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THE STABILITY OF THE CONVEYOR BELT PONTOON

STABILITA PONTONU PÁSOVÉHO DOPRAVNÍKU

Abstract

To lead the conveyor belt transport cross water area the pontoon are used to support the carrying structure of the belts. The accident can happen when the pontoon turnover. For this reason the pontoon stability is investigated. The stability is described by the Reed's diagram. This can be constructed analytically or via numerical modeling. Both methods are described in the paper.

Abstrakt

Při dopravě sypkých materiálů přes vodní plochy se jako podpory pro dopravníkové trasy používají plovoucí pontony. Občas se stávají závažné nehody, když dojde k převrácení pontonu. Proto byla vyšetřována stabilita pontonu proti převržení. Stabilita je vyjádřena tzv. Reedovým diagramem. Ten může být konstruován analyticky nebo cestou numerického modelování. Obě tyto metody jsou popsány v příspěvku.

1 INTRODUCTION

In the process of mining and subsequent transport of the material on water the conveyor belts are lead on the floating pontoons. The question of stability of these pontoons is very important for the safety of such transport traces. The problem is close to the question of stability of boats.

2 THE LIFTING FORCE

The process of floating is determined by the concurrent acting of two forces - gravitational force and lifting force (due to hydrostatic pressure). These two forces are of the same value (the vessel floats).

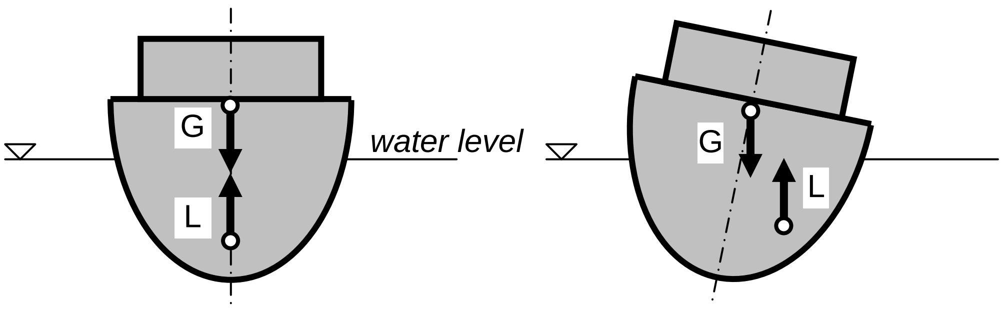

Fig. 1. The vessel in equilibrium and non-equilibrium position.

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In equilibrium position (the vessel board is horizontal) both forces lies on the same line (left on Fig. 1.) and resulting moment is zero. In non-equilibrium position the forces lies on the different lines (right on Fig. 1.). The resulting non-zero moment can have stabilizing (on Fig. 1.) or destabilizing effect. The amount of either stabilizing or destabilizing effect depends on the point of application of both forces.

The point of application of gravitational force is in the centre of gravity. The position of the centre of gravity is the subject of technical education and will not be discussed in this paper.

The lifting force is the result of hydrostatic pressure on the sides and bottom of the vessel (see Fig. 2.). The question of the point of application of the resulting force is not quite clear.

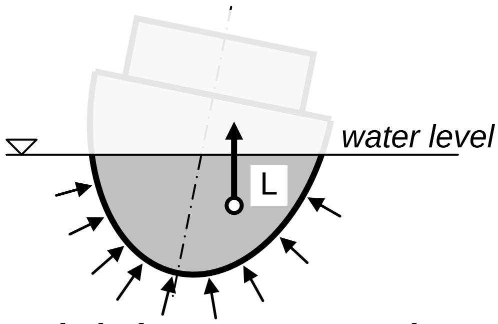

Fig. 2. The hydrostatic pressure on the vessel.

Suppose the triangular body (orthogonal triangle) in the water (see Fig. 3.).

Here: a, b and c is the dimensions of the triangle,
h is the depth of the triangle under the water level,
y is the vertical coordinate from the side a downward,
z is the coordinate along the side c,
β is the angle between the sides a and c.

The hydrostatic pressure with respect to the depth is:

$$p = \rho \cdot g \cdot (h + y)$$

where: ρ is the water density,
g is the gravitational acceleration ($g = 9{,}81\ m/s^2$),
(h+y) is the depth of the common point.

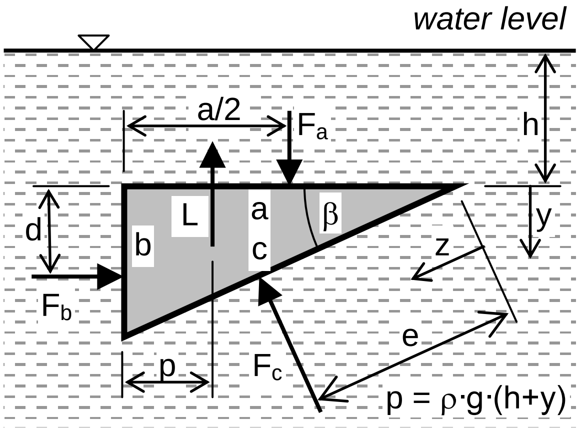

Fig. 3. The triangular body under the hydrostatic pressure.

