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NUMERICAL SIMULATIONS OF STEADY-STATE OF TWO-CRACK ROTOR SYSTEM SUPPORTED BY RADIAL ACTIVE MAGNETIC BEARINGS

NUMERICKÉ SIMULACE USTÁLENÉHO STAVU ROTOROVÉ SOUSTAVY SE DVĚMA TRHLINAMI ULOŽENÉ V RADIÁLNÍCH AKTIVNÍCH MAGNETICKÝCH LOŽISKÁCH

Abstract

The steady-state response of a two-crack rotor system is investigated by computational simulations in the paper. The rotor system supported by radial active magnetic bearings is excited by centrifugal forces of discs unbalances. A flexibility matrix of Bernoulli beam element takes into account coupling phenomena between directions of vibration. The phenomenon occurs in cracked rotors. The couplings express relations between vibration in different directions, i.e. bending-torsion, bending-longitudinal, and torsion-longitudinal. The flexibility matrix elements of the cracked rotor are derived using the concepts of fracture mechanics for transverse surface crack. Partial opening/closing of cracks implemented in motion equations is determined by sign of stress intensity factor for mode I. The factor is computed for the crack edge. The motion equations are nonlinear because the system response depends on breathing of cracks and nonlinear force coupling is introduced by radial active magnetic bearings. Parametric studies of the system response were carried out in order to examine influence of various angles between two cracks. Several recommendations for detection of cracks and monitoring of the cracked rotor are suggested.

Abstrakt

V této práci jsou užity výpočetní simulace ke zkoumání ustálené složky odezvy na buzení odstředivými silami nevyvážených disků rotorové soustavy se dvěma trhlinami, jenž je uložená v radiálních aktivních magnetických ložiskách. Matice poddajnosti Bernoulliho nosníkového prvku je upravena tak, aby se uvážily všechny vazby v kmitání, které existují v rotoru s trhlinou, tj. ohybově-torzní, ohybově-podélné a torzně-podélné. Prvky matice poddajnosti s trhlinou jsou odvozeny na základě teorie lomové mechaniky pro příčnou povrchovou trhlinu. Částečné otevření/zavření trhlin zahrnuté do pohybové rovnice je řízeno podle znaménka součinitele intenzity napětí I. módu zatížení vypočítaného na hraně trhliny. Pohybové rovnice rotorové soustavy jsou nelineární kvůli odezvě závislé na dýchání trhlin a nelineární silové vazbě zavedené radiálními aktivními magnetickými ložisky. Byla provedena parametrická studie s cílem zkoumat vliv různých hodnot úhlu mezi trhlinami na ustálený stav odezvy rotorové soustavy. Taktéž jsou prezentovány doporučení pro detekci a monitorování rotoru se dvěma trhlinami.

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1 INTRODUCTION

Transverse fatigue cracks can originate by cyclic bending loading of a shaft. The fatigue cracks occur in places of crack initiators (constructional and structural notches). The cracks influence rotor response due to change of shaft flexibility [1].

Many works deal with influence of transverse surface crack on dynamic properties of rotor systems. The issue of rotor with several cracks is rarely solved. Sekhar [2] presented numerical study of bending vibrations for rotor system including two transverse open cracks. The shaft of the rotor is discretized into finite beam elements in the computational model. Transverse cracks are introduced by local flexibility matrix. The flexibility matrix is derived from strain energy concentration in vicinity of crack tips. Eigenvalues and system stability analyses of two-crack rotor system were provided as parametrical. Parameters were crack depth and crack location. In [3], Darpe et al. studied a simple Jeffcott rotor with two transverse surface cracks. Breathing behaviour of cracks has been examined by means of nonlinear model using signs of the stress intensity factor for Mode I. The effects of mutual cracks orientation on unbalance responses and on the breathing behaviour of two cracks are presented. The computational simulations proved influence of the mutual crack orientation on the response is higher at lower speeds.

The topic of actively controlled rotor system with a crack was investigated in few articles as Ewins et al. [4], [5] and Kasarda et al. [6]. The Jeffcott rotor supported by rigid bearings is controlled by active magnetic bearing in these papers. Ferfecki [7] presented results of numerical simulations on a actively controlled rotor system with shaft containing a transverse crack or a slant crack or combination of both. Influence of changing of controller parameters on steady-state response caused by unbalance is negligible.

In the work, computational simulation is utilized to study the steady-state response of unbalanced rotor with two surface transverse cracks. The investigated rotor system is supported by two identical radial active magnetic bearings controlled by current PD feedback controllers. The presence of both transverse cracks is described by a local flexibility matrix introduced by Papadopoulos and Dimarogonas, in [8]. The local flexibility matrix is considered for six degrees of freedom in one node of shaft element. Breathing of cracks is implicated in the equations of motion and partial opening and closing of cracks is determined by signs of stress intensity factors along crack edges. Response of rotor system is obtained by direct integration of nonlinear motion equations. The influence of various orientations of cracks is discussed using orbit plots, vibration responses and frequency spectra.

