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# THE INFLUENCE OF THE PARTIAL SURFACE WETTING ON THE FLOW FIELD IN A PIPE WITH CIRCULAR CROSS-SECTION

# VLIV ČÁSTEČNĚ SMÁČIVÉHO POVRCHU NA PROUDOVÉ POLE V POTRUBÍ KRUHOVÉHO PRŮŘEZU

**Abstract**

In this paper the study of laminar flow in a pipe with a slip boundary is presented. The influence of the partial surface wetting on shear and velocity profile as well as pressure drop has been investigated numerically. Steady, isothermal, incompressible flow was modelled in 2D and 3D geometry. Wall boundary condition was modified through the user defined function to account for partial surface wettability based on the theory proposed by Pochylý [1-10]. The results obtained by numerical modelling in Fluent were compared with theoretical assumptions.

**Abstrakt**

Článek prezentuje výsledky numerického modelování laminárního proudění v potrubí kruhového průřezu s částečně smáčivou stěnou. Byl vyšetřován vliv okrajové podmínky na průběh smykového napětí po průřezu, rychlostní profil a tlakový spád. Proudění bylo modelováno jako stacionární, izotermní a nestlačitelné ve 2D i 3D geometrii. Okrajová podmínka na stěně byla modifikována pomocí uživatelsky definované funkce umožňující zahrnout do výpočtu adhesní součinitel *k* podle teorie definované prof. Pochylým [1-10]. Výsledky z numerického modelování byly dále srovnány s teoretickými předpoklady.

## 1 INTRODUCTION

Most of innovations and improvements in all branches of industry are aimed to reduction of power consumption and so increasing of efficiency. In hydraulic equipment the energy losses can be classified as friction and local losses. Friction losses are closely connected with the shear stress generated due to the fluid viscosity. The flow of real fluids exhibits viscous effect and they tend to "stick" to solid surfaces.

Solid walls can contribute to the pipe friction by their roughness. In turbulent flow the roughness can increase the amount of turbulence and so energy losses in the flow. Friction losses can significantly be reduced by application of hydrophobic surfaces. Wetting dynamics is involved in a wide range of surface phenomena, including adhesion, surface forces, and the boundary conditions for

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fluid flow. The theory of mathematical definition of partially wettable surface by means of adhesion coefficient $k$ was introduced by Pochylý [1-10]. This approach will be further applied on the laminar pipe flow.

## 2 THEORETICAL BACKGROUND

### 2.1 Laminar flow with no slip condition on the wall

The theory of laminar flow with no slip condition on walls is well known. In that case, the control volume contains a stationary surface with zero velocity and maximum shear stress. The shear stress is proportional to rate of shear strain according to the Newton's law of viscosity:

$$\tau = \eta \frac{dc}{dr} = -\frac{1}{2}\frac{\Delta p}{l} r \qquad (1)$$

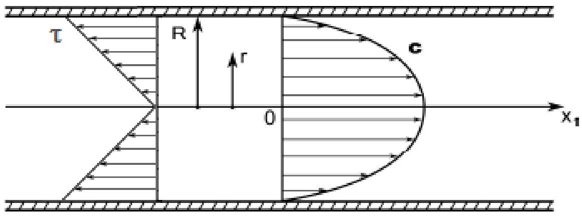


**Fig. 2** Straight pipe section with shear and velocity distribution in case of fully wettable wall (no slip).

Velocity distribution in cylindrical coordinate system can be defined as a function of the distance from the center of the pipe:

$$c = \frac{\Delta p}{4\eta l}\left(R^2 - r^2\right) \qquad (2)$$

Laminar fully developed flow has a parabolic profile. Velocity reaches its maximum at the pipe centerline $r = 0$:

$$c_{\max} = \frac{\Delta p R^2}{4\eta l} \qquad (3)$$

Considering the parabolic distribution, mean velocity is defined as one half of velocity maximum:

$$c_{mean} = \frac{\Delta p R^2}{8\eta l} \qquad (4)$$

The pressure drop in a fluid flowing through a long cylindrical pipe can be calculated from the Hagen–Poiseuille law:

$$\Delta p = \frac{8\eta l Q}{\pi R^4} \qquad (5)$$

### 2.2 Laminar flow with slip condition on the wall

No slip boundary works well for most of the flow conditions. But in many cases there is a need to define more general condition that can account for a slip. This type of boundary condition was mentioned by Navier nearly 200 years ago. The Navier slip condition defines that there is a slip at solid boundary proportional to magnitude of tangential stress:

$$c = \beta \frac{dc}{dr} \qquad (6)$$

where $\beta$ is a slip coefficient (slip length) [m]. The slip condition may be relevant in case of partially wettable walls.

