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THE INFLUENCE OF THE PARTIAL SURFACE WETTING ON THE FLOW FIELD BETWEEN THE TWO COAXIAL CYLINDERS

VLIV ČÁSTEČNĚ SMÁČIVÉHO POVRCHU NA PROUDOVÉ POLE V MEZEŘE MEZI DVĚMA SOUOSÝMI VÁLCI

Abstract

The influence of the partial surface wetting on the flow field between the two coaxial cylinders, the inner of which rotates, has been investigated numerically. Wall boundary condition was modified to account for partial wettability based on the equation proposed by Pochylý for the general curved surface [1-10]. FLUENT software was applied to model the stationary viscous fluid flow in a narrow gap. Different boundary conditions were applied on the rotating wall of the inner cylinder. The results obtained by numerical modelling were compared with theoretical assumptions.

Abstrakt

Vliv částečně smáčivého povrchu na proudové pole v mezeře mezi dvěma souosými válci, z nichž vnitřní rotuje, byl zkoumán s využitím numerického modelování. Modifikace okrajové podmínky vychází z rovnice, kterou pro částečně smáčivý obecně zakřivený povrch definoval Pochylý [1-10]. V software FLUENT bylo simulováno stacionární proudění nestlačitelné viskózní tekutiny v úzké mezeře, kdy na stěnu vnitřního válce byla aplikována okrajová podmínka zahrnující vliv částečné smáčivosti. Výsledky z numerické simulace byly následně porovnány s teoretickými vztahy.

1 INTRODUCTION

Wall boundary conditions are applied to bound fluid and solid regions. Numerical modelling of internal flows assumes most often *no- slip* boundary condition on walls. Particles close to a surface do not move along with a flow when adhesion is stronger than cohesion. This assumption fits well for most of solid surfaces as was verified by many physical experiments in the past. However, 200 years ago Navier pointed out that the liquid can in some cases slip on the wall and he stated the proportion between the velocity on the surface and the shear stress by the formulae:

$$c_x = \beta \cdot \frac{dc_x}{dy}. \tag{1}$$

where y is the coordinate normal to the wall and β is called the slip length.

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For an ideal gas, the slip length is often approximated as $\beta \approx 1{,}15 l$, where l is the mean free path. Some highly hydrophobic surfaces have also been observed to have a nonzero but nanoscale slip length [11].

Development of new materials and technologies has attracted attention to non wettable or partially wettable surfaces. Partial wettability contributes to pressure drop reduction in the fluid flow and thus can improve the efficiency of devices with special surface treatment. Numerical modelling can be applied as a tool to investigate the influence of partial wettability of walls in different technical applications.

2 COMPARISON OF THEORETICAL ASSUMPTIONS AND CFD RESULTS

2.1 Theoretical approach

Definition of boundary condition for the generally curved surface, which is partially wettable, applies following assumptions: if the fluid is slipping on the surface with velocity c, adhesive shear stress vector σ_A lays in the plane defined by the outer normal vector n to the surface and the velocity vector c (Fig 1). The adhesive shear stress vector σ_A tangentional to the surface can be defined as follows:

$$\sigma_A = (\sigma \times n) \times n = -k \cdot c \tag{2}$$

where:
k is adhesive coefficient [$\mathrm{Pa \cdot s \cdot m^{-1}}$].

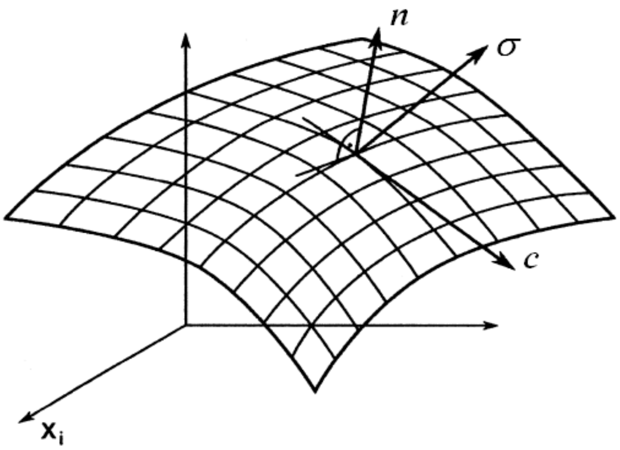

Fig. 1 Shear stress on general curved surface.

