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COMPARATIVE STUDY OF LUMPED AND CONTINUUM MODELS OF THERMAL FIELD IN SMA ACTUATOR

POROVNÁVACIA ŠTÚDIA MODELOV SO SÚSTREDENÝMI A ROZLOŽENÝMI PARAMETRAMI TEPELNÉHO POĽA SMA AKTUÁTORA

Abstract

The article deals with thermal analysis, modeling and simulation of the Shape Memory Alloy (SMA) actuator. New parametric lumped model of actuator thermal field was developed in Matlab-Simulink program in order to control SMA actuator. The characteristics of the parametric model were compared with the continuous model made in ANSYS FEM program. The goal is to verify lumped model using FEM model in steady-state analysis as well as in dynamic analysis.

Abstrakt

Článok sa zaoberá tepelnou analýzou, modelovaním a simuláciou aktuátora zo zliatiny s tvarovou pameťou. V prostredí Matlab-Simulink bol vyvinutý nový model teplotného poľa aktuátora so sústredennými parametreami pre účely riadenia SMA aktuátora. Charakteristiky z tohto parametrického modelu boli porovnané s kontinuálnym modelom MKP v programe ANSYS. Cieľom je verifikovať model so sústredenými parametrami pomocou MKP modelu pre ustálený ako aj dynamický režim.

1 INTRODUCTION

Shape Memory Alloy is a smart material converting thermal energy to mechanical work, which can be used in many different mechatronic systems [1, 2] like actuator. The thermal actuator made of Nickel–Titanium (NiTi) wire is heated by Joule loss heat caused by electric current and naturally air-cooled. Critical part of SMA actuator from thermal point of view is connection NiTi wire to crimp - see Fig. 1. This part of NiTi wire is cooled more than the rest of the wire due to connections to crimp. It is necessary to investigate the influence of actuator crimps to temperature distribution along NiTi wire, because unsuitably designed connection can caused loss of power and other characteristics in this part of SMA actuator.

In previous authors works, the improved crimp with separated mechanical anchor and electrical connection was designed by Finite Element Method (FEM) [3-6]. The improved crimp increases the end-connection temperature depending on crimp parameters. The goal of crimp design is to obtain the thermal distribution in actuator that provides phase transformation of SMA over the whole active length of the actuator.

In order to control SMA actuator new thermal parametric lumped model of SMA has been developed in Matlab-Simulink and is presented in this paper. The parameters of the lumped model will be set up in accordance with continuous model made in FEM code ANSYS. New thermal lumped

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model will describe the thermal field along the NiTi wire actuator with influence crimps, natural convection and conduction in wire. Obtained results of lumped model simulation will be compared with the results obtained by continuous model in steady-state and transient analysis.

The crimp of SMA actuator is consisted of three basic parts (see Fig. 1), namely crimp body of tube shape made of Cu, mechanical anchor of cylindrical shape made of Teflon - insulating material (this geometry point is denoted as A2) and electrical anchor of cylindrical shape made of Fe - conducting material (this geometry point is denoted as A1). Passive part of NiTi wire of length L, which is between mechanical and electrical anchor, have a strong effect on temperature T_2 in point A2 and on temperature distribution on active part of NiTi wire. As is shown in [5,6], crimp with separated mechanical and electrical anchor of NiTi wire has better steady-state and dynamic characteristics of actuator. The SMA model, presented in this paper, does not contain phase change of transformation and material properties are considered constants. These simplifications will be removed in our next research.

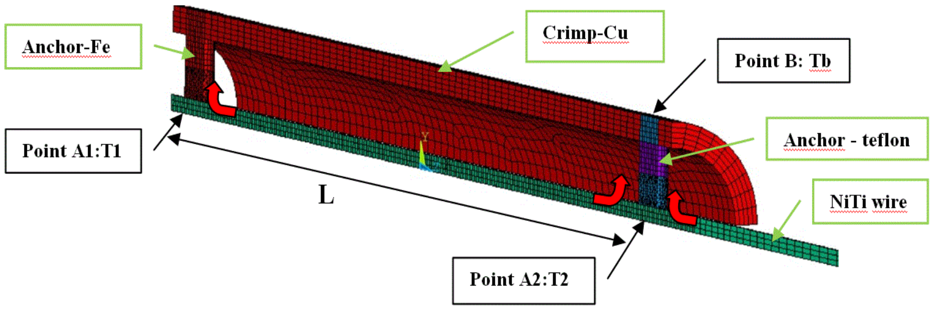

Fig. 1 Electrical and mechanical connection of NiTi wire - detail of actuator.