The theory of mathematical definition of partially wettable surface by means of adhesion coefficient $k$ for a general curved surface has been presented by Pochylý [1-7]. Rinka applied this theory on various types of flow, including the laminar pipe flow [8].

Boundary condition on the slip wall for r = R is defined as follows:

$$n = -1; \; -\left.\frac{\partial c}{\partial r}\right|_{r=R} = \frac{k}{\eta} c \tag{7}$$

In this case the outer surface of the control volume moves with the slip velocity defined as:

$$c_0 = \frac{\Delta p R}{2lk} \tag{8}$$

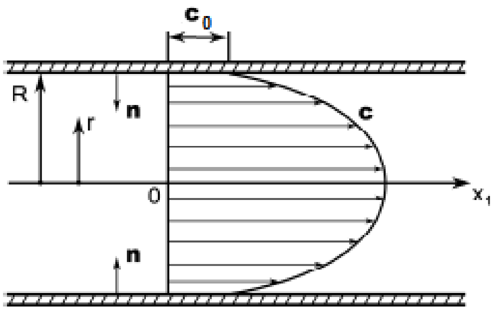


**Fig. 3** Velocity profile for partially wettable surface of the tube.

The velocity profile is modified according to the following formula:

$$c = \frac{\Delta p}{4\eta l}\left(R^2 - r^2\right) + \frac{\Delta p R}{2lk} = \frac{\Delta p}{2l}\left(\frac{R^2}{2\eta} + \frac{R}{k} - \frac{r^2}{2\eta}\right) \tag{9}$$

At the centerline of the pipe the liquid is moving fastest and for $r$ = 0 we obtain:

$$c_{\max} = \frac{\Delta p R^2}{4l\eta} + \frac{\Delta p R}{2lk} == \frac{\Delta p}{2l}\left(\frac{R^2}{2\eta} + \frac{R}{k}\right) \tag{10}$$

Mean velocity is defined as the summation of the slip velocity $c_0$ and mean velocity in case of no slip:

$$c_{mean} = \frac{\Delta p R^2}{8\eta l} + \frac{\Delta p R}{2lk} = \frac{\Delta p}{2l}\left(\frac{R^2}{4\eta} + \frac{R}{k}\right) \tag{11}$$

The pressure drop $\Delta p$ can be again calculated based on the flow rate $Q$:

$$\Delta p = \frac{8l\eta Q}{\pi R^4\left(1 + \frac{4\eta}{Rk}\right)} \tag{12}$$

### 2.2 Comparison of theoretical assumptions with CFD results

Standard fully wettable surfaces exhibit zero velocity, which is connected with the noslip boundary condition on the wall. In Fluent the no-slip condition is applied by default, but it is possible to define a tangential velocity component in terms of the translational or rotational motion of the wall boundary, or model a "slip" wall by specifying a certain value of shear. Such definition may cause a problem in complex geometry where the velocity may differ in various parts in which the flow is modelled. That is why a different approach was applied based on [8] and the UDF (User Defined Function) was defined, that enables to define the wall shear stress according to (7). Partial wettability is included into the mathematical model by means of the adhesion coefficient $k$, the derivation of which is described below [1-7]. The main objective of the work was to define the function enabling to account for adhesion coefficient $k$ and so for a slip on the wall. User defined function was formulated both for 2D and 3D pipe flow and can be generally applied to more complex problems. Numerical simulations were carried out in software package Fluent. For the 2D definition of problem three types of computational grid were prepared consisting of 10000, 40000 and 160000 cells, 3D grid contained 1142712 cells. The main dimensions of computational geometry as well as the physical properties of water are given in Tab. 1.

