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APPLICATION OF ROBUST CONTROL ALGORITHMS IN VECTOR ORIENTED CONTROL OF INDUCTION MOTOR

APLIKACE ROBUSTNÍCH ALGORITMŮ PŘI VEKTOROVĚ ORIENTOVANÉM ŘÍZENÍ ASYNCHRONNÍHO MOTORU

Abstract

The paper describes the design of robust control algorithms for the vector oriented control of an induction motor using the state variable aggregation method. The advantage of these control algorithms is their high robustness and simplicity. The control algorithms were designed for the MIMO (three-input/three-output) non-linear mathematical model of an induction motor in an orthogonal coordinate system synchronously rotating with a reference frame. The designed control algorithms were verified by computer simulation in the program MATLAB – Simulink.

Abstrakt

Příspěvek popisuje návrh robustních algoritmů řízení pro vektorově orientované řízení asynchronního motoru pomocí metody agregace stavových proměnných. Její výhodou je vysoká robustnost a jednoduchost. Algoritmy řízení byly navrženy pro trojrozměrový (tři vstupy/tři výstupy) nelineární matematický model asynchronního motoru V pravoúhlém souřadném systému rotujícím synchronně S prostorovým vektorem statorového proudu. Navržené algoritmy řízení byly ověřeny počítačovou simulací V programu MATLAB – Simulink.

1 INTRODUCTION

The paper deals with a design of robust control algorithms, which operates in sliding modes using a non-linear control synthesis called the state variable aggregation method. This method allows the design of the non-robust control (knowledge of the mathematical model of controlled subsystem and disturbances is required), the robust control with a high gain and the robust sliding mode control (SMC) as well [5, 6, 10]. The vector-field oriented control is achieved by adjusting the magnitude and angular frequency of rotor flux linkage [1 - 3, 8, 9]. It means that the rotor flux linkage must be adjusted synchronously with the reference frame and the magnitude must be controlled by stator currents on a constant value [3]. Control variables are the reference frame angular velocity and components of the stator current vector. State variables are the rotor angular velocity and components of the pseudocurrent vector.

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2 MATHEMATICAL MODEL OF INDUCTION MOTOR

An induction motor is a generally complex non-linear subsystem, which can be described in orthogonal coordinate system *d,q* synchronously rotating with the reference frame by the following equations [3]:

voltage equations

$$\boldsymbol{u}_s = R_s \boldsymbol{i}_s + \frac{\mathrm{d}\boldsymbol{\Psi}_s}{\mathrm{d}t} + \mathrm{j}\omega_s \boldsymbol{\Psi}_s \tag{1}$$

$$\boldsymbol{u}_r = R_r \boldsymbol{i}_r + \frac{\mathrm{d}\boldsymbol{\Psi}_r}{\mathrm{d}t} + \mathrm{j}\omega_{sk} \boldsymbol{\Psi}_r \tag{2}$$

flux linkage equations

$$\boldsymbol{\Psi}_s = L_s \boldsymbol{i}_s + L_m \boldsymbol{i}_r \tag{3}$$

$$\boldsymbol{\Psi}_r = L_r \boldsymbol{i}_r + L_m \boldsymbol{i}_s \tag{4}$$

electromagnetic torque

$$m_e = \frac{3}{2} p \,\mathrm{Im}\left\{\boldsymbol{i}_r^{*} \boldsymbol{\Psi}_s\right\} \tag{5}$$

where $\boldsymbol{u}_s$ is the stator voltage vector [V], $\boldsymbol{u}_r$ – the rotor voltage vector (for squirrel cage rotor $\boldsymbol{u}_r = 0$) [V], $\boldsymbol{i}_s$ – the stator current vector [A], $\boldsymbol{i}_r$ – the rotor current vector [A], $\boldsymbol{\Psi}_s$ – the stator flux linkage vector [Wb], $\boldsymbol{\Psi}_r$ – the rotor flux linkage vector [Wb], R_s – the stator resistance [Ω], R_r – the rotor resistance [Ω], L_s – the stator self inductance [H], L_s – the rotor self inductance [H], L_m – the mutual inductance [H], p – the number of poles [-], Im – the imaginary number, *– the complex conjugate, ω_s – the reference frame angular velocity [rad·s^{-1}], ω_{sk} – the slip angular velocity [rad·s^{-1}].