2 THE LUMPED MODEL

Typical distribution of temperature along the NiTi wire is shown in Fig. 2. This distribution can be also obtained by solving differential equation of heat transfer with appropriate boundary conditions including temperatures T_1 and T_2 - this model is called lumped model.

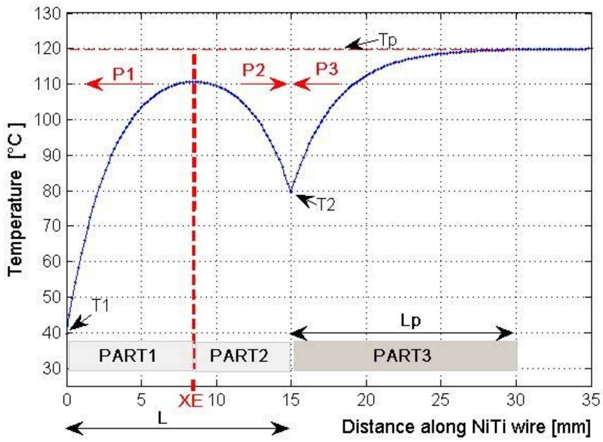

Fig. 2 The distribution of temperature along NiTi wire with boundary temperatures T_1 and T_2.

To model the distribution of temperature along NiTi wire using differential equation (lumped model), it is necessary to know the maximum temperature of NiTi wire denoted as T_P (this temperature is developed as potential of Joule loss heat $P = R_e I^2$) and temperatures T_1 and T_2 in both connection points A1 and A2, respectively. The temperature T_b of Cu body of crimp was considered close to the temperature T_1 of electrical contact due to very good thermal conductivity of Anchor -Fe. The direction of thermal flows from NiTi wire to both connections (points A1 and A2) are shown by red arrows, Fig. 1 and Fig. 2. The variables and parameters used in next equations are summarized in the Tab.1.

Tab. 1 Variables and parameters used in analysis of SMA.

T_0 [°C]	Ambient temperature	d [m]	Diameter of NiTi	α [$Wm^{-2}°C^{-1}$]	Convection coefficient
T_1 [°C]	Temperature in point A1	S [m^2]	Cross-section	λ [$Wm^{-1}°C^{-1}$]	Thermal conductivity
T_2 [°C]	Temperature in point A2	A [m^2]	Convective surface	c [$J\ kg^{-1}°C^{-1}$]	Specific heat
T_b [°C]	Temperature in point B	V [m^3]	Volume	R [$°C\ W^{-1}$]	Thermal resistance
T_P [°C]	Temperature potencial NiTi	L [m]	Length	τ [s]	Time constant
T_N [°C]	Corrected temperature NiTi	a [m^{-1}]	Coefficient	t [s]	Time
T_S [°C]	Average temperature	x [m]	Distance	P [W]	Thermal power

2.1 Analytical solution

In the case that electric current, which is used as thermal power source, is constant or is varied very slowly (i.e. the system is considered steady-state), the distribution of temperature can be described as following ordinary differential equation

$$\frac{\partial^2 T}{\partial x^2} - \frac{\alpha A(T-T_0)}{\lambda V} = -\frac{P}{\lambda V} \tag{1}$$

with boundary conditions (see Fig. 2): point A1: $T(0) = T_1$, point A2: $T(L) = T_2$.

The solution of the equation (1) with boundary conditions is [7]:

$$T(x) = C_1 e^{-ax} + C_2 e^{ax} + C_3 \tag{2}$$

where the coefficients a, C_1 , C_2 and C_3 are:

$$C_1 = \frac{(T1-T2)-C_2(1-e^{aL})}{1-e^{-aL}} \tag{3}$$

$$C_2 = \frac{(T_1-C_3)(1-e^{-aL})-(T_1-T_2)}{e^{aL}-e^{-aL}} \tag{4}$$

$$C_3 = T_0 + \frac{P}{\lambda V a^2} = T_0 + \frac{P}{\alpha A} = T_P \tag{5}$$

$$a^2 = \frac{\alpha A}{\lambda V} = \frac{4\alpha}{\lambda d} \tag{6}$$

The constants „1/a“ [m] is characteristic length and depends on material and geometry of SMA wire. The distance L between two anchor points will be viewed in relation to the 1/a.

