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ROBUSTNESS OF CONTROL SYSTEM TUNED BY MULTIPLE DOMINANT POLE METHOD
AND DESIRED MODEL METHOD

ROBUSTNOST REGULAČNÍHO OBVODU SEŘÍZENÉHO MEDOTOU NÁSOBNÉHO
DOMINANTNÍHO PÓLU A METODOU POŽADOVANÉHO MODELU

**Abstract**

In the article two analytical analog controller PI tuning methods are shortly described and compared from the point of view of the control system robustness for the first order plus time delay plant. For comparison the multiple dominant pole method and the desired model method were chosen. The program Matlab/Simulink for verification of the control system robustness was used.

**Abstrakt**

V příspěvku jsou stručně popsány a porovnány z hlediska robustnosti regulačního obvodu dvě analytické metody seřízení regulátoru PI pro proporcionální soustavu se setrvačností prvního řádu a dopravním zpožděním. Pro porovnání byla vybrána metoda násobného dominantního pólu a metoda požadovaného modelu. Pro ověření robustnosti byl použit program Matlab/Simulink.

## 1  CONTROL OF FOPTD PLANTS

Consider the control system in Fig. 1 with the first order plus time delay (FOPTD) plant

$$G_S(s) = \frac{k_1}{T_1 s + 1} e^{-T_{d1} s} \tag{1}$$

and with the PI controller

$$G_R(s) = k_P \left( 1 + \frac{1}{T_I s} \right) \tag{2}$$

where $k_1$ is the plant gain, $T_1$ – the time constant, $T_{d1}$ – the time delay, $k_P$ – the controller gain, $T_I$ – the integral time, $W(s)$ – the desired variable transform, $V(s)$ – the disturbance variable transform, $Y(s)$ – the controlled variable transform.
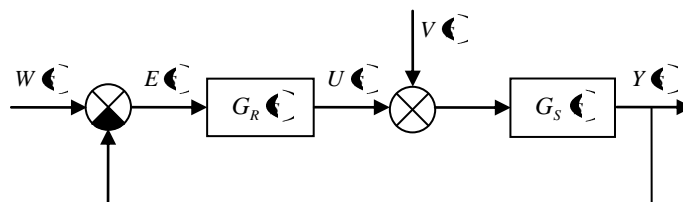


**Fig. 1** Control system.

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[**] Prof. Ing. Miluše VÍTEČKOVÁ, CSc., VŠB - Technical University of Ostrava, Faculty of Mechanical Engineering, Department ATŘ-352, 17. listopadu 15, Ostrava - Poruba, 708 33, Czech Republic, (+420) 59 732 4493, e-mail miluse.viteckova@vsb.cz

## 1.1 Multiple dominant pole method

The multiple dominant pole method (MDPM) is simple analytical tuning method, which ensures the non-oscillating control process [2, 5, 9]. It supposes that the multiple dominant pole determines the control system behavior and influence of the nondominant poles and zeros can be neglected. The triple dominant pole $s_3^*$ and the adjustable controller parameters $k_P^*$ a $T_I^*$ (Tab. 1) can be obtained by solving of the system of three equations

$$\frac{d^i N(s)}{ds^i} = 0, \quad i = 0, 1, 2 \tag{3}$$

where $N(s)$ is the characteristic quasi polynomial of the control system with the controller (2) and the plant (1).

The controller adjustable parameters are given in Tab. 1. The MDPM is in detail described e.g. [2, 9].