**Tab. 1** Dimensions of the computational domain and physical constants of water.

| **Physical constants** | | **Water** | |
|---|---|---|---|
| Density | $\rho =$ | 998.2 | $kg{\cdot}m^{-3}$ |
| Dynamic viscosity | $\eta =$ | 0.001003 | Pa·s |
| Kinematic viscosity | $\nu =$ | 1.00481E-06 | $m^2{\cdot}s^{-1}$ |
| **Dimensions of the computational domain** | | | |
| Pipe inner diameter | $D =$ | 15 | mm |
| Pipe length | $L =$ | 1000 | mm |

The flow was modelled for low Reynolds numbers corresponding to velocity lower then its critical value given by:

$$v = \frac{\nu \cdot \mathrm{Re}_{kr}}{D} = \frac{1{,}00481 \cdot 10^{-6} \cdot 2320}{0{,}015} = \underline{\underline{0{,}155\, m.s^{-1}}} \tag{13}$$

From the theory it is known, that the Hagen–Poiseuille law is valid only in case of fully developed laminar flow with parabolic velocity profile. That is why the calculations were repeated several times, at first the constant mean velocity was defined at the inlet, but in the next steps the outflow velocity profile was written in a file and then used again as the inlet boundary condition. This procedure was repeated until the agreement of inlet and outlet profiles was achieved.

The influence of the grid density was investigated in 2D problem. The convergence of the solution was not achieved on the grid consisting of 160000 cells, numerical inaccuracy led to deformation of velocity profile. In case of 10000 and 40000 cells, further adaption near the wall was required when a certain value of $k$ was achieved. Increasing of value of $k$ caused divergence of the solution, but after the grid adaption it was possible to continue in simulations with higher values of adhesion coefficient $k$.

Fig. 4 shows velocity profiles for the chosen values of $k$ both for no slip and slip condition on the wall. Presented profiles were calculated for $Re$ = 1500 and a 3D geometry. A good agreement of the numerical simulation with the theoretical assumptions can be observed. Velocity profiles obtained from 2D and 3D problem are compared in Fig. 7.

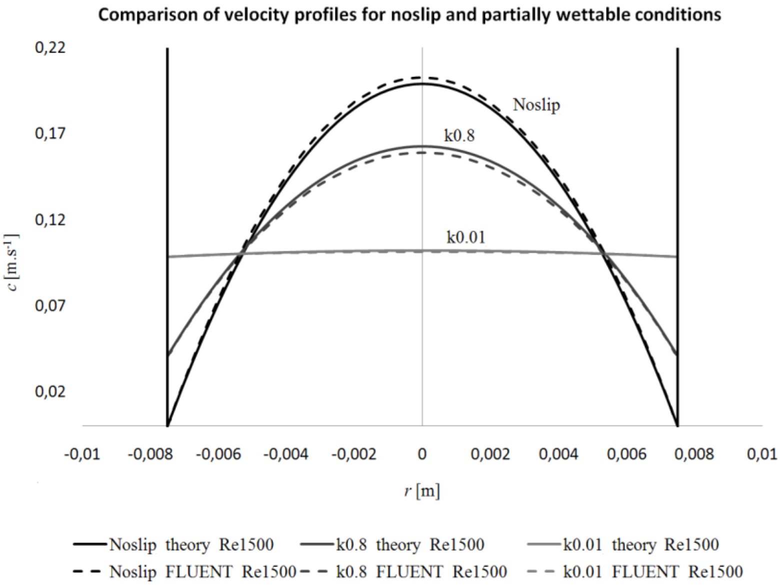


**Fig. 4** Comparison of velocity profiles obtained from theory and FLUENT simulation.

Besides shear stress and velocity profiles also the pressure drop was evaluated. For a no slip condition on the wall the pressure drop can be calculated from the Hagen–Poiseuille law (5) and in case of partially wettable surface according to (12). In numerical experiment the inlet and outlet static pressure was defined as an area average in case of fully developed laminar flow. The simulations were carried out for various values of adhesion coefficient and compared with theoretical results. Comparison of theory and numerical simulation in 3D pipe geometry is presented in Fig. 5.