2 EQUATIONS OF MOTION OF A TWO-CRACK ROTOR SYSTEM

Transverse surface cracks introduce an increase of a local flexibility of a shaft, which can be described by the local cracked flexibility matrix and by the coupling between lateral, longitudinal and torsional vibration. Let's consider a cracked shaft element. The crack is situated at a distance *x* from the first node as shown in Fig. 1a.

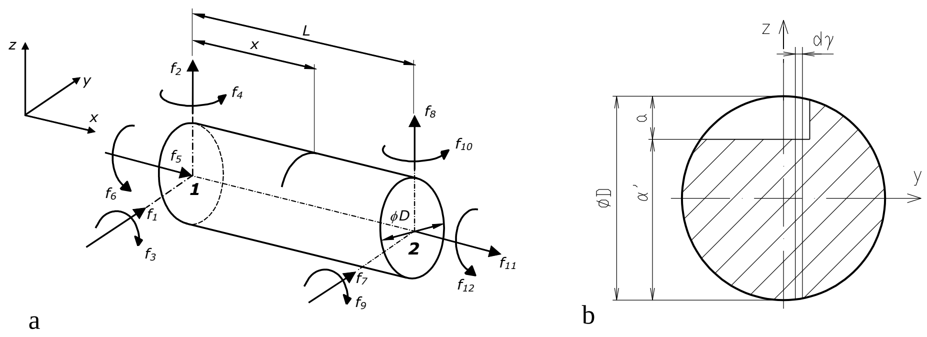

Fig. 1 (a) A cracked shaft element (b) The cross-section of shaft with the crack.

The shaft element has the diameter D and the length L. The generalized nodal forces for the first node are shear forces f_1 and f_2 in y and z direction respectively, bending moments f_3 and f_4 about y and z axes respectively, axial force f_5, and torsional moment f_6. The generalized nodal forces for the second node are shear forces f_7 and f_8 in y and z direction respectively, bending moments f_9 and f_{10} about y and z axes respectively, axial force f_{11}, and torsional moment f_{12}.

Matrix of local flexibility of the cracked shaft element is determined on basis concepts of fracture mechanics. Flexibility matrix is derived using Castingliano's theorem. Additional displacements due to the crack are found using strain energy. Strain energy density function of the cracked element is stated by stress intensity factors for Mode I, II, and III. The stress intensity factors are not expressed for the crack in a shaft with circular cross section, see Fig. 1b. Therefore the factors have to be compounded from elementary rectangular strips ($\alpha' \cdot \mathrm{d}\gamma$) and their total value is given by integration over surface of the crack with depth a. All elements of a local cracked flexibility matrix can be found in the article of Darpe, et al. [9]. The total flexibility matrix of a cracked shaft element is given by sum of the flexibility matrix without the crack and the local flexibility matrix of the crack. The total flexibility matrix of a cracked shaft element has to be transformed to the fixed coordinate system, because the equations of motion for the system are expressed in this coordinate system.

During a rotating of shaft, the transverse crack can be opened, closed or these two states can change one to another. Vibration of rotor systems with opened crack is treated similar as vibration of non-symmetrical cross-section shaft. If the crack keeps opening and closing during a shaft rotation, we refer it as „breathing" crack. This may occur if vibration amplitude excited by centrifugal forces is lower than static deflection.

In case of the breathing crack, elements of the flexibility matrix of a shaft element with the crack are dependent on size of opening or closing of the crack. Implementation of the breathing crack to mathematical model is provided by „crack closure line" method (CCL). The method was proposed in the paper [9], where CCL is defined as a virtual line perpendicular to the edge of the crack and simultaneously the line separates opened and closed part of the crack. The crack front is divided by several uniformly spaced points, where the total stress intensity factor is determined, considering Mode I of loading. Positive value of stress intensity factor shows, there is tensile loading in examined point thus the crack is opened and vice versa. The mathematical model of the breathing crack using signs of stress intensity factors in the described points gives the most accurate approach known in the present. The approach is applicable in whole range of operational revolutions and also in case of transient problems.

It is assumed the model of the cracked rotor system has following properties: (i) shaft is represented by a flexible beam-like body that is discretized into finite elements, (ii) shaft element is derived by using Bernoulli's beam theory, (iii) stationary part is rigid, (iv) discs are considered to be absolutely rigid axisymmetric bodies, (v) inertia and gyroscopic effects of the rotating parts are taken into account, (vi) material damping of the shaft and other types of damping are assumed linear, (vii) connection of stationary part and the shaft is provided by the radial magnetic bearings and massless axial bearing, (viii) there are transverse cracks in the shaft, crack edges are straight and the cracks affects only stiffness matrix of the rotor system, (ix) the rotor is loaded by discrete and distributed forces of constant or periodic time behaviour and (x) the rotor rotates at constant angular velocity.