When the stator current is explicitly known, it is evident that only the rotor circuit voltage equations are required to describe the dynamic behavior of the electrical part of the induction motor. Then the stator current can be considered as a control input to the rotor circuit. If we substitute (4) into (2), we get

$$L_r \frac{\mathrm{d}\boldsymbol{i}_r}{\mathrm{d}t} = -R_r \boldsymbol{i}_r - \mathrm{j}\omega_{sk} L_r \boldsymbol{i}_r - \mathrm{j}\omega_{sk} L_m \boldsymbol{i}_s - L_m \frac{\mathrm{d}\boldsymbol{i}_s}{\mathrm{d}t} \tag{6}$$

In order to avoid difficulties, we could define pseudocurrent variables corresponding to the rotor flux linkage by [3]

$$\boldsymbol{\Psi}_r = L_r \boldsymbol{i}_m \tag{7}$$

From the relations (4) and (7), we obtain

$$\boldsymbol{i}_m = \frac{L_m}{L_r} \boldsymbol{i}_s + \boldsymbol{i}_r \tag{8}$$

After substitution (8) into (6) and further their modification we obtain

$$\frac{\mathrm{d}\boldsymbol{i}_m}{\mathrm{d}t} = -\frac{1}{T_r} \boldsymbol{i}_m - \mathrm{j}(\omega_s - \omega_r)\boldsymbol{i}_m + \frac{k_r}{T_r} \boldsymbol{i}_s \tag{9}$$

where $T_r = \dfrac{L_r}{R_r}$ and $k_r = \dfrac{L_m}{L_r}$

In according with the pseudocurrent defined in (7), the electromagnetic torque m_e is obtained from (5)

$$m_e = \frac{3}{2} p L_m \left(i_{sq} i_{md} - i_{sd} i_{mq}\right) \tag{10}$$

The dynamic behavior of the induction motor can be described by the torque equation

$$J \frac{\mathrm{d}\omega_r}{\mathrm{d}t} = -B\omega_r + m_e - m_z \tag{11}$$

where m_z is the load torque [N·m], J – the total inertia [kg·m^2], B – the dumping factor [kg·s^{-1}], m_e – the motor torque [N·m], ω_r – the rotor angular velocity[rad·s^{-1}].
After substitution (10) into (11) and itemizing of (9) in the component form we obtain [3]

$$\frac{\mathrm{d}\omega_r}{\mathrm{d}t} = -\frac{B}{J}\omega_r + \frac{3p^2L_m}{2J}\left(i_{sq}i_{md} - i_{sd}i_{mq}\right) - \frac{p}{J}m_z \tag{12a}$$

$$\frac{\mathrm{d}i_{md}}{\mathrm{d}t} = -\frac{1}{T_r}i_{md} + \left(\omega_s - \omega_r\right)i_{mq} + \frac{k_r}{T_r}i_{sd} \tag{12b}$$

$$\frac{\mathrm{d}i_{mq}}{\mathrm{d}t} = -\frac{1}{T_r}i_{mq} - \left(\omega_s - \omega_r\right)i_{md} + \frac{k_r}{T_r}i_{sq} \tag{12c}$$

The equations (12) represent the 3rd order mathematical model of the induction motor. That mathematical model has three input variables - ω_s, i_{sd}, i_{sq} and three output variables - ω_r, i_{md}, i_{mq}. It should be noted that the above description is related to orthogonal coordinate system d,q synchronously rotating with a reference frame.