The position X_E of local extreme between mechanical and electrical connections (see Fig. 2) can be determine by derivative the equation (2)

$$x_E = \frac{1}{2a} ln \frac{C_1}{C_2} = \frac{L}{2} + \frac{1}{2a} ln \frac{(T_P-T_1)-e^{-aL}(T_P-T_2)}{(T_P-T_2)-e^{-aL}(T_P-T_1)} \tag{7}$$

T_P is the potential temperature according to (5).

As it will be seen in next part of the paper, the average temperatures on part 1 (denoted as T_{S1}) and part 2 (denoted as T_{S2}) of passive part NiTi wire (see Fig. 2) are used to compute the heat, which flows from passive part of NiTi wire to crimp through both connections. The values of the above mentioned averaged temperatures can be derived using integration of equation (2) and have form

$$T_{S1} = \frac{1}{x_E}\int_0^{X_E} C_1 e^{-ax} + C_2 e^{ax} + T_P \, dx \tag{8a}$$

$$T_{S1} = T_P + \frac{1}{a x_E}(C_1 - C_2) \tag{8b}$$

$$T_{S2} = \frac{1}{L - x_E}\int_{X_E}^{L} C_1 e^{-ax} + C_2 e^{ax} + T_P \, dx \tag{9a}$$

$$T_{S2} = T_P + \frac{1}{a(L - x_E)}\left(C_2 e^{aL} - C_1 e^{-aL}\right) \tag{9b}$$

Above mentioned solution of equation (1) is valid for all length L, but for length L> 4,6/a the solution (2) can be simplified and the expression e^{-aL} can be crossed out. This simplification represents error less than 1%. Other simplification can be carried out for cases, where the length L is too large, for example for $L \geq 10/a$. For these cases, the influence of coefficient C_2 on the solution is relatively small, so we can set $C_2=0$ and then the temperature distribution have form

$$T(x) = T_P - (T_P - T_2)e^{-ax} \tag{10}$$

The average temperature in this part of active NiTi wire depends on length L_P. Appropriately chosen length L_P does not have effect on accuracy of obtained results (i.e. error of solution less than 1%).

$$T_{S3} = T_P - \frac{1 - e^{-aL_P}}{aL_p}(T_P - T_2) \tag{11}$$

The distribution of temperature along NiTi wire on Fig. 2 is described by equations (2) and (10) with boundary conditions T_1=40 °C , T_2=80 °C a T_P=120 °C. There are only 3 parameters of system a=316 m^{-1} and L=L_P=15 mm.

2.2 Set up boundary conditions

The temperatures T_1, T_2 and T_P represent the boundary conditions of governing equation of the problem, which have to be set up. The temperature T_P is set up in accordance with Joule loss heat P, ambient temperature T_0, convection coefficient α and convective surface A - equation (5).

To set up the temperatures T_1 and T_2, the analysis of thermal flows through mechanical and electrical anchor is used. In these analyses, three thermal flows have to be defined, namely thermal flow P_1, that flows from part 1 of NiTi to electrical anchor, thermal flow P_2 that flows from part 2 of NiTi to mechanical anchor and thermal flow P_3 that flows from part 3 of NiTi to mechanical anchor. These three thermal flows can be expressed using average temperatures defined by equations (8-9) and (11) and have form

$$P_3 = \alpha_p A_3 (T_P - T_0) - \alpha_3 A_3 (T_{S3} - T_0) \tag{12}$$
$$P_2 = \alpha_p A_2 (T_P - T_0) - \alpha_2 A_2 (T_{S2} - T_0) \tag{13}$$
$$P_1 = \alpha_p A_1 (T_P - T_0) - \alpha_1 A_1 (T_{S1} - T_0) \tag{14}$$

Parameter A_x and α_x=f(T_{sx}) is convective surface of NiTi and convection coefficient of individual part of NiTi wire, respectively. The convection coefficient depends on average surface temperature of individual part of NiTi wire.