Let us consider the stationary flow of the viscous fluid in a narrow gap between the two coaxial cylinders. The diameter of the inner and outer cylinder are R_1 and R_2 and the rotational speed of the inner cylinder is ω_1 (Fig. 2). At first we assume the full wettability of both surfaces (*no-slip* condition).

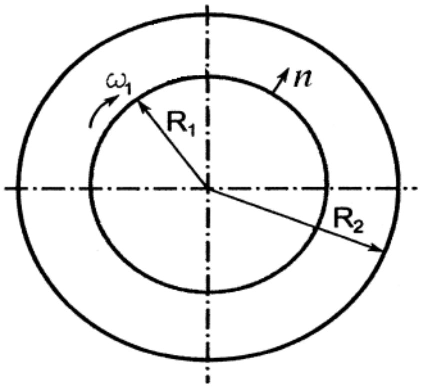

Fig. 2 Coaxial cylinders.

We will introduce the cylindrical coordinate system *r*, *φ*, *z*. Navier – Stokes equations for incompressible flow can be transformed to cylindrical coordinates assuming the stationary flow in concentric circles:

$$\frac{\partial^2 c}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial c}{\partial r}-\frac{c}{r^2}=0. \tag{3}$$

This Euler type equation yields the solution [12]:

$$c=\frac{A}{r}+B\cdot r, \qquad \frac{\partial c}{\partial r}=-\frac{A}{r^2}+B. \tag{4}$$

Boundary condition for the inner cylinder is:

$$c\big|_{R_1}=u \tag{5}$$

For the outer cylinder we can define:

$$c\big|_{R_2}=0 \tag{6}$$

Substituting these boundary conditions to the equation (3), we can derive the integration constants *A*, *B* and obtain the relation for velocity *c*:

$$c=\frac{u\cdot R_1}{R_2^2-R_1^2}\cdot\left(\frac{R_2^2}{r}-r\right). \tag{7}$$

Now we consider partial wettability of the inner cylinder. Wall boundary condition given by (8) allows accounting for partial wettability while wall boundary condition of the outer cylinder will remain same as in previous case (6):

$$\eta\left(\frac{\partial c}{\partial r}\bigg|_{R_1}-\frac{c\big|_{R_1}}{R_1}\right)=k\cdot(c\big|_{R_1}-u). \tag{8}$$

Based on these boundary conditions and after definition of integration constants, velocity *c* is given by:

$$c=\frac{u\cdot R_1^2}{R_1\cdot(R_1^2-R_2^2)-2\cdot\dfrac{\eta}{k}\cdot R_2^2}\left(r-\frac{R_2^2}{r}\right). \tag{9}$$

If we assume viscous force $\Pi_{r\varphi}$, acting on the unit area $1\cdot r\cdot \mathrm{d}\varphi$ of the cylinder surface given by the radius *r*, we can express the total moment M_K of viscous forces by:

$$M_k=-\frac{4\cdot\pi\cdot u\cdot\eta\cdot R_1^2\cdot R_2^2}{R_1\cdot(R_2^2-R_1^2)+2\cdot\dfrac{\eta}{k}\cdot R_2^2}. \tag{10}$$

2.2 Comparison of theoretical assumptions with CFD results

Numerical simulation was applied to investigate the influence of different wettability of the inner rotating cylinder on the flow field. Computational domain was defined in agreement with Fig. 2. Detailed information on the dimensions and physical properties of the liquid (water) are given in Tab.1. Computational grid consisted of 16000 quad cells with 32 cells in radial direction, which was enough to evaluate the velocity profile in the gap.

Tab. 1 Dimensions of the computational domain and physical constants of water.