Considering the cracked rotor, the equations of motion for the rotor system supported by the radial magnetic bearings are expressed in the fixed coordinate system as follows

$$\mathbf{M}\ddot{\mathbf{q}}(t) + (\mathbf{B} + \eta_\mathrm{V}\mathbf{K}_\mathrm{SH} + \omega\mathbf{G})\dot{\mathbf{q}}(t) + \{\mathbf{K}[\mathbf{q}(t)] + \omega\mathbf{K}_\mathrm{C}\}\mathbf{q}(t) = \mathbf{f}_\mathrm{M}(\mathbf{q},\mathbf{i}) + \mathbf{f}_\mathrm{A}(t) + \mathbf{f}_\mathrm{V}\,,$$
$$\mathbf{q}_\mathrm{OP} = \mathbf{q}_\mathrm{OP}(t)\,, \qquad \mathbf{q}(0) = \mathbf{q}_0\,, \qquad \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}_0\,. \tag{1}$$

$\mathbf{M}$, $\mathbf{B}$, $\mathbf{G}$, $\mathbf{K}$, $\mathbf{K}_\mathrm{SH}$ and $\mathbf{K}_\mathrm{C}$ denote mass matrix, damping matrix (external damping and material damping of stationary part), matrix of gyroscopic effects, stiffness matrix of the rotor system with the cracks, stiffness matrix of the shaft and circulation matrix of the rotor system, $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ are vectors of generalized displacements, velocities and accelerations, $\mathbf{q}_\mathrm{OP}, \mathbf{q}_0, \dot{\mathbf{q}}_0$ denote vectors of boundary and initial conditions, $\mathbf{f}_\mathrm{M}$, $\mathbf{f}_\mathrm{A}$, $\mathbf{f}_\mathrm{V}$, $\mathbf{i}$ are vectors of magnetic forces, generalized forces exerting on the system (external and constraint forces), controlling currents in coils of electromagnet, η_V is material damping coefficient of the shaft, ω is angular velocity of the shaft and t denotes time.

Magnetic forces are incorporated into the equations of motion (1) by nonlinear vector of additional loading $\mathbf{f}_M$. If the breathing crack model is used, then the stiffness matrix of the system is dependent on the vector of generalized displacements. For this purpose the stiffness matrix is updated after every single degree of rotation.

3 THE INVESTIGATED ROTOR SYSTEM

The examined rotor system (Fig. 2a) consists of shaft (SH) driven by electromotor (EM) and two discs (D1, D2) mounted at the end of the shaft. The shaft and electromotor are connected by coupling (CO). The rotor is supported by two radial active magnetic bearings (MB1, MB2) and these are connected to base platform (BP). There is the first crack (CR1) between disc 1 and magnetic bearing 2 and there is the second crack (CR2) between magnetic bearing 1 and magnetic bearing 2.

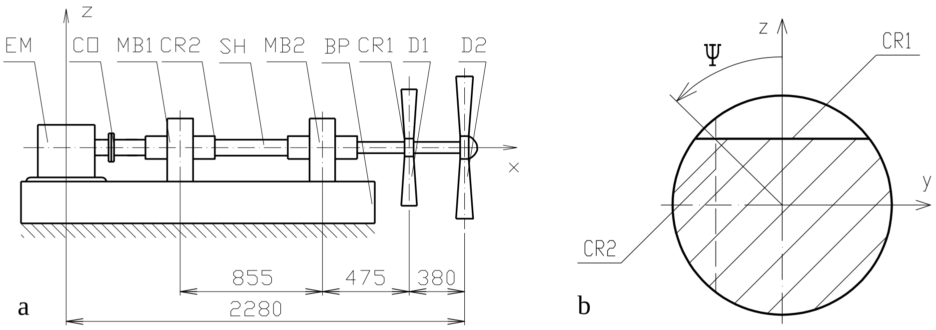

Fig. 2 (a) Scheme of the testing rotor system with the two transverse cracks
(b) Orientation of the cracks.

The rotor system is supported by the two identical radial active magnetic bearings consisting of four electromagnets, which are uniformly distributed around the bearing circumference. The magnetic bearings are in horizontal and vertical direction controlled by two independent current PD feedback controllers. The discrete axial bearing stiffness is in the axial direction $k_x = 1 \cdot 10^5\,\mathrm{N\,m^{-1}}$ and torsional stiffness is $k_t = 1\,\mathrm{N\,m\,rad^{-1}}$. Unless otherwise indicated, eccentricities (ε = 0.005 mm) of both discs are identical and placed on the symmetry axis of the crack CR1 (positive coordinate of z axis in Fig. 1b and Fig. 2b). We suppose, angle between cracks axes is not changing. The angle is denoted by ψ (see Fig. 2b). More detailed data regarding to the investigated rotor, radial active magnetic bearings and PD controllers are given in [10].