All three thermal flows (12-14) flow from NiTi through anchors to crimp and in steady-state they are transported by convection to surrounding air. The convection from the crimp is defined by convection coefficient α_b, convective surface A_b and ambient temperature T_0, and for the temperature of crimp T_b we can write

$$T_b = \frac{1}{\alpha_b A_b}(P_1 + P_2 + P_3) + T_0 \tag{15}$$

The temperature T_1 can be determined using known temperature of crimp (T_b), thermal flow that flows through electrical anchor (P_1) and geometry (L_{A1} and S_{A1}) and material property (λ_{A1}) of electrical anchor and has form

$$T_1 = \frac{L_{A1}}{\lambda_{A1} S_{A1}} P_1 + T_b \tag{16}$$

Similarly we can determined the temperature T_2, which has form

$$T_2 = \frac{L_{A2}}{\lambda_{A2} S_{A2}} (P_2 + P_3) + T_b \tag{17}$$

The only input parameters for solving boundary temperatures T_1 and T_2 using above mention equations are geometry and material parameter of actuator, Joule loss heat P and ambient temperature T_0. To obtain these boundary temperatures, iteration process had to be programmed in code Matlab/Simulink.

2.3 Simulation in Matlab-Simulink

Above mentioned equations have been used to developed mathematical model of SMA actuator in order to obtain temperature distribution in NiTi wire.

Tab. 2 Design parameters and estimate of time constants.

part / material	diameter	length	convection/conduction	time constant
	[mm]	[mm]	[W/m^2/K] / [W/m/K]	[s]
crimp /Cu	D / ΔD = 2/0.5	L_S = 15	α = 17 / λ= 385	37.8
anchor / Teflon	d = 0.4	h = 0.6	λ= 0.25	3.02
anchor / Fe	d = 0.4	h= 0.6	λ= 70	0.02
wire/NiTi active	d = 0.3	L_N = 100	α = 69 / λ= 8	3.22

To simulate thermal conditions in SMA actuator, model was supplemented by first order dynamics of the structural parts (anchors 1, 2 and body of crimp), Fig. 3. Estimated time constants are in Tab. 2 and in the literature [5, 6]. The principle of the estimation consists in calculating the heat capacity of body C_T and thermal resistance R_T according to the way of thermal distribution.

Mechanical and electrical connection is realized through small length of NiTi wire and this small length and heat generated in this small part of NiTi wire must be included into the simulation. Geometry parameters of Teflon anchor are designed with respect to insulation resistance and time constant. The simulation scheme is shown on Fig. 3, where iterations are substituted by feedback denoted as „Thermal flow estimation“. The computation of average temperatures is performed in accordance with equations (8-9) and (11) and thermal flows with equations (12-14). In this simulation scheme, structural stability of feedback has been verified.

NiTi model is system of first order with time constant 3,22s (see Tab. II) and with gain from equation (5), that represents thermal resistance 154 °C/W. Model NiTi wire is used twice in the scheme. In the first case, the model output is temperature T_P, which is the potential for thermal field (named Potential NiTi). In the second case the model output is the corrected temperature T_n. In this case power P is corrected by the calculated heat flow P_3 before entering the block named NiTi, Fig.3. Heat flow P_3 comes from the active part of NiTi and flows into the the anchor. There is the same crimp on both sides of the actuator and therefore there are twice P_3.

