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MATHEMATICAL MODELING OF GAS FLOW INCLUDING HEAT TRANSFER IN SPIRAL HEAT EXCHANGER

MATEMATICKÉ MODELOVÁNÍ PROUDĚNÍ SPALIN VČETNĚ PŘESTUPU TEPLA VE SPIRÁLOVÉM VÝMĚNÍKU TEPLA

Abstract

The paper presents the numerical solution of gas flow in a spiral heat exchanger, which flowing water is heated. Gaseous combustion products are derived from the combustion of natural gas in micro-turbine, which reaches tens of power [kW]. The paper defines a mathematical model of gas flow in the exchanger, including consideration of heat transfer through the conductive spiral heat exchanger. Conductive heat exchanger areas are different wall and the insulation layer that surrounds the heat exchanger itself. Inlet boundary conditions for gas and water were got from the experimental measurements. Then defined mathematical model was solved numerically in programming software ANSYS Fluent13. The results of numerical simulations are presented in the basic distribution of current values in the individual sections of exchanger. Subsequently, variables are evaluated to determine the energy analysis of the heat exchanger.

Abstrakt

Příspěvek prezentuje numerické řešení proudění spalin ve spirálovém výměníku tepla, kterým se ohřívá proudící voda. Plynné spaliny jsou získány ze spalování zemního plynu V mikroturbíně, která dosahuje výkonu řádově desítky [kW]. V příspěvku je definován matematický model proudění spalin ve výměníku včetně uvažování přestupu tepla skrz vodivé oblasti spirálového výměníku. Vodivými oblastmi výměníku tepla jsou jednotlivé stěny a vrstva izolace, která obklopuje samotný výměník. Vstupní okrajové podmínky pro spaliny a vodu byly získány na základě experimentálního měření. Definovaný matematický model byl následně řešen numericky V programovém prostředí ANSYS Fluent13. Výsledky numerické simulace jsou prezentovány rozložením základních proudových veličin V jednotlivých řezech výměníkem. Následně jsou vyhodnoceny veličiny K stanovení energetické analýze výměníku tepla.

1 INTRODUCTION

This paper deals with the possibility of using waste heat gases that are produced by small micro-turbine using numerical modeling in programming environment ANSYS Fluent13. One way to

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increase the thermal efficiency of energy processes based on combustion of gaseous fuels is design of various types of heat exchangers that use waste heat produced by combustion in various stages of energy process. By comparing the heat exchangers of various structures are examined in a number of authors [1], [2] who have just pointed out to design heat factor, particularly in relation to a reduction in pressure loss of flowing air. Efforts to increase the thermal (or overall) efficiency of the process are mainly located in the field of heat directly from the combustion equipment such as casing microturbine or combustion chamber.

Little attention is now devoted to the possibilities of obtaining the residual heat from the combustion of small and medium-sized micro turbines. There are studies that focus on heat transfer, depending on the thickness of the wall heat exchange surface, the shape of ribs and their number. Similarly is conceived the design of heat exchanger due to the size or efficiency of energy units. Waste gases, although already a large part of its potential transmit the intake combustion air, but still contain a relatively large portion of heat which is not used. Reducing the temperature of outgoing gases at the outlet of the vent (chimney) can provide increased heat, respectively overall efficiency of the process, which is undeniable contribution to the energy. Energy recovery of waste heat and reducing gas flow temperature of outgoing gases (flue gases) into the atmosphere is also a positive environmental benefit.

Among partial benefits include utilization of part waste heat for heating water and reduce gas outlet temperature of the atmosphere during application in cogeneration units. This paper provides information about the newly proposed type exchanger. It is a heat exchanger, where heated fluid (water) flow through the spiral pipe flows and cooled gas flow in interior space heat exchanger. a characteristic of the spiral heat exchanger is shown in Fig 1. The figure shows the inputs and outputs of individual media. Subsequently, from the flow of water in spiral tube is clear that heat is in some stage as same stream in the next phase as opposite stream flow, so it is not a classic same stream or opposite stream flow of heat exchanger.