In the computational model the shaft was represented by a beam-like body that was discretized into fifty four finite elements of equal length. Discs are implemented by moments of inertia and masses applied in nodes accordant with centres of discs.

The task was to analyze the effect of mutual position of two cracks on steady-state vibration excited by centrifugal forces, in conjunction with rotor system actively controlled. Numerical experiments were provided in computer system MATLAB, considering the described system with the two breathing cracks sketched in Fig. 2a. Depth of the cracks is 17.6 mm; for CR1 it is 20 % of shaft diameter and for CR2 it is 18.5 % of shaft diameter. Value of revolutions of 18.1 $\mathrm{rad \cdot s^{-1}}$ is more than twice lower than first bending critical revolutions.

4 RESULTS OF THE COMPUTATIONAL SIMULATIONS

Orbits of the shaft centre in D2 location are shown in Fig. 3 to 4. The orbits are plotted for variety angles between the two cracks (CR1 and CR2) ψ = 0°, ψ = 90°, ψ = 180° and ψ = 270°. The orbit is shaped into complex curve and there is small loop in the upper part of the orbit. The loop is corresponding to the second multiplication of rotation revolutions. The loop position depends on the orientation of the cracks. The loop is the most significant in case of crack axes orientation of 180°.

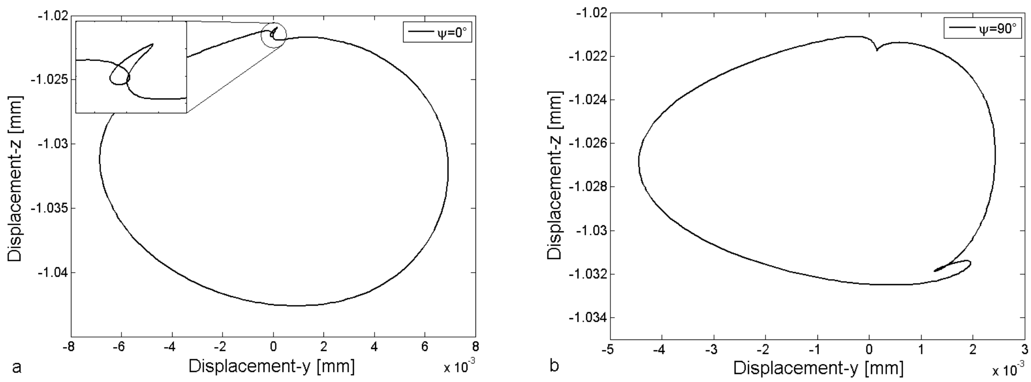

Fig. 3 Trajectory of D2 centre in steady-state for the rotor system with two cracks of depth 17.6 mm; (a) ψ = 0° (b) ψ = 90°.

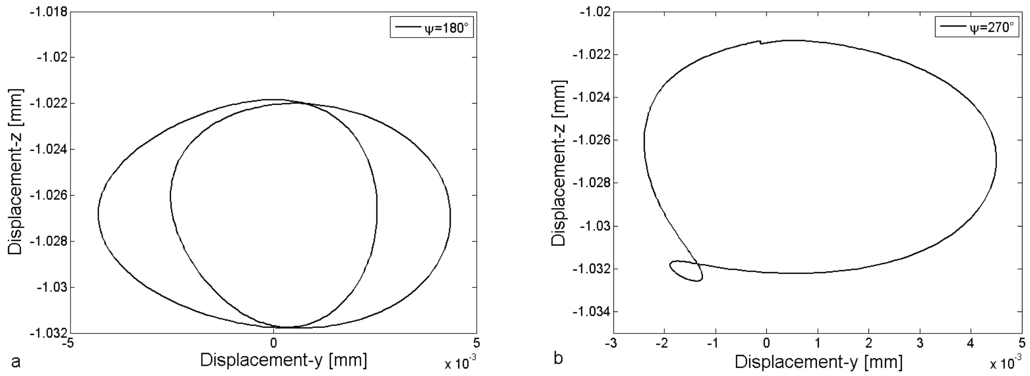

Fig. 4 Trajectory of D2 centre in steady-state for the rotor system with two cracks of depth 17.6 mm; (a) ψ = 180° (b) ψ = 270°.

After Fourier's transformation to frequency domain, we obtain graphs shown in Fig. 5 to 8. For ψ = 0°, so directions of crack axes are identical; first and second multiple of revolution frequency have similar value, considering frequency spectrum of horizontal displacements. Third multiple is also significant and 2.5 times lower. In case of vertical displacements, first four multiples (w, $2w$, $3w$, $4w$) of revolution frequency are significant. Values of multiple amplitudes are decreasing with increasing frequency.