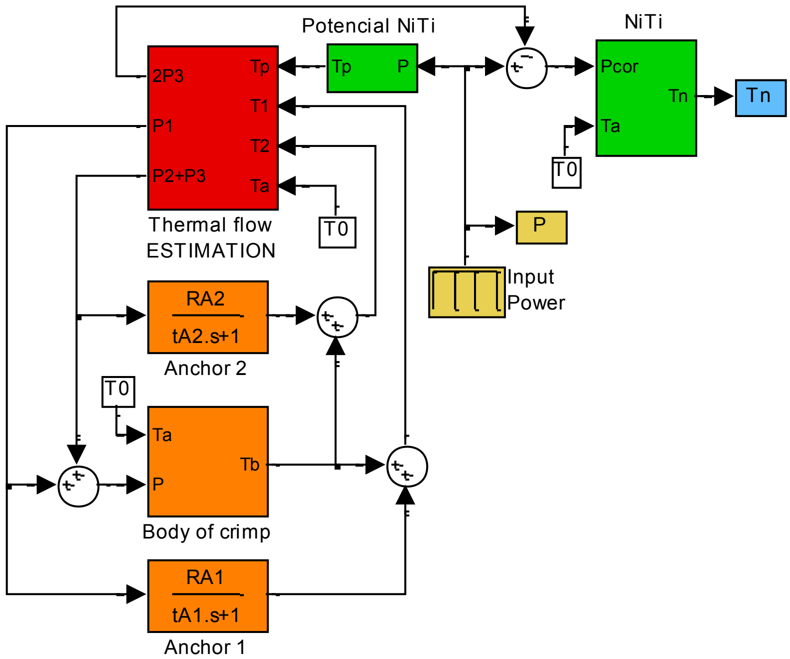

Fig. 3 Simulation scheme of thermal properties of SMA actuator.

Thus we obtain the corrected temperature T_n, which is applicable to the rest of the SMA model. This model is suitable for automatic control purposes, using the methods described in [10, 11]. The responses of temperature in selected points are shown on Fig. 4 and Fig. 5.

Two cases with different mechanical connection has been compared.

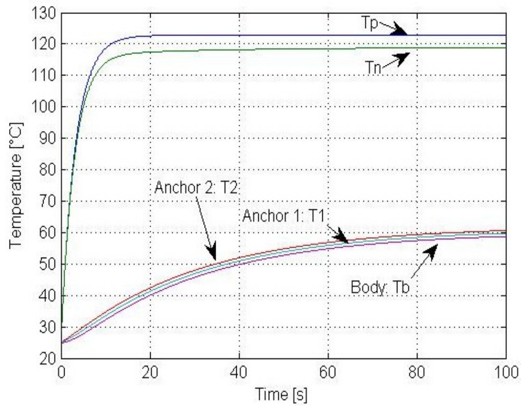

Fig. 4 Responses of temperature for anchor made from thermal conductor.

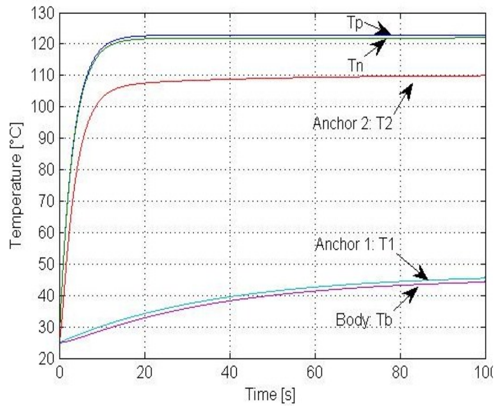

Fig. 5 Responses of temperature for anchor made from thermal insulator.

The geometry of both cases is the same, but the material is used different. In the first case, mechanical anchor is made from thermal conductive material (this is very often case in engineering practice [3], [8]) and in the second case from thermal insulator - Teflon. Thermal power (Joule loss heat) P=640 mW and ambient temperature T_a=T_0=25 °C has been used in both cases. These two parameters defined potential temperature T_P=122.8 °C. The comparison of time responds of selected points of both cases is shown in Fig. 4.

As is seen from Fig. 4 and Fig. 5, when the material of mechanical anchor has been chosen insensitively, the temperature T_2 is under 70°C during the whole operation of actuator. This low value T_2 has effect on temperature distribution on active part of NiTi wire, and considerable part of the active part of NiTi wire does not get transformation temperature (austenit is transformed to martensit between temperatures 90°C and 110°C, it depends on loading conditions).

The high conductivity of the anchor causes the temperature T_2 "follows very bad dynamic" body of the crimp and transients subside a few tens to hundreds of seconds. This is uncomfortable for the control of actuator and literally poor at measuring experiments. The corrected temperature of NiTi in Fig. 4 has the steady value of T_n = 118.9 ° C. Temperature difference the T_p and T_n is nearly 4 °C. But it is up to 20 % of the transformation temperature span.

The second case (material of mechanical anchor is insulator) shows important improvement in steady-state and dynamic behavior of SMA actuator. Time responds of T_2 is comparable with time responds of T_P. In steady-state, the temperature T_2 achieves 110.2°C, i.e. the temperature in each point of active part of NiTi wire get over transformation temperature. This is a satisfactory result.