After substitution

$$x_1 = \omega_r \qquad u_1 = \omega_s \qquad v = m_z$$
$$x_2 = i_{md} \qquad u_2 = i_{sd} \tag{13}$$
$$x_3 = i_{mq} \qquad u_3 = i_{sq}$$

we obtain the mathematical model of the induction motor in the standard form [10]

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},v) + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{u} \tag{14a}$$

$$\boldsymbol{x} = \left[x_1, x_2, x_3\right]^T \qquad \boldsymbol{u} = \left[u_1, u_2, u_3,\right]^T \tag{14b}$$

$$\boldsymbol{f}(\boldsymbol{x},v) = \begin{bmatrix} -a_1x_1 & -a_2v \\ -x_1x_3 & -a_3x_2 \\ x_1x_2 & -a_3x_3 \end{bmatrix} \tag{14c}$$

$$\boldsymbol{B}(\boldsymbol{x}) = \begin{bmatrix} 0 & -b_1x_3 & b_1x_2 \\ x_3 & b_2 & 0 \\ -x_2 & 0 & b_2 \end{bmatrix} \tag{14d}$$

$$a_1 = \frac{B}{J},\ a_2 = \frac{p}{J},\ a_3 = \frac{1}{T_r},\ b_1 = \frac{3p^2L_m}{2J},\ b_2 = \frac{k_r}{T_r} \tag{14e}$$

where T is the transposition symbol.

3 DESIGN OF ROBUST CONTROL

Because the model of the induction motor (14) is in the standard form, we can use the state variables aggregation method directly [10] for the design of a robust control.

On the basis of the state variable aggregation method the optimal feedback control

$$\boldsymbol{u}^* = \boldsymbol{B}^{-1}(\boldsymbol{x})\left[\boldsymbol{T}^{-1}\boldsymbol{e} + \dot{\boldsymbol{x}}^w - \boldsymbol{f}(\boldsymbol{x},v)\right] \tag{15}$$

can be obtained, which minimizes the following quadratic objective functional

$$J = \int_0^\infty \left(\boldsymbol{e}^T\boldsymbol{e} + \dot{\boldsymbol{e}}^T\boldsymbol{T}^2\dot{\boldsymbol{e}}\right)\mathrm{d}t \tag{16}$$

The optimal feedback control (15) causes the optimal closed-loop control system

$$\dot{\boldsymbol{e}} + \boldsymbol{T}^{-1}\boldsymbol{e} = \boldsymbol{0},\ \boldsymbol{e} = \boldsymbol{x}^w - \boldsymbol{x} \tag{17}$$

where $\boldsymbol{T}$ is the diagonal matrix of positive time constants, $\boldsymbol{e}$ – the error vector.

The approximate optimal control can be obtained on the basis of the formula [10]

$$\boldsymbol{u}^{\times} = \boldsymbol{\Theta}\int_0^{\infty} \boldsymbol{m}^{w}\, \mathrm{d}\tau + \boldsymbol{u}_0 = \boldsymbol{\Theta}\left(\boldsymbol{T}^{-1}\int_0^{t} \boldsymbol{e}\, \mathrm{d}\tau + \boldsymbol{e} - \boldsymbol{e}_0\right) + \boldsymbol{u}_0\,,\ \ \boldsymbol{m}^{w} = \dot{\boldsymbol{e}} + \boldsymbol{T}^{-1}\boldsymbol{e} \tag{18}$$

where $\boldsymbol{\Theta}$ is the diagonal matrix of gains, $\boldsymbol{u_0}$ – initial control.

The relation (19) expresses the robust control algorithm with high gain and it can be written in the form [10]

$$\boldsymbol{u}^{\times} = \boldsymbol{\Theta}\boldsymbol{m} + \boldsymbol{u}_0\,,\ \ \boldsymbol{m} = \boldsymbol{T}^{-1}\int_0^{t} \boldsymbol{e}\, \mathrm{d}\tau + \boldsymbol{e} - \boldsymbol{e}_0 \tag{19}$$

