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COMPENSATION TUNING OF ANALOG AND DIGITAL CONTROLLERS FOR FIRST ORDER PLUS TIME DELAY PLANTS

KOMPENZAČNÍ SEŘÍZENÍ ANALOGOVÝCH I ČÍSLICOVÝCH REGULÁTORŮ PRO SOUSTAVY SE SETRVAČNOSTÍ PRVNÍHO ŘÁDU A DOPRAVNÍM ZPOŽDĚNÍM

Abstract

The article is devoted to the simple compensation tuning of analog and digital PI and PID controllers for the first order plus time delay plants. The described method makes controller tuning possible so that the control process is non-oscillatory without an overshoot for all input variables. The use is shown in the example.

Abstrakt

Článek je věnován jednoduché kompenzační metodě seřízení analogových i číslicových regulátorů PI a PID pro proporcionální regulované soustavy se setrvačností prvního řádu a dopravním zpožděním. Popisovaná metoda umožňuje seřízení regulátoru tak, aby regulační proces byl nekmitavý a bez překmitu pro všechny vstupní veličiny. Použití je ukázáno na příkladě.

1 INTRODUCTION

Great attention is devoted all the time to PI and PID controller tuning. It is given in that industrial practice demands simple, reliable and at the same time robust controllers. Among these controllers there belongs PI and PID controllers. Their operation is easily understandable and technical semi–skilled staff make up their tuning on the basis of the recommended methods. Unfortunately, at present there are a huge number of different kinds of controller tuning methods, in which the inexperienced user is not oriented [1, 7, 8 – 10].

The described tuning method in the article is simple and relative robust. It enables analog and digital PI and PID controller tuning so that the control process will be non-oscillating without overshoots. It is suitable for plants with the transfer function

$$G_P(s) = \frac{k_1}{T_1 s + 1} \mathrm{e}^{-T_d s} \tag{1}$$

where k_1 is the plant gain, T_1 – the time constant, T_d – the time delay, s – the complex variable in the L-transform.

It is supposed that the quantization error is negligibly small and therefore the terms discrete and digital can be considered as equivalent.

2 CONTROLLERS AND THEIR TUNING

The D-transform is used to have a uniform approach to analog and digital controllers. Detailed information about it can be found, e.g. in [2, 3, 6, 11, 12].

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On the assumption that there is a D/A converter with the behavior of a sampler and a zero order holder, the plant D-transfer function has the form [11]

$$G_P(\gamma) = \frac{ak_1}{T\gamma + a}(T\gamma+1)^{-d}, \; a = 1 - \mathrm{e}^{-\frac{T}{T_1}}, \; d = \frac{T_d}{T} \tag{2}$$

where T is the sampling period, d – the relative discrete time delay (meanwhile the integer is supposed).

Further, the use of the standard controllers PI and PID are supposed, their transfer functions are given in Tab. 1 [11], where K_P is the controller gain, T_I – the integral time, T_D – the derivative time, z – the complex variable in the Z-transform, γ – the complex variable in the D-transform.

Tab. 1 Transfer functions of standard PI and PID controllers.

	L-transform	Z-transform	D-transform
PI	$K_P\left(1+\frac{1}{T_I s}\right)$	$K_P\left(1+\frac{T}{T_I}\frac{z}{z-1}\right)$	$K_P\left(1+\frac{T\gamma+1}{T_I\gamma}\right)$
PID	$K_P\left(1+\frac{1}{T_I s}+T_D s\right)$	$K_P\left(1+\frac{T}{T_I}\frac{z}{z-1}+\frac{T_D}{T}\frac{z-1}{z}\right)$	$K_P\left(1+\frac{T\gamma+1}{T_I\gamma}+\frac{T_D\gamma}{T\gamma+1}\right)$

The compensation tuning of the PI and PID controllers is a very often used method in technical practice. It is given by its simplicity. The dynamic simplification is taken during the compensation and therefore it enables the use of analytical approaches.

The derivation procedure for computation of the adjustable controller parameters will be shown for the PID controller.

The D-transfer function of the PID controller with a cascade structure (with an interaction) has the form

$$G_C(\gamma) = K'_P\left(1+\frac{T\gamma+1}{T'_I\gamma}\right)\left(1+\frac{T'_D\gamma}{T\gamma+1}\right) = \frac{K'_P[(T'_I+T)\gamma+1][(T'_D+T)\gamma+1]}{T'_I\gamma(T\gamma+1)} \tag{3}$$

Consider the control system in Fig. 1, where W is the transform of the desired variable, v – the transform of the disturbance variable, Y – the transform of the controlled variable.