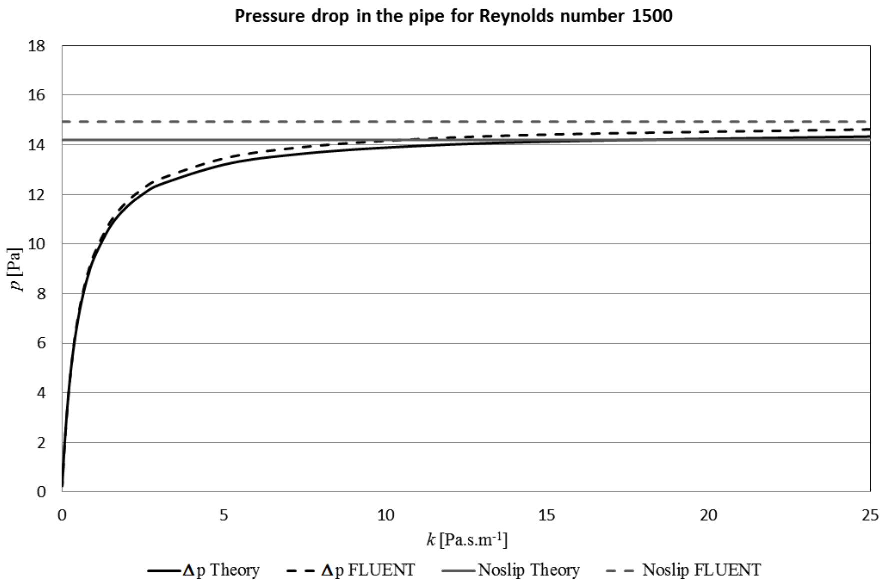


**Fig. 5** Comparison of pressure drop for wettable (no slip) and partially wettable (slip) boundary conditions on the pipe wall for $Re$ = 1500.

We can observe exponential variation of pressure drop on adhesion coefficient *k*. The adhesion coefficient should theoretically vary from near zero to infinity. But it can be seen, that only a small range of *k* brings significantly lower value of pressure drop. When a certain limit of *k* is reached, the pressure drop is approaching the value of no slip condition. The difference between the theoretical and numerical prediction is within the 5%. The results from 3D and 2D simulations are compared in Fig. 8. The shear stress profile for chosen values of adhesion coefficient *k* is illustrated in Fig. 6.