For ψ = 90°, so axes of the cracks are perpendicular to each other; frequency spectrum changes significantly in both directions, see Fig. 6. Amplitude accordant with revolution frequency dominates in both directions of vibration. Amplitude accordant with second multiple of revolution frequency is 2.6 times lower than the revolution frequency amplitude in case of horizontal direction, see Fig. 6a. As for vertical direction, this frequency does not appear at all. Orbit is a half smaller comparing with ψ = 0° and shape has slightly changed, see Fig. 3.

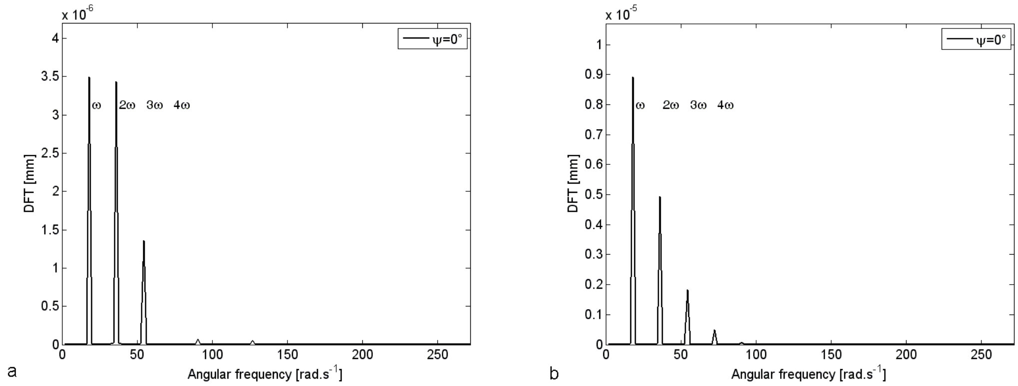

Fig. 5 (a) Fourier's transformation of displacements in horizontal direction in D2 location (b) Fourier's transformation of displacements in vertical direction in D2 location.

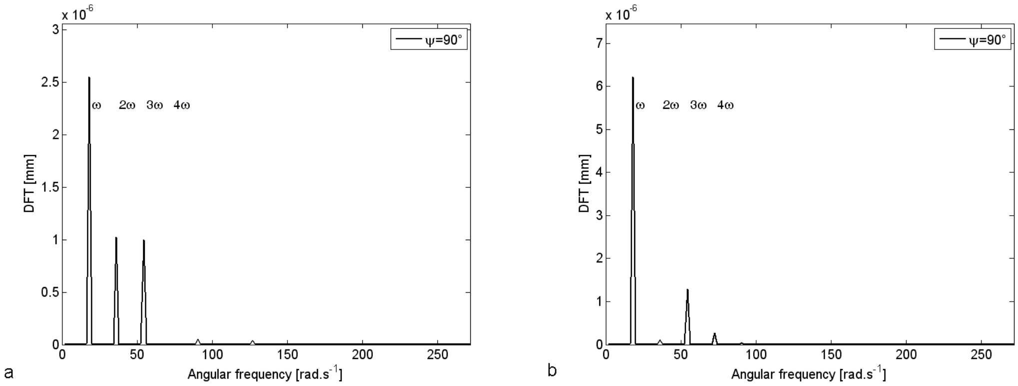

Fig. 6 (a) Fourier's transformation of displacements in horizontal direction in D2 location (b) Fourier's transformation of displacements in vertical direction in D2 location.

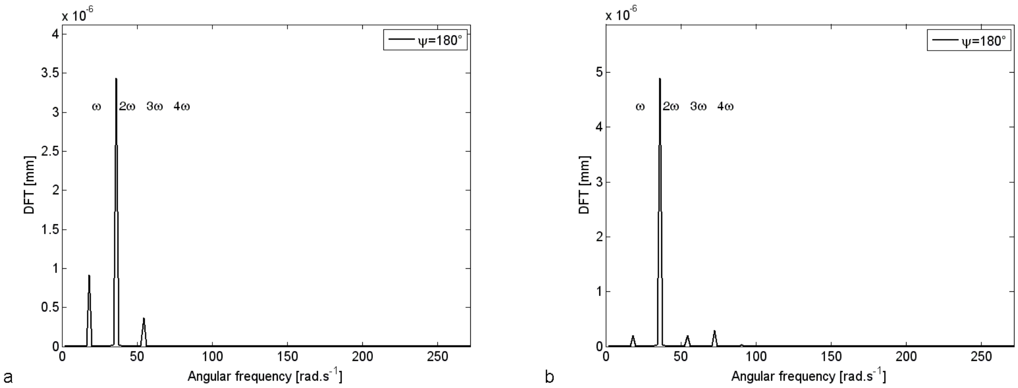

Fig. 7 (a) Fourier's transformation of displacements in horizontal direction in D2 location (b) Fourier's transformation of displacements in vertical direction in D2 location.