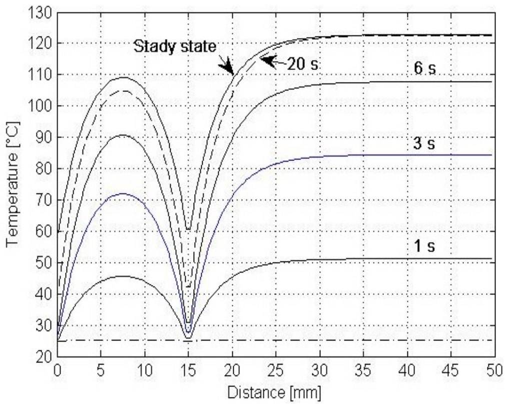

Fig. 6 Temperature distribution along NiTi wire for anchor made from thermal conductor.

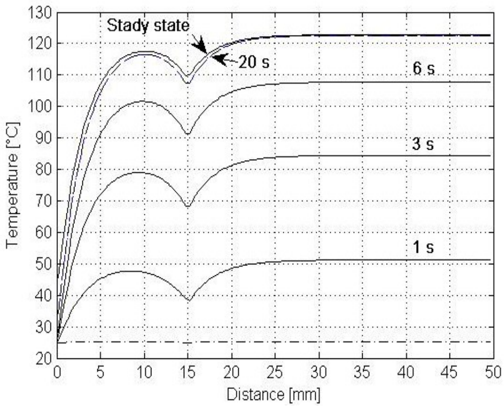

Fig. 7 Temperature distribution along NiTi wire for anchor made from thermal insulator.

Simulation performed in Matlab-Simulink does not compute temperature distribution, but only temperatures T_1, T_2 and T_P. The temperature distribution is computed as a next step from temperatures T_1, T_2 and T_P used as boundary conditions and from equations (2) and (10). Obtained results for temperature distributions for both cases are shown on Fig. 6 and Fig. 7.

3 CONTINUUM MODEL OF SMA WIRE

As reference for lumped model, continuum model made in FEM code ANSYS [12] was chosen. Only 1/8 symmetric part of SMA wire was modeled as 3D system - see Fig. 8.

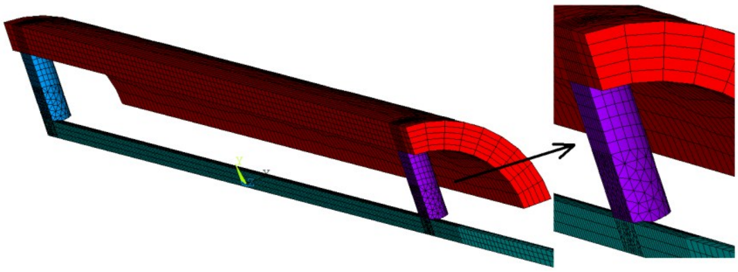

Fig. 8 Mesh of SMA wire in program ANSYS.

Because the goal of continuum model is verification of thermal lumped model, we modeled the system only as thermal system. We used two different element types: 3D Solid 90 element and surface effect elements Surf 152. The total number of elements was 15576. Solid 90 elements were used to model conduction heat flow in system and Surf 152 elements were used to model convective heat flow between system and its surrounding. In spite of capability of used element types, we used only constant value of convective coefficient due to equality of boundary conditions between lumped and continuum model.

Transient simulation was used to solve the problem. Three load steps were setup:

1. load step represented step loading of heat generation to operation loading state - duration of the step is 0.01s,
2. load step represented the time during which system is heat up - duration of the step is 30s,
3. load step had time integration switched off so this step represented only steady state for prescribed heat generation - duration of the step played no role.

Obtained results for last load step (steady-state) are shown in Fig. 9. The transient results are presented in next chapter, where they are compared with results obtained by lumped model.