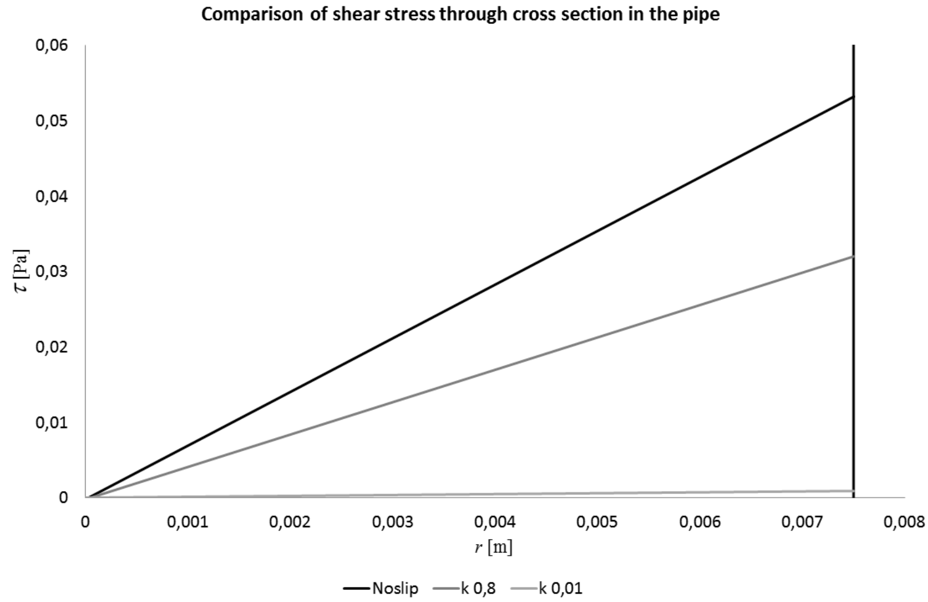


**Fig. 6** Shear stress profile in dependence on *k*.

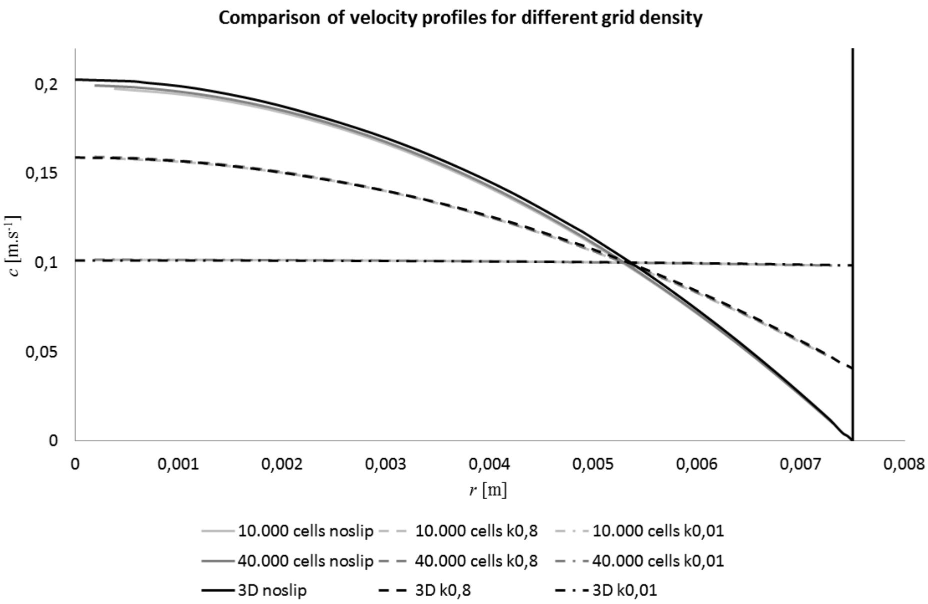


**Fig. 7** Comparison of calculated velocity profiles from 2D and 3D geometry.

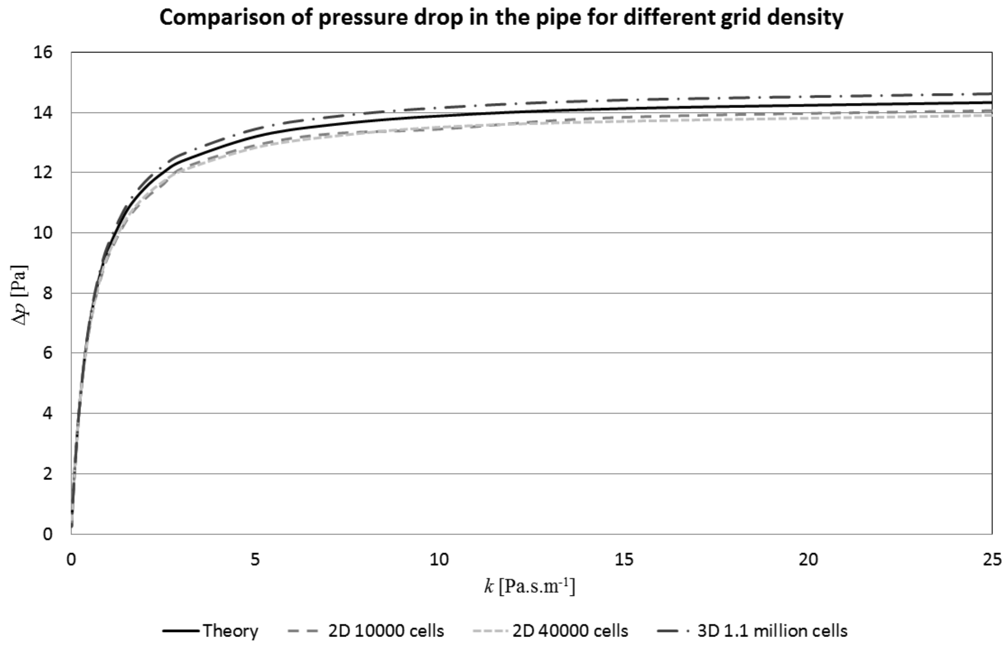


**Fig. 8** Calculated pressure drop in 2D a 3D geometry with different grid density.

## 3 CONCLUSION

The main objective of this work was to apply the theory presented by Pochylý [1-7] and Rinka [8] on the numerical modelling of the laminar pipe flow. The wall boundary condition was modified by user defined function that enabled to account for slip by means of adhesion coefficient *k*. For specified *k* FLUENT computes the tangential velocity $c_0$ at the boundary. Both shear stress and velocity profile change and the pressure drop decreases. The changes are significant in rather narrow range of adhesion coefficient. When a certain limit of *k* is reached, the pressure drop is approaching the value of no slip condition.

Mesh spacing near walls plays significant role in the accuracy of the computed wall shear stress. To evaluate the influence of the grid quality, the problem was modelled as 2D for various grid density, i.e. 10000, 40000 and 160000 cells. Very fine grid consisting of 160000 cells led to divergence, in case of 10000 and 40000 cells the results were in good agreement, but further adaption near the wall was required when a certain value of *k* was achieved. 3D solution was carried out for comparison. The difference between the theoretical and numerical prediction is within the 5% both for 2D and 3D case.

Slip condition with zero shear stress corresponds to inviscid flows. Software FLUENT enables to specify the non-zero shear on the wall as a constant, which is convenient for simple geometry. Definition of UDF for introduction of adhesion coefficient *k* opens further possibilities for modelling of slip condition in complex geometries of hydraulic elements and devices.