If angle of cracks axes is ψ = 180°; frequency spectrum changes significantly once again, see Fig. 7. Amplitude of second revolution frequency multiple dominates over the frequency spectrum in both directions. This implies, orbit is shaped into double loop, see Fig. 4a.

Amplitudes and distribution of revolution frequency multiples is shown in Fig. 8 for angle ψ = 270°. These characteristics are similar to those ones measured for angle ψ = 90°, see Fig. 6. Orbits are different, but orbit sizes are almost same; compare Fig. 3b and Fig. 4b.

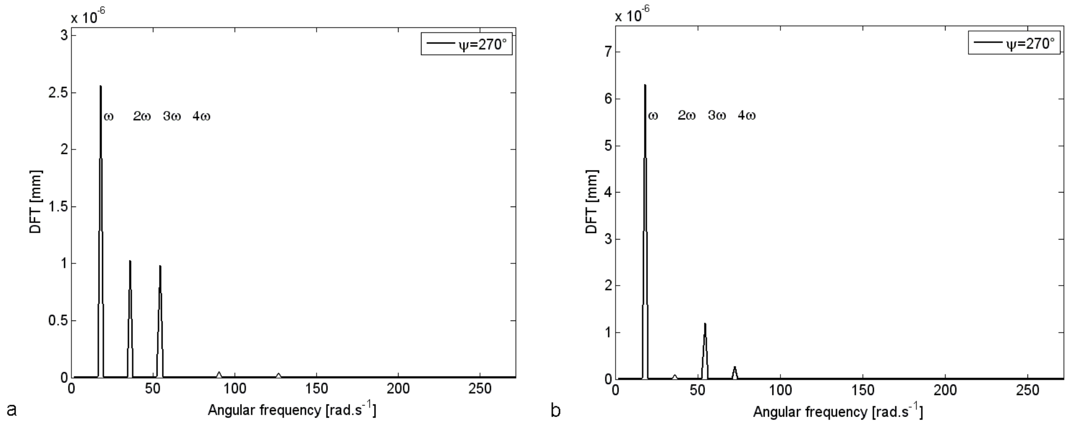

Fig. 8 (a) Fourier's transformation of displacements in horizontal direction in D2 location (b) Fourier's transformation of displacements in vertical direction in D2 location.

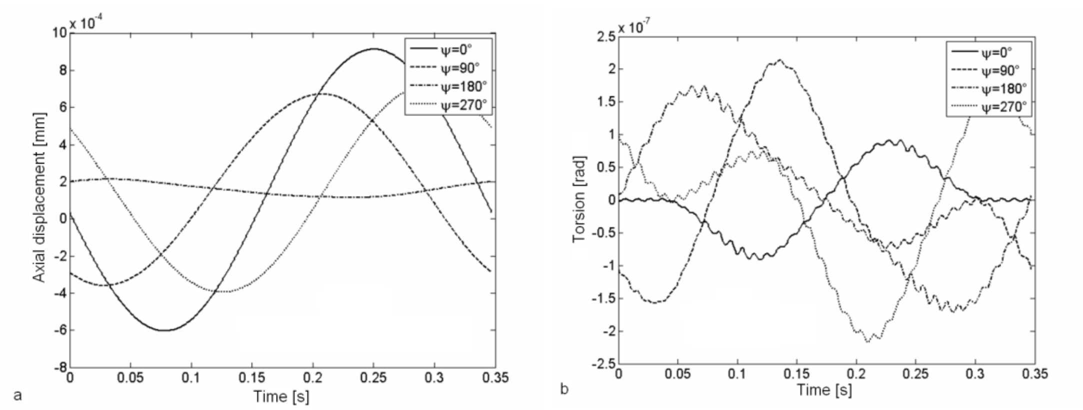

Fig. 9 (a) Time behaviour of axial displacements of D2 centre and (b) time behaviour of torsion of D2 centre.

Excitation by centrifugal forces initiated also axial and torsional vibration of the rotor, see Fig. 9. The phenomenon is caused by the presence of the transverse cracks. Frequency spectrums obtained by Fourier's transformation are shown in Fig. 10 to 13.

In case of axial vibration, amplitude of revolution frequency was dominant for all cases of angle between axes of cracks (ψ = 0°, ψ = 90°, ψ = 180° and ψ = 270°). Just for angle of cracks ψ = 180°, second multiple of revolution frequency was excited with magnitude 6.7 times smaller, see Fig. 12a. The highest amplitude of revolution frequency is excited for angle ψ = 0° and it is lowest for angle ψ = 180° and values of amplitudes of revolution frequency for angles ψ = 90° and ψ = 270° are between these extremes.