Physical constants		**Water**	
Density	$\rho =$	998.2	$kg \cdot m^{-3}$
Dynamic viscosity	$\eta =$	0.001003	$Pa \cdot s$
Kinematic viscosity	$\nu =$	1.00481E-06	$m^2 \cdot s^{-1}$
Dimensions of the computational domain			
Outer radius	$R_1 =$	25	mm
Inner radius	$R_2 =$	15	mm
Gap width	$s =$	10	mm

To avoid generation of Taylor vortices, Taylor number was applied to define the rotational speed of the inner cylinder. Its critical value, above which the Taylor vortices develop, is defined as Ta = 41.3. Maximum rotational speed is limited by:

$$Ta = \frac{\omega \cdot R_1 \cdot s}{\nu} \cdot \sqrt{\frac{s}{R_1}} \leq 41{,}3 \tag{11}$$

This yields $\omega_{max} = 0.339\ s^{-1}$ and $n_{max} = 3.24\ min^{-1}$.

Stationary laminar flow was modelled using the *no-slip* boundary conditions on both cylinders. Rotational speed was set to $n = 2\ min^{-1}$. Comparison of velocity profile derived from (7) and obtained with numerical modelling is illustrated in Fig. 3.

Velocity profiles are identical, velocity on the inner cylinder corresponds to circumferential velocity. *No-slip* boundary condition for viscous fluids states that at a solid boundary the fluid has zero velocity relative to the boundary. The wall of inner cylinder is fully wettable and the liquid does not slip on it.

If we assume the inner cylinder as partially wettable, we expect that the fluid can „slip“ on the surface. Thus the fluid velocity on the wall will be lower than that of the cylinder and will depend on the value of adhesion coefficient k according to (9).

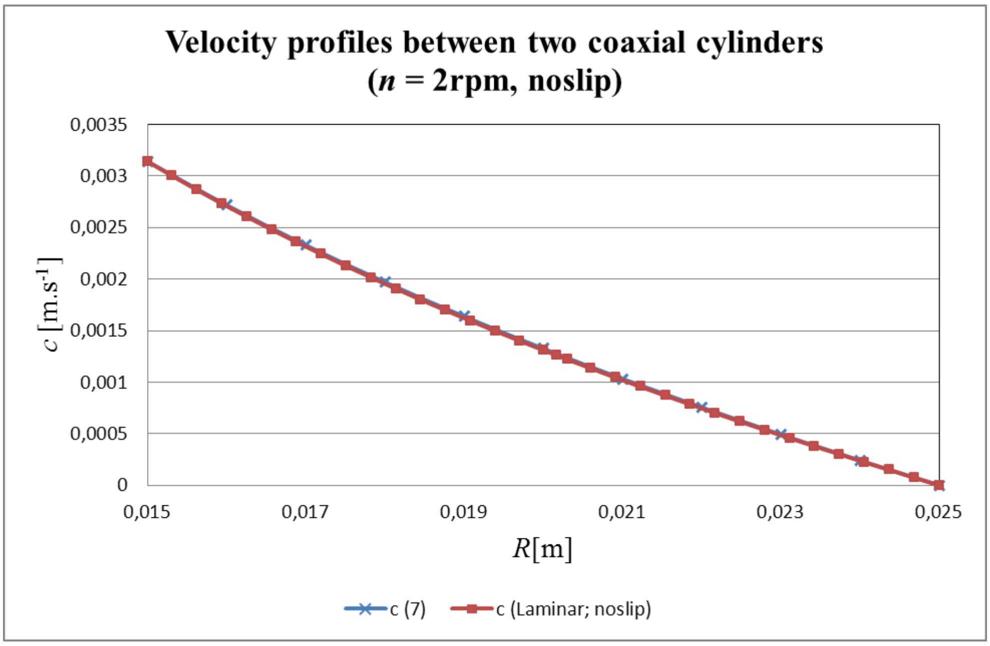

Fig. 3 Comparison of velocity profiles at n = 2 rpm, inner cylinder is fully wettable.

In FLUENT software, the *no-slip* boundary condition is enforced at walls by default. If required, a tangential velocity component can be specified in terms of the translational or rotational motion of the wall boundary, or a "*slip*" wall can be modelled by specifying shear. To define adhesion coefficient k, user defined function (UDF) was used to account for (8). The condition was transferred from cylindrical to cartesian coordinate system.

Fig. 4 shows the comparison of velocity profile derived from (9) and the profile obtained from numerical modelling with the application of the UDF in FLUENT. Velocity profiles are identical, which verifies the proper definition of UDF.