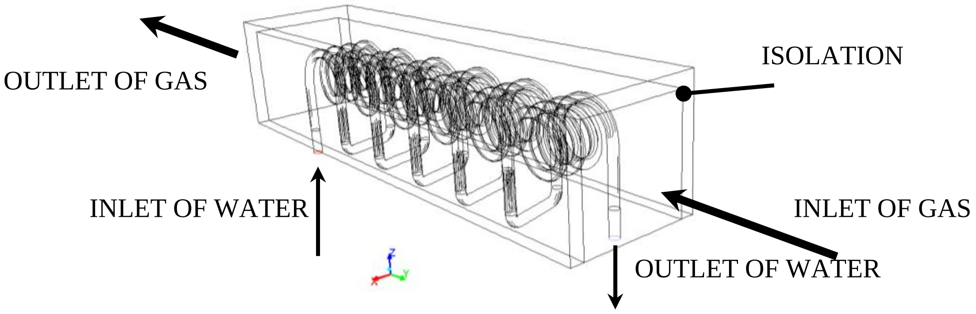

Fig. 1 The geometry of the spiral heat exchanger (water-gas).

The actual design of spiral heat exchanger is shown in Fig 2. The picture shows a spiral heat exchanger, including the supply and exhaust gases. Then from the picture is visible inlet and outlet of heated water and itself micro-turbine is captured to burning natural gas.

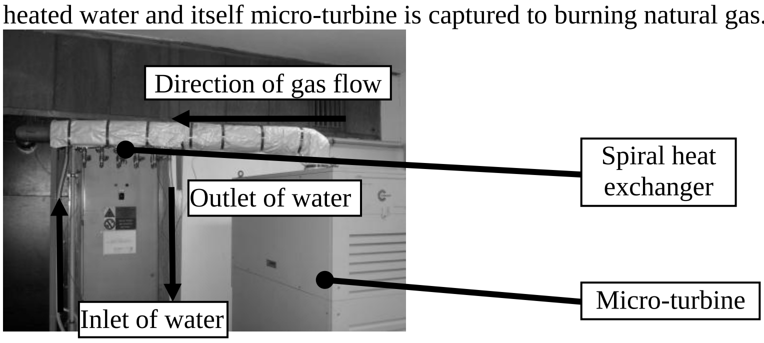

Fig. 2 Spiral heat exchanger including a turbine.

Mathematical modeling of fluid flow includes real now widely applied in many industry applications (From the metallurgical industry, chemical industry to environmental issues). In this paper a mathematical model is characterized by flowing gaseous mixture and water in the spiral heat exchanger. a characteristic of the heat exchanger is described above. In addition, in the mathematical model of heat transfer is considered the individual heat exchanger walls, and defining by their thickness and material properties. Moreover, outer layer of insulation is considered as the by creating a conductive area in which is taken into account the calculation of heat transfer. Insulation layer is shown in Fig 1. Current approaches to solving the problems are presented by several authors [3], [4].

2 DEFINITION OF MATHEMATICAL MODEL OF GAS AND WATER FLOW

Based on the characteristics of the issue of gas and water flow in the heat exchanger is defined turbulent Realizable k-ε model with heat transfer. Below are defined the basic balance equations of a mathematical model:

Mass equation:

$$\frac{\partial\left(\rho\overline{u_j}\right)}{\partial x_j} \tag{1}$$

where:

