1,2)=1	x(1,2)=1
x(1,3)=0	x(1,3)=0	x(1,3)=0
x(1,4)=1	x(1,4)=1	x(1,4)=1
x(2,1)=1	x(2,1)=1	x(2,1)=1
x(2,2)=0	x(2,2)=0	x(2,2)=0
x(2,3)=1	x(2,3)=1	x(2,3)=1
x(2,4)=0	x(2,4)=0	x(2,4)=0
x(3,1)=0	x(3,1)=0	x(3,1)=0
x(3,2)=1	x(3,2)=1	x(3,2)=1
x(3,3)=1	x(3,3)=1	x(3,3)=1
x(3,4)=1	x(3,4)=1	x(3,4)=1
d=1.3	d=1.5	d=1.65
e(1)=0.0001	e(1)=0.0001	e(1)=0
e(2)=0	e(2)=0	e(2)=0
e(3)=0.0001	e(3)=0.0001	e(3)=0
e(4)=0.0001	e(4)=0	e(4)=0

The first iteration was performed after entering all the input information and introducing the restricted conditions and objective function. The model was defined in a way the solver is notified –

in case the objective function may be increased – of the end customer where a more suitable method of supplying in terms of reliability may be provided. In tab. 2 this fact is suggested from the values of variables e_j. If the value of thevariable e_j is zero, a suitable method of supplying with backup coverage has been found and the value ofreliability of supplying of such customer cannot be increased any further. If its value is other than zero, there is space for increasing the reliability. The assignment method is shown in lines with the x marking (stock, customer) where e.g. x(1.1)=1 corresponds to assignment of the first customer to the first stock. If the value is zero, the respective customer is not assigned to the stock. The first column suggests that the objective function value was 1,3003 and no suitable method of backup coverage has been found for customers 1, 3 and 4. Therefore, we will appropriately adjust the model with the right side of conditions being put to equal to 1.3, by which keeping the achieved level of reliability will be provided in respect of these customers. Then, we will perform another iteration (see the second column in tab. 2). Here, we have found out that the objective function value has increased to 1.5. We have performed another iteration and found out to obtain an objective function value of 1.65, which is the best solution as yet and a suitable method of supplying with backup coverage has also been achieved for all the four customers, with the resulting supplying method being shown in pic. 1.



Pic. 1 Optimum method of customer supplying.

5 CONCLUSION

The article submitted deals with the issue of solving tasks regarding the optimization of the structure of a distribution system with backup coverage where the timely shipment delivery reliability is the optimization criterion. In practice, there are a number of situations where the end customer requires to be supplied in time. To prevent delays in shipment deliveries in case of any complication with the stock primarily designated for supplying, backup coverage may be considered. It needs to be pointed out that the backup coverage requirement naturally entails a requirement for maintaining higher stock capacities. The article includes a mathematical model to solve the given task and an example verifying its functionality. The solution was performed using the Xpress – IVE optimization software.

REFERENCES

[1] DANĚK, J.; PLEVNÝ, M. *Výrobní a logistické systémy.* Plzeň: Západočeská univerzita, 2005. 222 s. ISBN 80-7043-416-3.

[2] PALÚCH, S.; PEŠKO, Š. *Kvantitatívne metódy v logistike.* Žilina: Žilinská univerzita v Žilině, 2006. 185 s. ISBN 80-8070-636-0.

[3] JANÁČEK, J.; JANÁČKOVÁ, M.; SZENDREYOVÁ, A.; GÁBRIŠOVÁ, L.; KOHÁNI, M.; JÁNOŠÍKOVÁ, Ľ. *Navrhovanie územne rozľahlých obslužných systémov.* Žilina: Žilinská univerzita v Žilině, 2010. 404 s. ISBN 978-80-554-0219-2.

[4] JANÁČEK, J. *Matematické programování*. Žilina: Žilinská univerzita v Žilině, 2003. 225 s. ISBN 80-8070-054-0.

[5] KOHÁNI, M., KOZEL P. *Využití kaskádového přístupu při koordinaci spojů.* Pardubice:Kvantitativní metody v dopravních a logistických systémech I. Univerzita Pardubice, 2010. ISBN 978-80-7395-297-6.

This article originated in connection with the grant of the Faculty of Mechanical Engineering Technical University of Ostrava SP2012/113 Development of new methods to support planning and management of transport processes.