User defined function enables to change the value of adhesion coefficient k and so to simulate various wettability conditions on the wall. Different values of adhesion coefficient k ranging from 0,05 to 2 were applied. Modification of velocity profile based on the value of k was investigated, as illustrated in Fig. 5.

The behaviour of the flow field in a gap between the two cylinders, the inner of which rotates, corresponds to assumptions of fluid slip on the surface of the inner cylinder. The lower is the value of adhesion coefficient k, the lower is also the circumferential velocity and the fluid is slipping on the surface. At the same time the moment of the viscous forces will be decreasing.

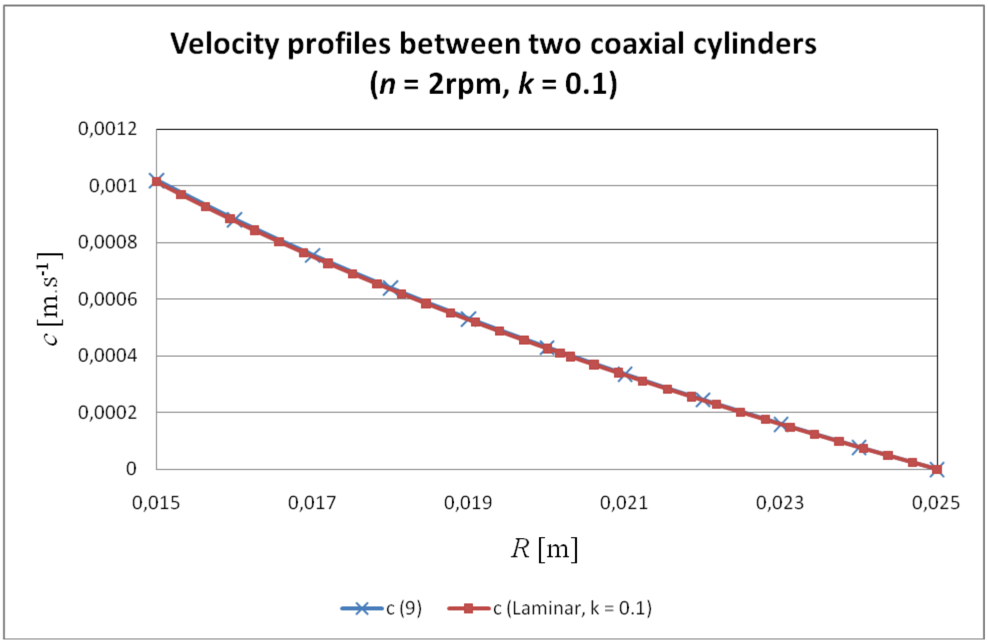

Fig. 4 Comparison of velocity profiles for n = 2 rpm, inner cylinder is partially wettable.