### REFERENCES

[1] POCHYLÝ, F., RINKA, L. *Povrchová energie v hraniční vrstvě kapky vody a tuhého povrchu*. Výzkumná zpráva VUT-EU13303-QR-34-07, Brno (CZ) VUT FSI, prosinec, 2007, 32 s.

[2] POCHYLÝ, F., HABÁN, V. *Smáčivost kapalin vůči pevným povrchům*. Výzkumná zpráva VUT-EU13303-QR-14-06, Brno (CZ) VUT FSI, 2006.

[3] POCHYLÝ, F., FIALOVÁ, S., HABÁN, V., RINKA, L. *The Wettability of the Liquid-Solid Interface*, FIV 2008-FLOW INDUCED VIBRATION, p. 47-52, ISBN 80-87012-12-7, (2008), Institute of Thermomechanics, AS CR.
[4] POCHYLÝ, F., FIALOVÁ, S., RINKA, L. *Adhesive Forces at a Solid-Liquid Interface,* Zeszyty naukowe, Vol. 1, (2008), No. 10, p. 245-252, ISBN 1897-2683, Wydawnictwo politechniki Świętokrzyskiej w Kielcach.
[5] RINKA, L., POCHYLÝ, F. *Analýza adhésních sil na rozhraní tekutiny a tuhého povrchu.* Výzkumná zpráva VUT-EU13303-QR-21-08, Brno (CZ) VUT FSI, prosinec 2008.
[6] FIALOVÁ, S., POCHYLÝ, F., RINKA, L. *The Boundary Condition of the Adhesive Forces Effect on the Solid/Liquid Interface.* In *Engineering Mechanics 2009-book of Extended Abstracts.* První edice, 2009. Praha, Akademie věd České republiky. 2009. p. 48-53. ISBN 978-80-86246-35-2.
[7] POCHYLÝ, F., FIALOVÁ, S., HABÁN, V., RINKA, L. *Praktičeskoje ispolzovanije smačivajemosti. NASOSY&OBORUDOVANIJE.* 2009. (54) 2009(1). p. 20-21.
[8] RINKA, L. *Interakce kapaliny s tuhou stěnou v závislosti na smáčivosti povrchu.* Brno: VUT, 2009. 38 s. Fakulta strojního inženýrství.
[9] FLUENT: FLUENT 12 - *User's guide.* Fluent Inc. 2007 [online]. Dostupné z < http://spc.vsb.cz/portal/cz/documentation/manual/index.php >.
[10] BIRD, R. B., STEWART, W. E., LIGHTFOOT, N. N: *Přenosové jevy*. Praha: Academia, 1968, 800 s.

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