ρ - the density $\left[\frac{kg}{m^3}\right]$

$\overline{u_j}$ - the component of medium velocity $\left[\frac{m}{s}\right]$

x_j - the cartesian coordinates in the system $[m]$

Momentum equations:

$$\frac{\partial\left(\rho\overline{u_i}\,\overline{u_j}\right)}{\partial x_j}=-\frac{\partial p}{\partial x_i}+\frac{\partial}{\partial x_j}\left[(\mu+\mu_t)\frac{\partial\overline{u}_i}{\partial x_j}\right] \tag{2}$$

where:

p - the pressure $[Pa]$

μ - the dynamic viscosity $[Pa\cdot s]$

μ_t - the turbulent viscosity $[Pa\cdot s]$

Equation of turbulent kinetic energy:

$$\frac{\partial\left(\rho\overline{u_j}k\right)}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+G_k+G_b-\rho\varepsilon-Y_M \tag{3}$$

where:

k - the turbulence kinetic energy $\left[\frac{m^2}{s^2}\right]$

σ_k - the empirical constant $[-]$, $\sigma_k=1$

G_k - the generation of turbulence kinetic energy due to the mean velocity gradients $\left[\frac{kg}{m\cdot s^3}\right]$

G_b - the generation of turbulence kinetic energy due to buoyancy $\left[\frac{kg}{m\cdot s^3}\right]$

Y_M - the dilatation dissipation term $\left[\frac{kg}{m\cdot s^3}\right]$

Generation of turbulence kinetic energy due to the mean velocity gradients G_k, generation of turbulence kinetic energy due to buoyancy G_b, and dilatation dissipation term Y_M are defined in [6].

Equation rate of dissipation:

$$\frac{\partial(\rho\overline{u_j}\varepsilon)}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial\varepsilon}{\partial x_j}\right]+\rho C_1 S\varepsilon-\rho C_2\frac{\varepsilon^2}{k+\sqrt{\nu\varepsilon}}+C_{1\varepsilon}\frac{\varepsilon}{k}-C_{3\varepsilon}G_b \qquad (4)$$

where:

$$C_1=\max\left[0.43,\frac{\eta}{\eta+5}\right],\eta=S\frac{k}{\varepsilon},S=\sqrt{2S_{ij}S_{ij}},S_{ij}=\frac{1}{2}\left(\frac{\partial\overline{u_j}}{\partial x_i}+\frac{\partial\overline{u_i}}{\partial x_j}\right),\mu_t=\rho C_\mu\frac{k^2}{\varepsilon},C_\mu=0.09,C_{1\varepsilon}=1.44$$

$$C_{3\varepsilon}=\tanh\left|\frac{v}{\upsilon}\right|$$

ε - the rate of dissipation $\left[\frac{m^2}{s^3}\right]$

v - the component of the flow velocity parallel to the gravitational vector $\left[\frac{m}{s}\right]$

υ - the component of the flow velocity perpendicular to the gravitational vector $\left[\frac{m}{s}\right]$

Defined equations represent the basic equations describing the turbulent flow of real fluids. Other equations in the resulting mathematical model are equations of energy, heat transfer equations in conductive areas (insulation) and the equation for the transfer of gaseous species, see [5], [6].

The resulting mathematical model of a spiral heat exchanger can be described as a 3D stationary mathematical model of turbulent flow of gaseous mixture and water and heat transfer when the flow of gaseous mixture is considered as compressible. Gas flow through heat exchanger is composed of (CO_2, H_2O, N_2, O_2). Calculation of the gas density is defined by the ideal gas equation for compressible gas. Other physical properties (viscosity, specific heat capacity, thermal conductivity) for gas mixtures are defined by mixing laws. With regard to the calculation of heat transfer from the gas through wall spiral pipe into the water is considered the actual wall thickness of pipe including the physical properties of wall material. Furthermore, in the heat exchanger model is considered as an insulating layer of material as conductive area with defined physical properties (density, specific heat capacity, thermal conductivity).