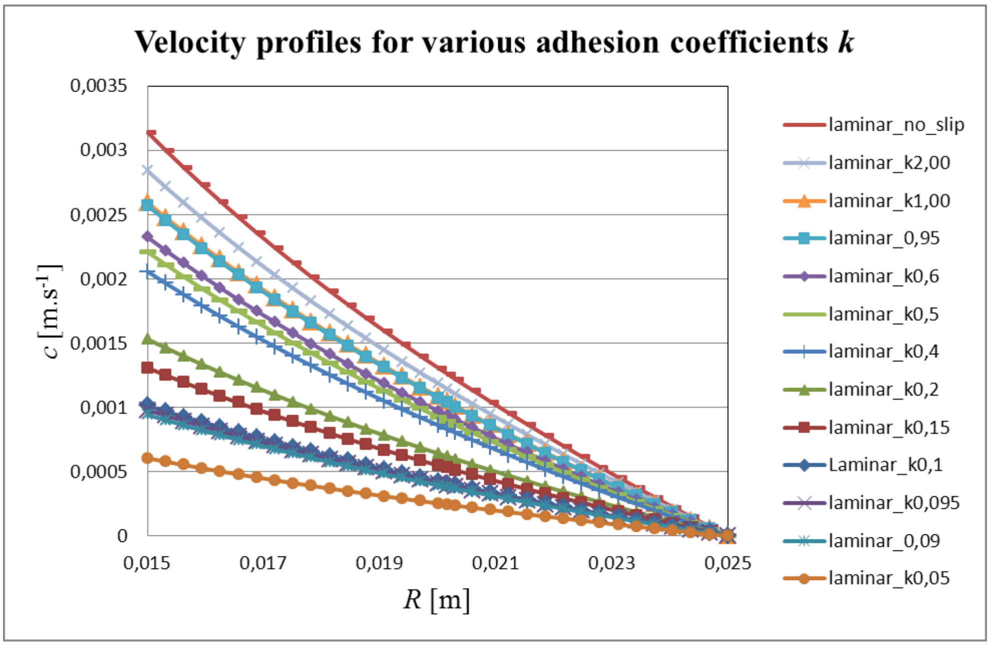

Fig. 5 Velocity profiles for various adhesion coefficients k at n = 2 rpm on inner cylinder.

Moment is developed by adhesion forces and its size can be determined by equation (10). By this equation we can determine the moment for unit length, which corresponds to the definition in FLUENT for our 2D geometry in Fig.2. Fig. 6 compares a size of the moment determined by equation (10) with the moment computed in FLUENT for various adhesion coefficients k on the inner cylinder wall.

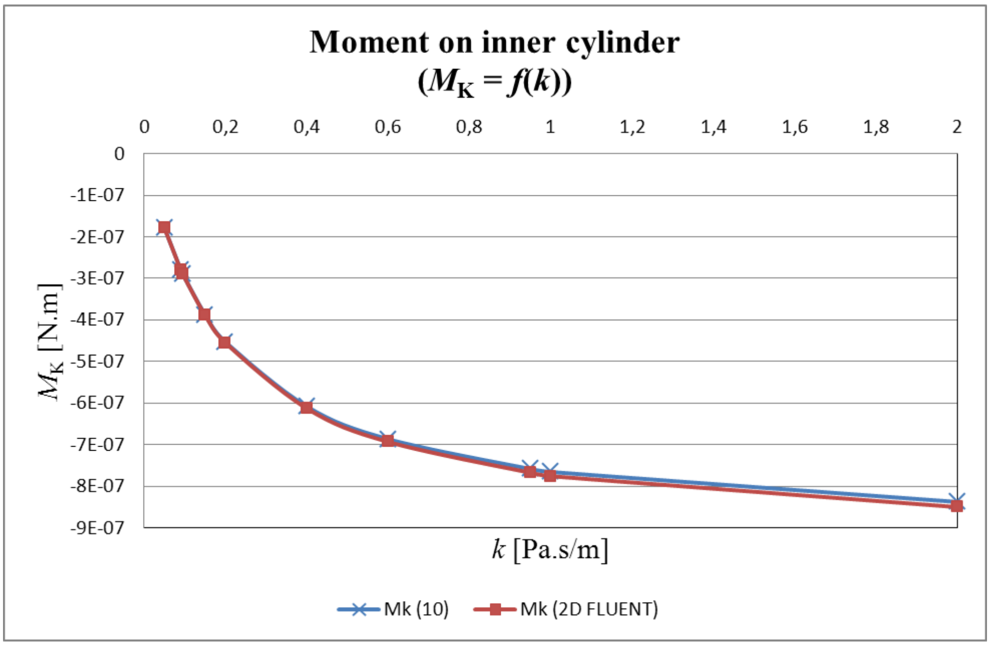

Fig. 6 Moment is compared on inner cylinder at n = 2 rpm, inner cylinder is partially wettable.

3 CONCLUSION

Numerical modelling of the viscous fluid flow between the two coaxial cylinders, the inner of which rotates, has been carried out. The main objective was to modify wall boundary conditions and to introduce the partial wall wettability into the model using the user defined function in FLUENT. Results from numerical solution are in very good agreement with results obtained from empirical formulas. It can be concluded that this approach could be applied on more complex flows in 3D geometries. The knowledge obtained from this simple 2D problem can enable prediction of partial wettability effects in various engineering applications. This can be useful e.g. for prediction of fluid flow for parameters in hydraulic components with partially wettable surface.