The robust control with high gain has a negative feature consisting in the high values of elements of the diagonal matrix $\boldsymbol{\Theta}$, which can cause high unacceptable values of the control variables. It can be a negative influence for the control system stability [5, 6, 10]. This can be avoided by adding non-linear function *sgn* into control algorithm in accordance with the following scheme [10]

$$\boldsymbol{u}^{sl} = \boldsymbol{U}^{m}\mathbf{sgn}(\boldsymbol{m}) + \boldsymbol{u}_0 \tag{20a}$$

$$\boldsymbol{U}^{m} = \mathrm{diag}\left[u_1^m, u_2^m, \ldots, u_m^m\right] \tag{20b}$$

$$\mathbf{sgn}(\boldsymbol{m}) = \mathrm{diag}\left[\mathrm{sgn}(m_1), \mathrm{sgn}(m_2), \ldots, \mathrm{sgn}(m_m)\right] \tag{20c}$$

$$\mathrm{sgn}(m_j) = \begin{cases} 1 & \text{for} \quad m_j > 0 \\ -1 & \text{for} \quad m_j < 0 \end{cases} \tag{20d}$$

where *sgn* is the sign function, u_j^m - marginal values of control variables.

The robust control algorithm designed for the induction motor (14) from Chapter 2 in component form is

$$u_1^{sl} = u_1^m \,\mathrm{sgn}\left(\frac{1}{T_1}\int_0^t e_1\, \mathrm{d}\tau + e_1 - e_{10}\right) + u_{10} \tag{21a}$$

$$u_2^{sl} = u_2^m \,\mathrm{sgn}\left(\frac{1}{T_2}\int_0^t e_2\, \mathrm{d}\tau + e_2 - e_{20}\right) + u_{20} \tag{21b}$$

$$u_3^{sl} = u_3^m \,\mathrm{sgn}\left(\frac{1}{T_3}\int_0^t e_3\, \mathrm{d}\tau + e_3 - e_{30}\right) + u_{30} \tag{21c}$$

The robust control algorithm with a sign function is characterized by the high activity of the control, which could be undesirable in some cases. To avoid the high activity of the control the *sgn* function can be replaced by the continuous *fce* function, because the approximate equality

$$\mathrm{sgn}(x) = \frac{x}{|x|} \approx \frac{x}{|x| + \varepsilon} = \mathrm{fce}(x) \tag{22}$$

holds, where ε is the small positive number, see Fig. 1.

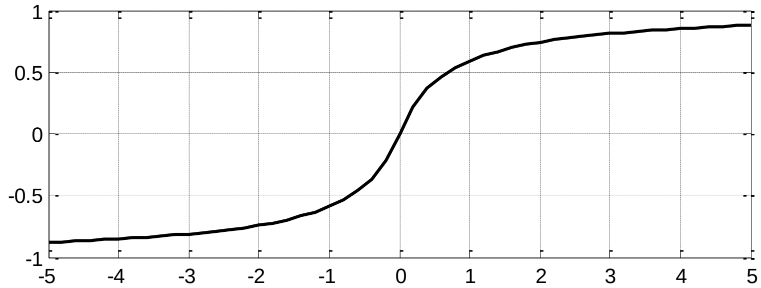

Fig. 1 Plot of *fce* function (ε = 0.7).

The *fce* function is very similar to the hyperbolic tangent function, which is used very often in sliding mode control, but the *fce* function is much easier for the computing than the hyperbolic tangent function.

4 SIMULATION VERIFICATION

The designed control algorithms were verified by the computer simulation in the program MATLAB – Simulink. The model parameters of the induction motor: $p = 2$, $R_s = 2.2842\ \Omega$, $R_r = 2.2878\,\Omega$, $J = 0.0179\,\text{kg}\cdot\text{m}^2$, $L_s = 28.3\,\text{mH}$, $L_r = 28.8\,\text{mH}$, $L_m = 86.8\,\text{mH}$. The parameters of the control algorithm were chosen: $T_1 = 0.5\,\text{s}$, $T_2 = T_3 = 0.02\,\text{s}$.

The rotor angular velocity ω_w was defined in the signal builder block as a variable required course (solid line in Figs 2a and 3a), the required values of current components were set $i_{md} = 10\ \text{A}$ and $i_{mq} = 0\ \text{A}$.

Figs 2a and 3a show that the shapes of real and required revolutions are close, though the model of the induction motor is not known. The robust control algorithm well removes the influence of disturbances as well. By comparing of Figs 1b,d,f and 2b,d,f it is obvious, that the second designed control algorithm is characterized by the lower activity of the control variables. The activity of a control algorithm generally depends on the value of ε (for $\varepsilon = 0$ *fce* = *sgn*).