3 DEFINITION OF PHYSICAL PROPERTIES AND BOUNDARY CONDITIONS

Physical properties

In the final mathematical model of a spiral heat exchanger is defined gaseous, liquid and solids material. If we define the gas flow then gaseous mixture contains of carbon dioxide (CO_2), water vapor (H_2O), nitrogen (N_2) and oxygen (O_2). As the heated liquid is water (H_2O). Solid materials are stainless steel (wall heat exchanger), copper (spiral tube) and Sibral (heat insulation). If we define the gaseous mixture flow then mixture is defined as compressible flow, ie. calculate the density of the gaseous mixture is defined by the ideal gas equation according to the following

$$\rho=\frac{p_{op}+p}{RT\sum_{i'}\frac{Y_{i'}}{M_{i'}}} \qquad (5)$$

where:

p_{op} - the operating pressure $[Pa]$

R - the universal gas constant $\left[\frac{J}{K\cdot mol}\right]$

$M_{i'}$ - the molecular weight of the gas $\left[\frac{kg}{kmol}\right]$

The remaining physical properties of the gas (viscosity, specific heat capacity and thermal conductivity) are defined by the mixing laws are discussed in [6].
For heated water we defined density (ρ), viscosity (μ), specific heat capacity (c_p) and thermal conductivity (λ) by piecewise linear functions.

In the case of density (ρ) we defined the following density functional formula for the calculation:

$$\rho(T) = \rho_n + \frac{\rho_{n+} - \rho_n}{T_{n+1} - T_n}(T - T_n) \tag{6}$$

Similarly we define the functional formula of the remaining physical properties of OO.
Then we define the physical properties of solid materials (Sibral, stainless steel and copper) that are listed in Tab. 1.

Tab. 1 Physical properties of solid materials.

Material	Stainless steel	Copper	Sibral
Density ρ [kg·m^{-3}]	8030	8978	130
Specific heat capacity c_p [J·kg^{-1}K^{-1}]	502	381	950
Thermal conductivity λ [W·m^{-1}K^{-1}]	40	387.6	0.07

Boundary conditions

The corresponding boundary conditions have to be defined on the individual boundary of computational model of the heat exchanger. In the spiral heat exchanger is used three types of boundary conditions:

- inlet boundary conditions – mass-flow boundary conditions ("mass-flow-Inlet")
- outlet boundary conditions – pressure boundary conditions ("pressure-outlet")
- walls - ("wall")

Inlet boundary conditions are defined for the inlet gas and water into a spiral heat exchanger as shown in Fig 1. Inlet boundary conditions are defined as mass-flow ("mass-flow-Inlet"). Sizes of inlet parameters on the inlet flow areas for the individual media are defined by measurement. If we define the gas mass flow $Q_m\left[kg \cdot s^{-1}\right]$, temperature $T(°C)$ and mass fractions $Y_{i'}[-]$ of gaseous components of flue gases. For water we define the water mass flow $Q_m\left[kg \cdot s^{-1}\right]$ and temperature $T(°C)$. Sizes of inlet parameters are listed in Tab. 2. Outlet boundary conditions are defined as the pressure ("pressure-outlet"), which define the zero pressure, and then discharge into the atmosphere. Marking the output boundary conditions for gas and water are similarly shown in Fig 1.

Tab. 2 Specification of inlet boundary conditions.

	Gases	Water
Mass flow rate $Q_m\left[kg \cdot s^{-1}\right]$	0.211565	0.05
Temperature $T(°C)$	291	13
Mass fractions of species $Y_{i'}[-]$	$Y_{CO_2} = 0.0223, Y_{H_2O} = 0.0176$	
	$Y_{O_2} = 0.1997,\ Y_{N_2} = 0.7604$	

The remaining boundary of spiral heat exchanger model is defined as the walls. For walls of the heat exchanger we defined stainless steel and for pipe spiral wall we defined copper. With regard to defining the calculation of heat transfer we corresponding to specify the thickness of walls, which were obtained from the actual heat exchanger, see Fig. 2.