Theory for laminar flow in tube with circular cross-section is well known. We can numerically solve this flow for *no-slip* boundary condition and for boundary condition with various adhesion coefficients k. Then we can compare velocity profiles and pressure drop along the tube for different boundary conditions. There is another way how to define boundary condition for partially wettable surfaces. In FLUENT software we can specify directly the value of shear stress on the wall. Then we can compare both boundary conditions, it means for specified shear as constant or UDF function with adhesion coefficient k.

At present we are not able to assign the precise value of adhesion coefficient k to surfaces created by various materials. Assignment of adhesion coefficient k was not under investigation in this paper. The main objective was to investigate the influence of k and thus the partial wettability on the flow field. In software Fluent the definition of k in wall boundary condition enables to account for different partially wettable surfaces by changing the value of constant k in UDF function.

REFERENCES

[1] POCHYLÝ, F., RINKA, L. *Povrchová energie v hraniční vrstvě kapky vody a tuhého povrchu*. Výzkumná zpráva VUT-EU13303-QR-34-07, Brno (CZ) VUT FSI, prosinec, 2007, 32 s.

[2] POCHYLÝ, F., HABÁN, V., JURAČKA, J. *Smáčivost kapalin vůči pevným povrchům.* In: *Zborník abstraktov a príspevkov z medzinárodnej konferencie „Strojné inžinierstvo 2006“. Článek ve sborníku z mezinárodní konference Strojné inžinierstvo 2006 pořádané Slovenskou technickou univerzitou v Bratislavě,* 23.11.2006, Bratislava, ISBN 80-227-2513-7.

[3] POCHYLÝ, F., HABÁN, V. *Smáčivost kapalin vůči pevným povrchům*. Výzkumná zpráva VUT-EU13303-QR-14-06, Brno (CZ) VUT FSI, 2006.

[4] POCHYLÝ, F., FIALOVÁ, S., HABÁN, V., RINKA, L. *The Wettability of the Liquid-Solid Interface,* FIV 2008-FLOW INDUCED VIBRATION, p. 47-52, ISBN 80-87012-12-7, (2008), Institute of Thermomechanics, AS CR, v.v. i.

[5] POCHYLÝ, F., FIALOVÁ, S., RINKA, L. *Adhesive Forces at a Solid-Liquid Interface,* Zeszyty naukowe, Vol. 1, (2008), No. 10, p. 245-252, ISBN 1897-2683, Wydawnictwo politechniki Świętokrzyskiej w Kielcach.

[6] POCHYLÝ, F., FIALOVÁ, S., HABÁN, V., RINKA, L. *Praktičeskoje ispolzovanie smačivajemosti.* PCVEXPO´08, p. 40-41, ISBN 978-5-7038-3214-1, (2008), MGTU im. N.E. Baumana.

[7] FIALOVÁ, S., POCHYLÝ, F., RINKA, L. *The Adhesive Force Determination on the Interface of Phases.* Mechanical Engineering 2008-proceedings of abstracts, p. 4-5, ISBN 978-80-227-2982-6, (2008), STU Bratislava.

[8] RINKA, L., POCHYLÝ, F. *Analýza adhésních sil na rozhraní tekutiny a tuhého povrchu.* Výzkumná zpráva VUT-EU13303-QR-21-08, Brno (CZ) VUT FSI, prosinec 2008.

[9] FIALOVÁ, S., POCHYLÝ, F., RINKA, L. *The Boundary Condition of the Adhesive Forces Effect on the Solid/Liquid Interface.* In *Engineering Mechanics 2009-book of Extended Abstracts.* První edice, 2009. Praha, Akademie věd České republiky. 2009. p 48-53. ISBN 978-80-86246-35-2.

[10] POCHYLÝ, F., FIALOVÁ, S., HABÁN, V., RINKA, L. *Praktičeskoje ispolzovanije smačivajemosti. NASOSY&OBORUDOVANIJE.* 2009. (54) 2009(1). p. 20-21.

[11] FLUENT: FLUENT 12 - *User's guide.* Fluent Inc. 2007 [online]. Dostupné z < http://spc.vsb.cz/portal/cz/documentation/manual/index.php >.

[12] BIRD, R. B., STEWART, W. E., LIGHTFOOT, N. N: *Přenosové jevy*. Praha: Academia, 1968, 800 s.

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