5 CONCLUSIONS

The contribution presents the possibility of using the robust control algorithm working in a sliding mode in the vector oriented control of the induction motor. There are described and compared two modifications of robust control algorithms: discontinuous (with *sgn* function) and with continuous substitutions derived from a sign function. All designed algorithms were verified by computer simulation using the simulation program MATLAB-Simulink.

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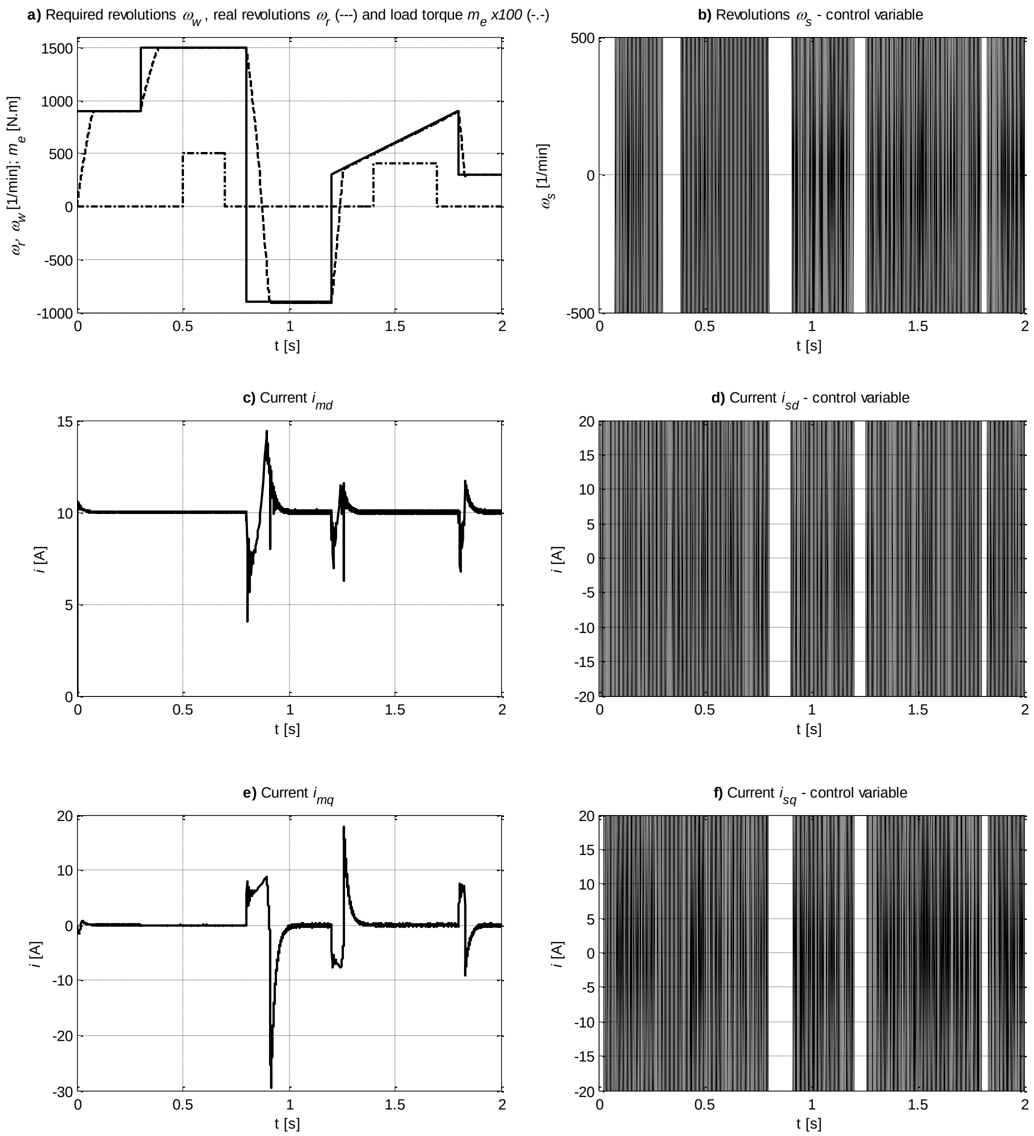

Fig. 2 Simulation results for a robust control algorithm with *sgn* function.