4 RESULTS OF NUMERICAL SIMULATION

In the first phase are evaluated graphical outputs of the numerical simulation of temperature distribution in the longitudinal section and cross sections of heat exchanger. Location longitudinal section and cross sections are shown in Fig. 3.

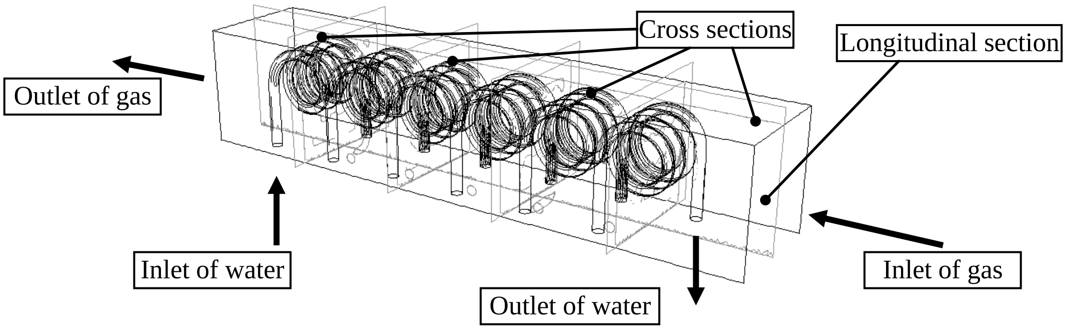

Fig. 3 Schematic representation of each section to evaluate.

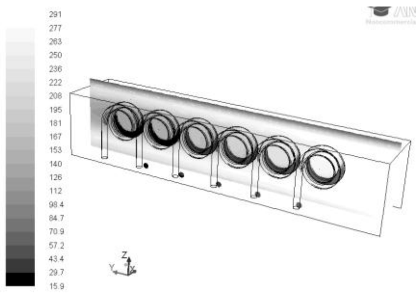

Fig. 4 Temperature field in the longitudinal section (T[°C]).

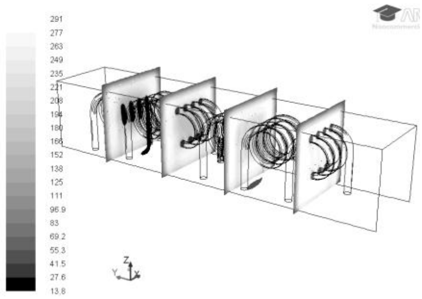

Fig. 5 Temperature field in the cross sections (T[°C]).

Temperature field in a longitudinal section through the center heat exchanger is shown in Fig. 4. Temperature range is 291°C to 16°C. The maximum temperature of 291°C corresponds to inlet gas temperature (white color) and towards the exit of gas temperature decreases (dark color). The Fig. 5 shows the temperature field in four cross sections. Temperature range is 291°C to 13°C. The maximum temperature again corresponds to the inlet gas temperature (white color) and minimum temperature of 13°C corresponds to the inlet water temperature. Cross sections show that the gas temperature gradually decreases towards the outlet (darker color). Area of minimum temperature (around 13°C) is shown in cross section (in the flow of water spiral tube), which passes just beyond the inlet of cold water (13°C) to the heat exchanger.

In the second stage, variables are evaluated to determine the energy analysis of the heat exchanger. Based on numerical simulation in software ANSYS Fluent 13.0 are the inlet and outlet to a heat exchanger for the gas and water evaluated following physical quantities (mean temperature T_s [°C] and specific heat capacity c_p [$J \cdot kg^{-1} \cdot K^{-1}$]), see. Tab. 3.

Tab. 3 Evaluated physical quantities at the inlet and outlet flue gas and water.