**Tab. 1** Adjustable parameters of controller PI for MDPM.

| The adjustable parameters of controller PI |
|:---:|
| $s_3^* = -\dfrac{2}{T_{d1}} - \dfrac{1}{2T_1} + \sqrt{\dfrac{2}{T_{d1}^2} + \dfrac{1}{4T_1^2}}$ |
| $k_P^* = -\dfrac{1}{k_1}\left[T_{d1}T_1 s_3^{*2} + (2T_1 + T_{d1})s_3^* + 1\right]e^{T_{d1}s_3^*}$ |
| $T_I^* = -\dfrac{T_{d1}T_1 s_3^{*2} + (2T_1 + T_{d1})s_3^* + 1}{s_3^{*2}(T_{d1}T_1 s_3^* + T_1 + T_{d1})}$ |

## 1.2 Desired model method

The desired model method (DMM) is more general method then it is mentioned here [5, 7 – 9]. In the article it is supposed its use only for the FOPTD plant (1) and for ensuring the non-oscillating control process without overshoot. It is the compensation method and therefore the integral time can be determined from the compensation condition $T_I^* = T_1$. The double pole $s_2^* = -1/T_{d1}$ and the controller gain $k_P^* = T_1/(k_1 e T_{d1})$ can be obtained from (3) for $i = 0,1$, i.e. from the system of two equations, where $N(s)$ is the characteristic quasi polynomial of the control system in Fig. 1 with the plant (1) and the controller (2) for $T_I^* = T_1$.

**Tab. 2** Adjustable parameters of controller PI for DMM.

| The adjustable parameters of controller PI | |
|:---:|:---:|
| $k_P^* = \dfrac{T_1}{k_1 e T_{d1}}$ | $T_I^* = T_1$ |

The controller adjustable parameters are given in Tab. 2. The DMM is in detail described e.g. [7, 8].

## 2 VERIFICATION OF ROBUSTNESS

For verification of both tuning methods the FOPTD plant was chosen

$$G_S(s) = \frac{0.5}{2.25s + 1}e^{-9.5s} \qquad \hat{k}_1 = 0.5, \qquad \hat{T}_1 = 2.25 \text{ s}, \qquad \hat{T}_{d1} = 9.5 \text{ s} \tag{4}$$

where the symbol ^ means nominal value.

The changes of nominal values of parameters $\hat{k}_1, \hat{T}_1$ and $\hat{T}_{d1}$ of the FOPTD plant were being considered $\pm 25\%$ :

$$k_1 = \delta\,\hat{k}_1$$
$$T_1 = \delta\hat{T}_1 \tag{5}$$
$$T_{d1} = \delta\hat{T}_{d1}$$

where $\delta = 0.75,\ 1$ and $1.25$.

The influence of the changes of the plant parameters $k_1$, $T_1$ and $T_{d1}$ (5) was examined by the help of the simulation program Matlab/Simulink.

### 2.1 Multiple dominant pole method

On the basis of Tab. 1 for the MDPM the PI controller parameters were obtained

$$s_3^* = -0.165 \qquad\qquad k_P^* = 0.304 \qquad T_I^* = 3.252 \text{ s} \tag{6}$$

By means of the computer and the program Matlab/Simulink the control system responses were obtained (Figs 2 – 5). The Figs 2, 3 and 4 show the control system responses successively for $\pm 25\%$ changes of the plant gain $k_1$, the time constant $T_1$ and the time delay $T_d$. Fig. 5 shows the hardest case of simultaneous changes of all plant parameters $\pm 25\%$.
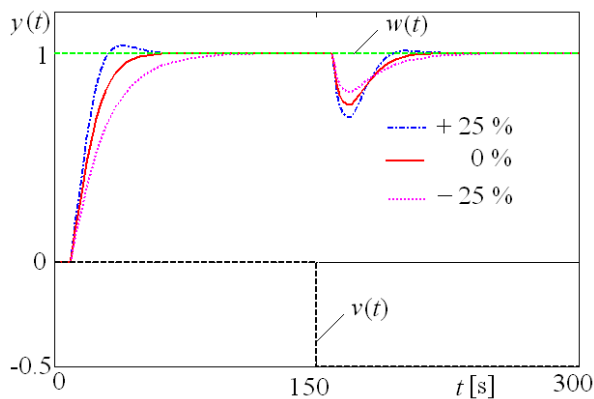