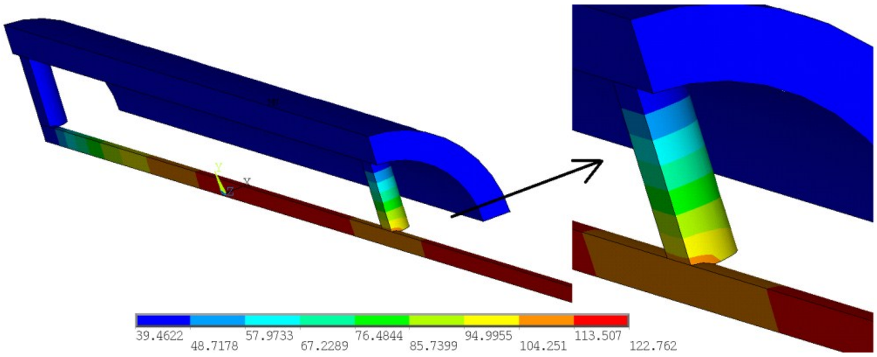

Fig. 9 Obtained temperature for last load step (steady-state) in [°C].

4 THE VERIFICATION OF LUMPED MODEL BY 3D FEM MODEL

The results obtained from lumped model are compared with results of 3D FEM model. The goal of result comparison is verification of mathematical description of simplified lumped model. Both models are defined using the same boundary and initial conditions as well as the same material parameters of individual components of SMA actuator.

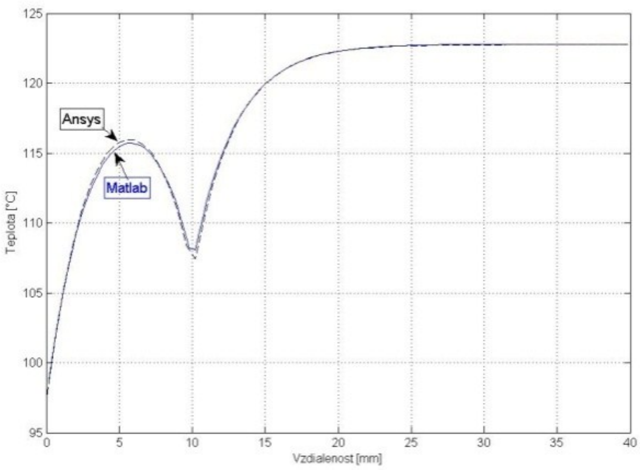

Fig. 10 Matlab-Ansys comparition, steady-state temerature in NiTi wire.

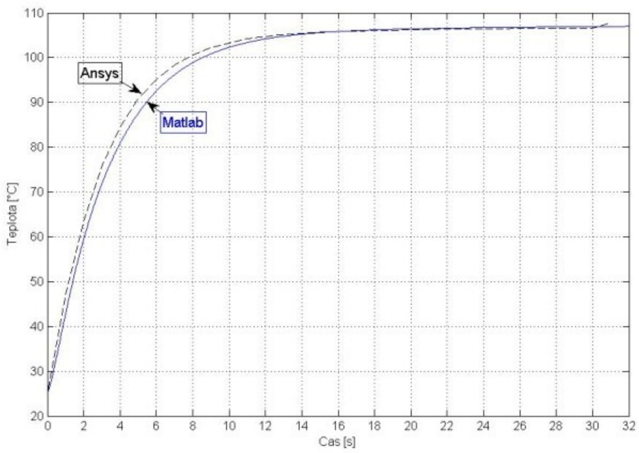

Fig. 11 Matlab-ANSYS comparition, time responds T_2.

The comparison of Matlab-Simulink model with ANSYS model is shown on Fig. 10 and Fig. 11. On Fig. 10, time responds of temperature in mechanical anchor is compared. On Fig. 11, the temperature distribution along NiTi wire (both active and passive parts) in steady-state is compared. As is seen from these figures, Matlab-Simulink lumped model is qualitatively and quantitatively comparable with more complicated 3D FEM ANSYS model. It means, that Matlab-Simulink lumped model can be used for determination of region, where the phase transformation does not realize.

5 CONCLUSIONS

The main goal of the paper was to present the capability and efficiency of lumped model of SMA actuator programmed in code Matlab-Simulink. Lumped model has been verified with 3D FEM model created in code ANSYS. The results of both models (lumped and 3D FEM) are qualitatively and quantitatively comparable but lumped model is more simpler. This simplicity of lumped model can be used in examination of influence of design parameters, materials and loading in steady-state as well as dynamic regime. The lumped model has the other advantage over FEM mode, it can be used very effectively in control of SMA actuator. The process of obtaining results in lumped model has been demonstrated on SMA actuator, where connection between NiTi wire and crimp has been realized by separated mechanical and electrical connections.

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