The open-loop D-transfer function on the basis of Fig. 1 for (2) and (3) can be determined

$$G_o(\gamma) = G_C(\gamma)G_P(\gamma) = \frac{k_1K'_P[(T'_I+T)\gamma+1][(T'_D+T)\gamma+1]}{T'_I\gamma(T\gamma+1)\left(\frac{T}{a}\gamma+1\right)}(T\gamma+1)^{-d} \tag{4}$$

For the compensation it must hold

$$T'_I + T = \frac{T}{a} \;\Rightarrow T_I'^{*}(T) = \frac{1-a}{a}T \tag{5}$$

After the compensation the open-loop transfer function of the control system in Fig. 1 has the form

$$G_o(\gamma) = \frac{k_1K'_P[(T'_D+T)\gamma+1]}{T_I'^{*}\gamma(T\gamma+1)}(T\gamma+1)^{-d} \tag{6}$$

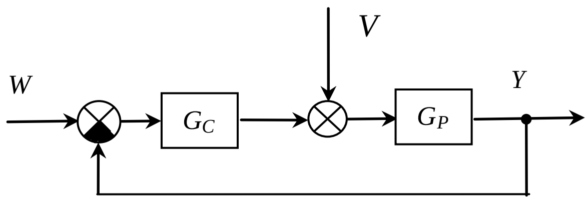

Fig. 1 Scheme of control system.

The controller gain $K_P'^*$ and the derivative time $T_D'^*$ can be determined by the multiple dominant pole method (MDPM), which supposes that the essential behavior is given by a stable multiple dominant pole and influences of the non-dominant poles and zeros are negligible [4, 5, 11, 13].

The triple dominant pole γ_3^*, the controller gain $K_P'^*$ and the derivative time $T_D'^*$ can be obtained from three equations

$$N(\gamma)=0,\ \frac{\mathrm{d}N(\gamma)}{\mathrm{d}\gamma}=0,\ \frac{\mathrm{d}^2N(\gamma)}{\mathrm{d}\gamma^2}=0 \tag{7}$$

where the control system characteristic polynomial $N(\gamma)$ can be determined from (6), i.e.

$$N(\gamma)=\gamma(T\gamma+1)^{d+1}+\frac{k_1K_P'}{T_I'^*}[(T_D'+T)\gamma+1] \tag{8}$$

By solving the three equations (7) it is obtained

$$\gamma_3^*=-\frac{2}{(d+2)T}=-\frac{2}{T_d+2T} \tag{9}$$

$$k_1K_P'^*=4T_I'^*(T)\frac{d+1}{(d+2)^2T}\left(\frac{d}{d+2}\right)^d=4\frac{(1-a)(d+1)}{a(d+2)^2}\left(\frac{d}{d+2}\right)^d \tag{10}$$

$$T_D'^*(T)=\frac{d^2T}{4(d+1)}=\frac{T_d^2}{4(T_d+T)} \tag{11}$$

The relations (5) and (9) – (11) directly hold for the PID digital controller with the cascade structure. For $T\to 0$ the corresponding relations for the PID analog controller with the cascade structure can be obtained

$$s_3^*=\lim_{T\to 0}\gamma_3^*=-\frac{2}{T_d}$$

$$k_1K_P'^*(0)=\lim_{T\to 0}[k_1K_P'^*(T)]=\frac{4T_1}{\mathrm{e}^2T_d} \tag{12}$$

$$T_I'^*(0)=\lim_{T\to 0}T_I'^*(T)=T_1 \tag{13}$$

$$T_D'^*(0)=\lim_{T\to 0}T_D'^*(T)=\frac{T_d}{4} \tag{14}$$

In relation (12) the equality

$$\lim_{d\to\infty}\left(1+\frac{2}{d}\right)^d=\mathrm{e}^2$$

was used.

The cascade structure of the PID digital controller (3) can be easily converted in the parallel structure (see Tab. 1)

$$G_C(\gamma) = K'_P i\left[1 + \frac{T\gamma + 1}{T'_I i\gamma} + \frac{T'_D\gamma}{i(T\gamma+1)}\right],\ i = 1 + \frac{T'_D}{T'_I} \tag{15}$$

where i is the interaction factor.