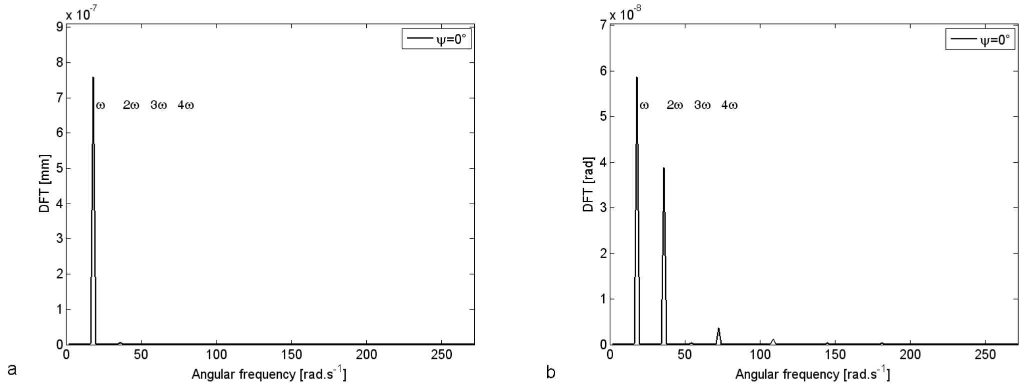

Fig. 10 (a) Fourier’s transformation of axial displacements in D2 location
(b) Fourier’s transformation of torsion in D2 location.

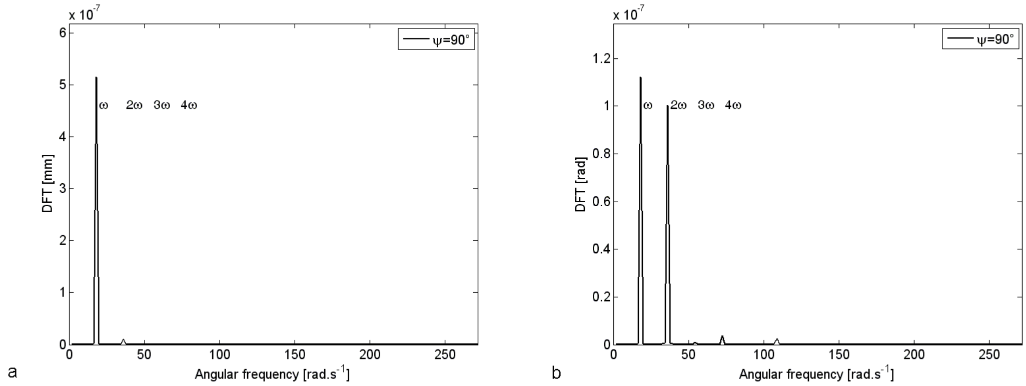

Fig. 11 (a) Fourier’s transformation of axial displacements in D2 location
(b) Fourier’s transformation of torsion in D2 location.

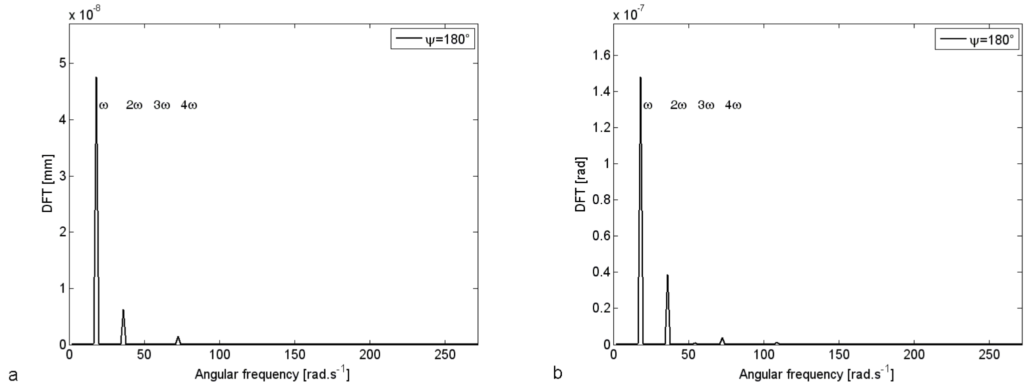

Fig. 12 (a) Fourier’s transformation of axial displacements in D2 location
(b) Fourier’s transformation of torsion in D2 location.

Considering torsional vibration, frequency spectrums are characteristic by significant amplitudes corresponding with first and second multiple of revolution frequency for all cases of angles between cracks axes ($\psi = 0°$, $\psi = 90°$, $\psi = 180°$ and $\psi = 270°$), see Fig. 10 to 13. Ratio of second to first multiple of revolution frequency is the highest for angles $\psi = 90°$ and $\psi = 270°$ and the lowest for $\psi = 180°$.