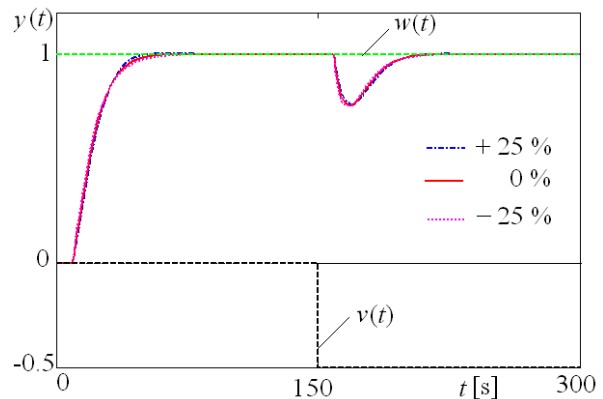**Fig. 2** Control system responses for changes of plant gain $k_1$ (MDPM).



**Fig. 3** Control system responses for changes of plant time constant $T_1$ (MDPM).
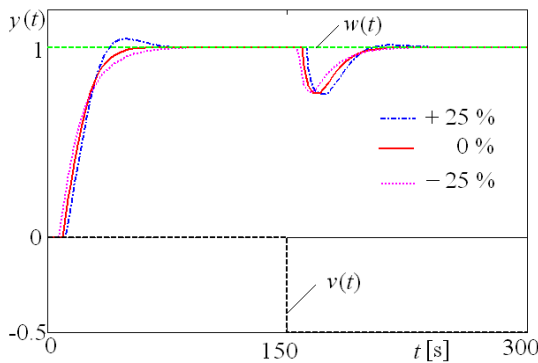


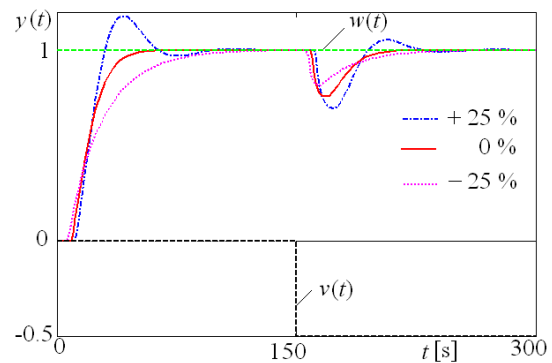**Fig. 4** Control system responses for changes of plant time delay $T_{d1}$ (MDPM).



**Fig. 5** Control system responses for changes of all plant parameters $k_1$, $T_1$ and $T_{d1}$ (MDPM).

255

On the basis of the control system responses in Figs 2 – 5 it is possible to say, that the control system tuned by the MDPM is robust. The change of the plant gain $k_1$ has the highest influence on the control system responses.

## 2.2 Desired model method

The adjustable controller parameters for the DMM were computed in accordance with Tab. 2

$$k_P^* = 0.174 \qquad\qquad T_I^* = 2.250 \text{ s}. \tag{7}$$

The responses of the control system tuned by the DMM are in Figs 6 – 9. The obtained courses are similar to the previous courses obtained by the MDPM and therefore the conclusions are the same.
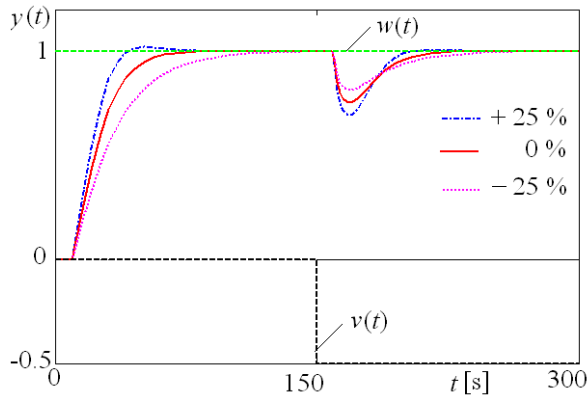