For the PID controller with the parallel structure in accordance with (5), (10), (11) and (15) there is obtained

$$i = 1 + \frac{T'_D(T)}{T'_I(T)} = \frac{4(1-a)(d+1) + ad^2}{4(1-a)(d+1)} \tag{16}$$

$$k_1 K_P^*(T) = k_1 K_P'^*(T) i = \frac{4(1-a)(d+1) + ad^2}{a(d+2)^2}\left(\frac{d}{d+2}\right)^d \tag{17}$$

$$T_I^*(T) = T_I'^*(T) i = \frac{4(1-a)(d+1) + ad^2}{4a(d+1)} T \tag{18}$$

$$T_D^*(T) = \frac{T_D'^*(T)}{i} = \frac{(1-a)d^2}{4(1-a)(d+1) + ad^2} T \tag{19}$$

The relations (17) – (19) directly hold for the PID digital controller with the parallel structure (Tab. 1). For $T \to 0$ the corresponding relations for the PID analog controller with the parallel structure can be obtained (Tab. 1)

$$k_1 K_P^*(0) = \lim_{T\to 0}[k_1 K_P^*(T)] = \frac{4T_1 + T_d}{e^2 T_d} \tag{20}$$

$$T_I^*(0) = \lim_{T\to 0} T_I^*(T) = T_1 + \frac{T_d}{4} \tag{21}$$

$$T_D^*(0) = \lim_{T\to 0} T_D^*(T) = \frac{T_d T_1}{4T_1 + T_d} \tag{22}$$

The relations (17) – (19) for the PID digital controller with a parallel structure have unpleasant forms for practical use. They can be fundamentally simplified but it is somewhat inaccurate.

By the use of the first order Padé approximation, i.e. the approximate equality

$$e^{-x} \approx \frac{1 - \frac{x}{2}}{1 + \frac{x}{2}} \tag{23}$$

in (5) it can be obtained

$$T_I'^*(T) \approx T_1 - \frac{T}{2} \tag{24}$$

The approximation (24) for $T_1/T \geq 2$ gives the error less than 3 % and for $T_1/T \geq 4$ the error is less then 1 %.

Similarly, by the use of the approximate equality (23) and the approximation

$$\frac{(d+1)}{(d+2)^2}\frac{d}{d+2} \approx \frac{1}{14 + (d-1)e^2} \tag{25}$$

in (10) it is obtained

$$k_1 K_P'^*(T) \approx \frac{2(2T_1 - T)}{(14 - e^2)T + e^2 T_d} \tag{26}$$

The approximation (25) for $d = T_1/T \geq 2$ gives the error less than 1 %.

The relation (11) needn't the approximation. It is obvious that for $T \to 0$ the corresponding relations (13) and (12) for the PID analog controller with the cascade structure can be obtained from relations (24) and (26).

The simplified relations for the PID digital controller with the parallel structure can be obtained from relations (16) – (19) after considering the approximation (23) – (26):

$$k_1 K_P^* \approx \frac{2(T_d + T)(2T_1 - T) + T_d^2}{(T_d + T)[(14 - e^2)T + e^2 T_d]} \doteq \frac{2(T_d + T)(2T_1 - T) + T_d^2}{(T_d + T)(6.611T - 7.389T_d)} \quad (27)$$

$$T_I^* \approx \frac{2(T_d + T)(2T_1 - T) + T_d^2}{4(T_d + T)} \quad (28)$$

$$T_D^* \approx \frac{(2T_1 - T)T_d^2}{2[2(T_d + T)(2T_1 - T) + T_d^2]} \quad (29)$$

The simplified relations are universal, for $T > 0$ hold for the standard PID digital controllers with the parallel structure and for $T = 0$ they hold for the standard PID analog controllers with the parallel structure.

By the similar approach after compensation accurate and simplified relations for the analog and digital PI controllers can be obtained.

The accurate relations are:

$$k_1 K_P^*(T) = \frac{1-a}{a(d+1)} \left(\frac{d}{d+1} \right)^d \quad (30)$$

$$T_I^*(T) = \frac{1-a}{a} T \quad (31)$$

It can be easily shown that

$$k_1 K_P^*(0) = \lim_{T \to 0} [k_1 K_P^*(T)] = \frac{T_1}{e T_d} \quad (32)$$

$$T_I^*(0) = \lim_{T \to 0} T_I^*(T) = T_1 \quad (33)$$

For simplification of the relations (30) and (31) the approximation (23) and the approximate equality

$$\frac{1}{d+1} \left(\frac{d}{d+1} \right)^d \approx \frac{1}{4 + (d-1)e} \quad (34)$$

were used.

The approximation (34) for $d = T_d/T \geq 1$ gives the error less than 0.5 %.

The simplified relations for the computation of the adjustable parameters of the PI digital controller have the forms

$$k_1 K_P^* \approx \frac{2T_1 - T}{2[(4 - e)T + e T_d]} \doteq \frac{2T_1 - T}{2.563T + 5.437T_d} \quad (35)$$

$$T_I^* \approx T_1 - \frac{T}{2} \quad (36)$$

The obtained simplified relations hold for the PI digital ($T > 0$) and analog ($T = 0$) controllers.