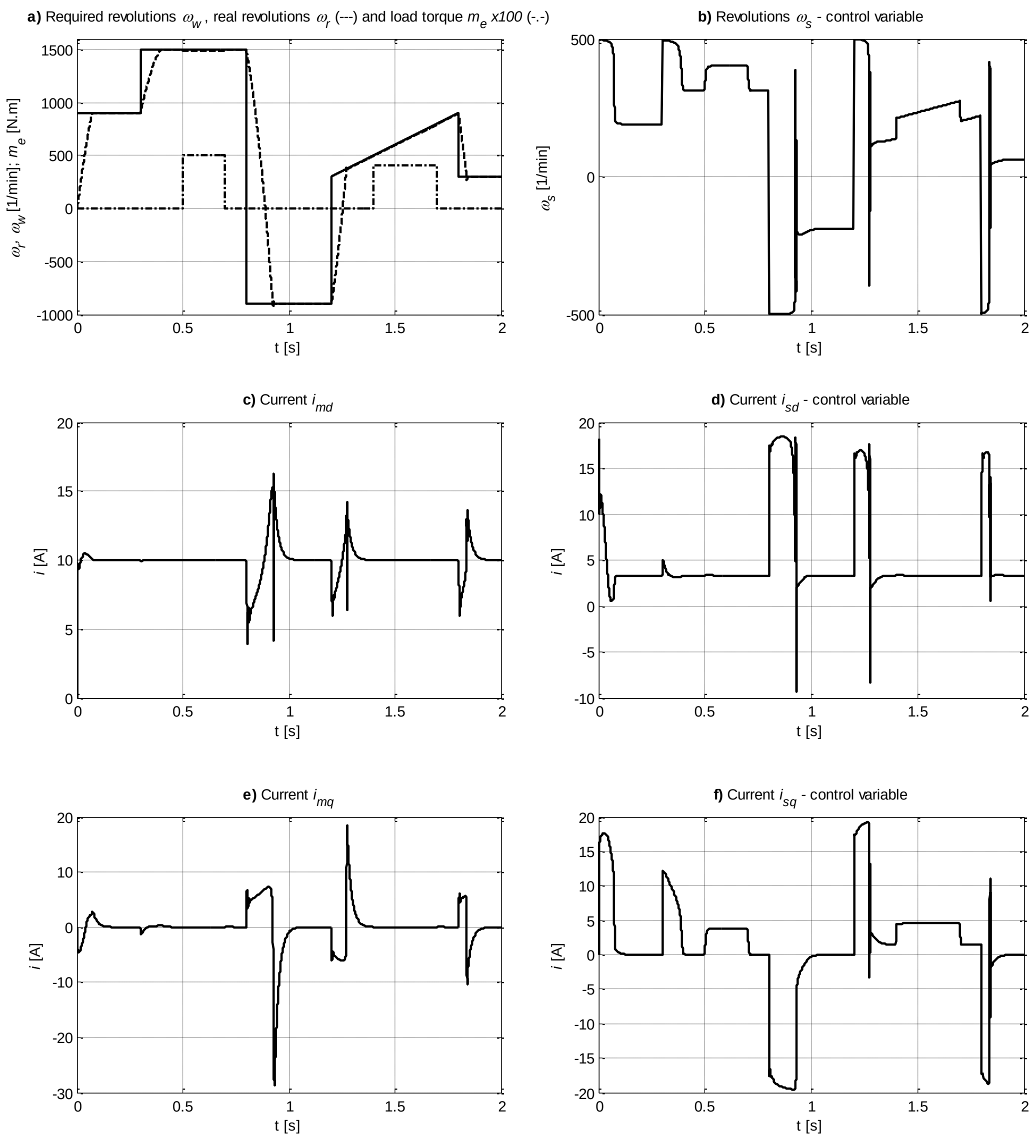

Fig. 3 Simulation results for a robust control algorithm with *fce* function.