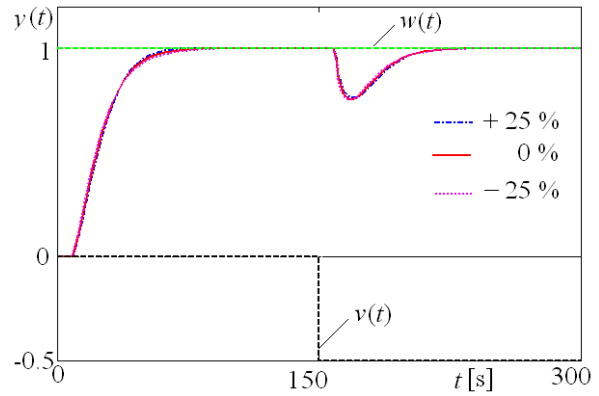**Fig. 6** Control system responses for changes of plant gain $k_1$ (DMM).



**Fig. 7** Control system responses for changes of plant time constant $T_1$ (DMM).
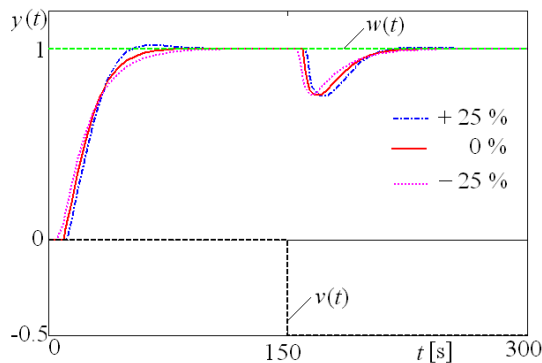


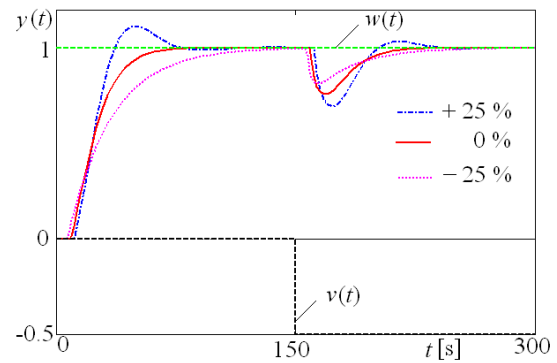**Fig. 8** Control system responses for changes of plant time delay $T_{d1}$ (DMM).



**Fig. 9** Control system responses for changes of all plant parameters $k_1$, $T_1$ and $T_{d1}$ (DMM).

The two performance indices – the settling time (5 %) and the criterion ITAE (integral of time multiplied by absolute error) for both methods were determined [1, 4, 6]. The obtained performance index values are given in Tab. 3. In accordance with expectation the performance indices for the MDPM are better then for the DMM. Both described methods are rather robust. The control system tuned by the MDPM has the triple dominant pole while the control system tuned by the DMM has only the double dominant pole and therefore the first method must give faster responses.

**Tab. 3** Control process performance indices for MDPM and DMM.

| Control process per-formance index | Multiple dominant pole method | | | Desired model method | | |
|---|---|---|---|---|---|---|
| | Change of all parame-ters $-25\%$ | Nominal parameters | Change of all parame-ters $+25\%$ | Change of all parame-ters $-25\%$ | Nominal parameters | Change of all parame-ters $+25\%$ |
| Settling time $t_r$ [s] | 73.22 | 40.45 | 57.33 | 88.55 | 51.78 | 65.23 |
| Criterion ITAE | 656.03 | 276.09 | 439.58 | 958.29 | 420.65 | 469.21 |

## 3 CONCLUSION

In the article the robustness of the MDPM and DMM for the PI analog controller and the FOPTD plant had been verified. The both methods are robust. The MDPM gives better values of the performance indices then the DMM but the DMM is simpler. For both methods changes of the plant gains have the highest influence on the control process and changes of the time constants have the lowest influence.

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