	Mean temperature T_s [°C]	Specific heat capacity c_p [$J \cdot kg^{-1} \cdot K^{-1}$]
Gas at the inlet	291	1055.3
Gas at the outlet	221.85	1039.5
Water at the inlet	13	4188.4
Water at the outlet	71.85	4190.6

Then the energy analysis is determined of the heat exchanger. Energy analysis is based on the calorimetric equation that describes the heat exchange of two bodies. Calorimetric equation assumes immobile body. The equations of index *c* it means heated fluid (water) and index *h* means cooled fluid (gas).

$$
\begin{aligned}
P_c &= Q_{m,c}\left(c^0_{p,c} \cdot t^0_c - c^I_{p,c} \cdot t^I_c\right) \\
P_h &= Q_{m,h}\left(c^0_{p,h} \cdot t^0_h - c^I_{p,h} \cdot t^I_h\right)
\end{aligned} \tag{7}
$$

where:

$c_{p,c}$ - the specific heat capacity of heated fluid (water) $\left[\frac{J}{kg \cdot K}\right]$

$c_{p,h}$ - the specific heat capacity of cooled fluid (gas) $\left[\frac{J}{kg \cdot K}\right]$

t^I_h - inlet of hot fluid (gas) $[°C]$

t^0_h - outlet of cooled fluid (gas) $[°C]$

t^I_c - inlet of cold fluid (water) $[°C]$

t^0_c - outlet of heated fluid (water) $[°C]$

The above defined quantities are evaluated from numerical simulations in Tab. 3. The mass flow rates of gas and water are defined in Tab. 2.

After substituting into relation (7) we get for water:

$$P_c = Q_{m,c}\left(c^0_{p,c} \cdot t^0_c - c^I_{p,c} \cdot t^I_c\right) = 0.05(4190.6 \cdot 71.85 - 4188.4 \cdot 13) = 12332 \text{ W}$$

After substituting into relation (7) we get for gas:

$$P_h = Q_{m,h}\left(c^0_{p,h} \cdot t^0_h - c^I_{p,h} \cdot t^I_h\right) = 0.211565(1039.5 \cdot 221.85 - 1055.3. \cdot 291) = -16180 \text{ W}$$

If the system was completely isolated, so the powers of cooled and heated fluids were identical. This does not apply in this case, because as the lower wall heat exchanger is not insulated, spiral tube is partially led out outside the heat exchanger, etc. .. The heated fluid (water) power P_c is positive, because the outlet water temperature is higher than the inlet $t^0_c \rangle t^I_c$. In other words, heated fluid receives heat, so power is positive. The cooled fluid (gas) is the conversely outlet temperature lower than the inlet $t^0_h \langle t^I_h$, power P_h is so negative, because fluid heat surrenders. In absolute terms, however, these powers positive achievements. The difference between these performances is a power dissipation of the heat exchanger $P_Z = |P_h| - P_c = 16180 - 12332 = 3848 \text{ W} \cdot$

5 CONCLUSION

Article a detailed define 3D mathematical model the flow of gaseous combustion products and water in a spiral heat exchanger including a heat conduction and convection. In the introduction is characterized by a spiral heat exchanger, which behaves as same stream and opposite stream flow heat exchanger. The following chapters the basic balance equations are defined by a mathematical model, including physical properties and boundary conditions. The values of the inlet boundary conditions for gas and water have been defined based on experimental measurements. The results of numerical simulations are presented by the temperature fields, which are evaluated in longitudinal section and in the center of heat exchanger and in the cross sections. From graphical output can be seen cooling of flow gas, when the gas at the inlet has the temperature T = 291°C. Gas temperature in the outlet section is in the range *T* = 123°C - 270°C and mean temperature is T_s = 221.85°C. In the second stage, variables are evaluated to determine the energy analysis of the heat exchanger (Mean temperature T_s [°C] and specific heat capacity c_p [$J \cdot kg^{-1} \cdot K^{-1}$]) at the inlet and outlet for gas and water.

Subsequently, from the calculated heat flow (heat, which receives water and heat which, divert gas) is the heat loss $P_Z = 3848\,\mathrm{W}$.

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