The universal relations for computation of the adjustable controller parameters are given in Tab. 2.

Tab. 2 Adjustable parameters of analog and digital PI and PID controllers.

Type		Controller < analog $T = 0$, digital $T > 0$
PI	T_I^*	$T_1 - \frac{T}{2}$
	K_P^*	$\frac{T_I^*}{k_1[(4-\mathrm{e})T + \mathrm{e}T_d]} \doteq \frac{T_I^*}{k_1(1.282T + 2.718T_d)}$
PID	T_I^*	$\frac{2(T_d + T)(2T_1 - T) + T_d^2}{4(T_d + T)}$
	K_P^*	$\frac{4T_I^*}{k_1[(14-\mathrm{e}^2)T + \mathrm{e}^2 T_d]} \doteq \frac{T_I^*}{k_1(1.653T + 1.847T_d)}$
	T_D^*	$\frac{(2T_1 - T)T_d^2}{8(T_d + T)T_I^*}$

With respect to the compensation the described method is suitable for the FOPTD plants, for which inequality

$$T_1 \le 8T_d \tag{37}$$

holds.

3 EXAMPLE

For the FOPTD plant with the L-transfer function

$$G_P(s) = \frac{1}{6s+1}\mathrm{e}^{-6s}$$

It is necessary to tune of the analog and digital PI and PID controllers so the servo and regulatory responses were non-oscillatory without overshoot. The time constant and the time delay are in seconds.

On the basis of Tab. 2 for $k_1 = 1$, $T_1 = 6$ s and $T_d = 6$ s it is obtained:

PI controller

- analog ($T = 0$): $K_P^* \doteq 0.37$; $T_I^* = 6$
- digital ($T = 2$): $K_P^* \doteq 0.26$; $T_I^* = 5$

PID controller

- analog ($T = 0$): $K_P^* \doteq 0.68$; $T_I^* = 7.5$; $T_D^* = 1.2$
- digital ($T = 2$): $K_P^* \doteq 0.43$; $T_I^* \doteq 6.13$; $T_D^* = 0.92$

The control system responses are shown in Figs 2 and 3.

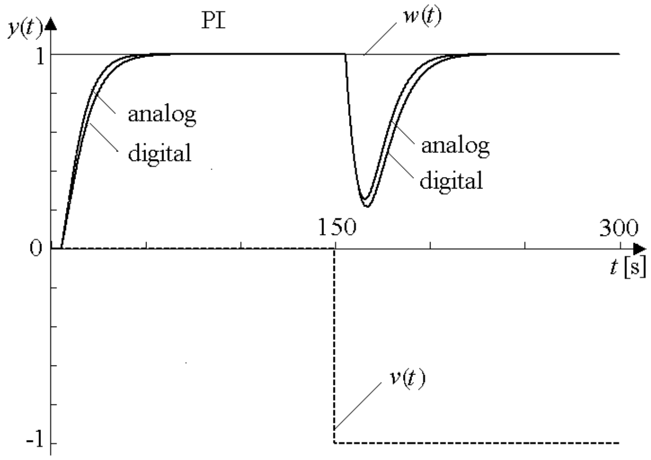

Fig. 2 Responses of control system with PI controller.

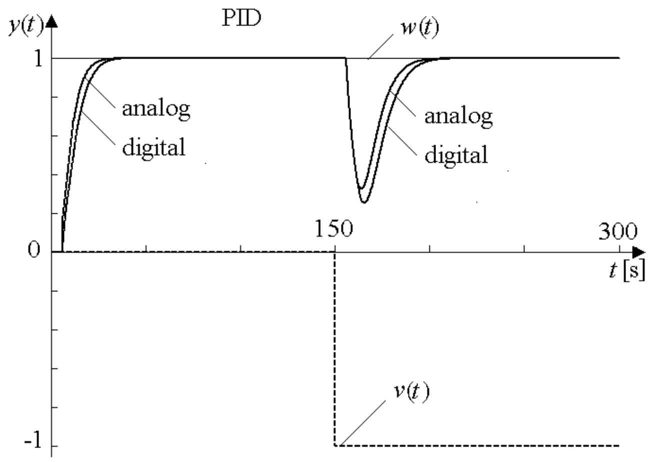

Fig. 3 Responses of control system with a PID controller.

4 CONCLUSIONS

In this article the new compensation method for the tuning of analog and digital PI and PID controllers is described in detail. The method ensures the servo and regulatory responses non-oscillatory without overshoot. It is fully derived. Its use is easy and effective. In case of need the responses can be accelerated by increasing the controller gain.

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