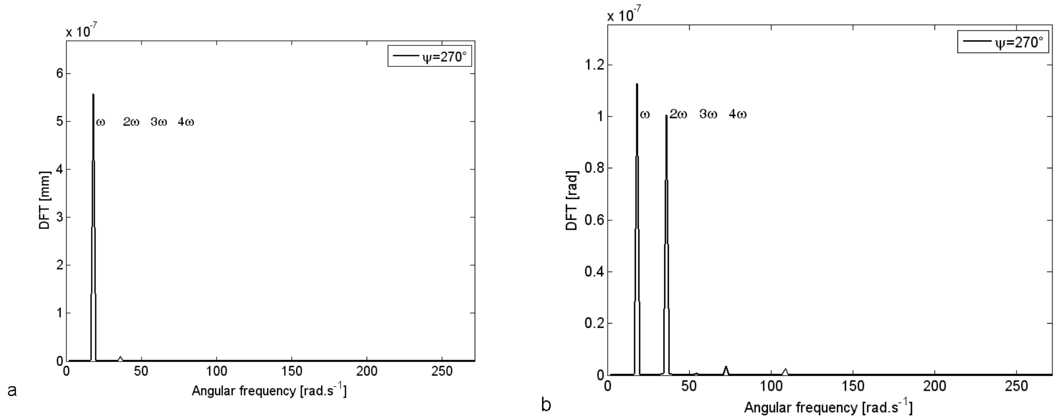

Fig. 13 (a) Fourier's transformation of axial displacements in D2 location
(b) Fourier's transformation of torsion in D2 location.

5 CONCLUSIONS

The paper deals with influence of two transverse surface cracks on the rotor system supported by radial active magnetic bearings. Response of the rotor system was determined from the equations of motion, where the cracks were implemented by nonlinear breathing model. The breathing model employs of stress intensity factor for Mode I.

Time behaviours and frequency spectrums of bending, axial and torsional vibration were calculated. Significant influences of the cracks were found for low operating revolutions. Various multiples of revolution frequency were excited and also ratios of amplitudes corresponding with these multiples were essentially different. Therefore presented results provide proposals of detection of transverse surface cracks in the shaft.

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REFERENCES

[1] DIMAROGONAS, A. D. Vibration of Cracked Structures: A State of the Art Review. Engineering Fracture Mechanics. 1996, 55. Nr. 5, pp. 831-857. ISSN 0013-7944.

[2] SEKHAR, A. S. Vibration Characteristics of a Cracked Rotor with Two Open Cracks. Journal of Sound and Vibration. 1999, 223. Nr. 4, pp. 497-512. ISSN 0022-460X.

[3] DARPE, A. K. & GUPTA, K. & CHAWLA, A. Dynamics of a Two-Crack Rotor. Journal of Sound and Vibration. 2003, 259. Nr. 3, pp. 649-675. ISSN 0022-460X.

[4] EWINS, D. J. & ROBB, D. A. & ZHU, C. The Dynamics of a Cracked Rotor with an Active Feedback Control System. In Proceedings of the Eighth International Symposium of Magnetic Bearings. Mito, Japan: Department of Mechanical Engineering, Ibaraki University, 2000, pp. 323-330.

[5] EWINS, D. J. & ROBB, D. A. & ZHU, C. The Dynamics of a Cracked Rotor with an Active Magnetic Bearing. Journal of Sound and Vibration. 2003, 265. Nr. 3, pp. 469-487. ISSN 0022-460X.
[6] KASARDA, M. & MANI, G. & QUINN, D. D. Active Health Monitoring in a Rotating Cracked Shaft Using Active Magnetic Bearings as Force Actuators. Journal of Sound and Vibration. 2006, 294. Nr. 3, pp. 454-465. ISSN 0022-460X.
[7] FERFECKI, P. Dynamic Analysis of a Rotor System Supported by Radial Active Magnetic Bearings with the Shaft Containing Both Transverse Crack and Slant Crack. In Proceedings Engineering Mechanics 2008. Svratka, Czech Republic: Institute of Thermomechanics AS CR, 2008, pp. 1-10. ISBN 978-80-87012-11-6. In Czech.
[8] PAPADOPOULOS, C. A. & DIMAROGONAS, A. D. Coupled Longitudinal and Bending Vibrations of a Rotating Shaft with an Open Crack. Journal of Sound and Vibration. 1987, 117. Nr. 1, pp. 81–93. ISSN 0022-460X.
[9] DARPE, A. K. & GUPTA, K. & CHAWLA, A. Coupled Bending, Longitudinal and Torsional Vibrations of a Cracked Rotor. Journal of Sound and Vibration. 2004, 269. Nr. 1-2, pp. 33-60. ISSN 0022-460X.
[10] FERFECKI, P. Computational Modelling of a Rotor System Supported by Radial Active Magnetic Bearings. Ph.D. thesis. Ostrava: VŠB-TUO, Faculty of Mechanical Engineering, 2005. 120 pp. ISBN 80-248-0872-2.