The hydrostatic pressure, acting on the triangle sides, results into the forces F_a, F_b and F_c:

$$F_a = p_{(h)} \cdot S = p_{(h)} \cdot a \cdot \ell$$

$$F_b = \int_0^b p_{(y)} \cdot \ell \cdot dy$$

$$F_c = \int_0^c p_{(y)} \cdot \ell \cdot dz$$

where:

$p_{(h)}$ is the hydrostatic pressure in the depth h,
S is the area, on which the pressure acts,
ℓ is the length of the body (not seen on Fig. 3.),
$p_{(y)}$ is the hydrostatic pressure in the depth y.

The coordinate z and its differential dz are:

$$z = \frac{y}{\sin\beta} \quad \text{and} \quad dz = \frac{dy}{\sin\beta}$$

Subsequently the pressure forces are:

$$F_a = \rho \cdot g \cdot h \cdot a \cdot \ell$$

$$F_b = \int_0^b \rho \cdot g \cdot (h + y) \cdot \ell \cdot dy = \rho \cdot g \cdot \ell \cdot \left(h + \tfrac{1}{2} \cdot b\right) \cdot b$$

$$F_c = \int_0^b \rho \cdot g \cdot (h + y) \cdot \ell \cdot \frac{1}{\sin\beta} dy = \rho \cdot g \cdot \ell \cdot \frac{1}{\sin\beta} \cdot \left(h + \tfrac{1}{2} \cdot b\right) \cdot b = \rho \cdot g \cdot \ell \cdot \left(h + \tfrac{1}{2} \cdot b\right) \cdot c$$

For horizontal direction x:

$$\sum F_x = F_b - F_c \cdot \sin\beta = \rho \cdot g \cdot \ell \cdot \left(h + \tfrac{1}{2} \cdot b\right) \cdot b - \rho \cdot g \cdot \ell \cdot \frac{1}{\sin\beta} \cdot \left(h + \tfrac{1}{2} \cdot b\right) \cdot b \cdot \sin\beta = 0$$

For vertical direction y:

$$\sum F_y = F_c \cdot \cos\beta - F_a = \rho \cdot g \cdot \ell \cdot \frac{1}{\sin\beta} \cdot \left(h + \tfrac{1}{2} \cdot b\right) \cdot b \cdot \cos\beta - \rho \cdot g \cdot h \cdot a \cdot \ell =$$

$$= \rho \cdot g \cdot \ell \cdot \left(\frac{h + \tfrac{1}{2} \cdot b}{\tan\beta} \cdot b - h \cdot a \right) = \rho \cdot g \cdot \ell \cdot a \cdot \left(h + \tfrac{1}{2} \cdot b - h\right) = \tfrac{1}{2} \cdot \rho \cdot g \cdot \ell \cdot a \cdot b$$

(taking into account that $\frac{\sin\beta}{\cos\beta} = \tan\beta = \frac{b}{a}$)

The result corresponds to the Archimedes law for the lifting force:

$$L = V \cdot \rho \cdot g$$

where

$$V = \tfrac{1}{2} \cdot \ell \cdot a \cdot b$$

is the volume of the underwater body.

Farther the point of application of the pressure forces F_a, F_b and F_c is necessary to determine.

The pressure $p_{(h)}$ along the side a is constant, then the point of application of the force F_a is in the centre of the side a, in the distance a/2 from the left corner of the triangle.

The pressure $p_{(y)}$ along the sides b and c increases linearly, then the arm d of the force F_b or e of the force F_c resp. are:

$$F_b \cdot d = \int_0^b y \cdot p_{(y)} \cdot \ell \cdot dy = \rho \cdot g \cdot \ell \cdot \int_0^b (h + y) \cdot y \cdot dy = \rho \cdot g \cdot \ell \cdot \left(\tfrac{1}{2} \cdot h \cdot b^2 + \tfrac{1}{3} \cdot b^3\right)$$

If

$$F_b = \rho \cdot g \cdot \ell \cdot \left(h + \tfrac{1}{2} \cdot b\right) \cdot b$$

then

$$d = b \cdot \frac{3 \cdot h + 2 \cdot b}{6 \cdot h + 3 \cdot b}$$

Farther

$$F_c \cdot e = \int_0^c z \cdot p_{(y)} \cdot \ell \cdot dz = \rho \cdot g \cdot \ell \cdot \int_0^b (h + y) \cdot \frac{y}{\sin\beta} \cdot \frac{dy}{\sin\beta} = \rho \cdot g \cdot \ell \cdot \frac{1}{\sin^2\beta} \cdot \left(\tfrac{1}{2} \cdot h \cdot b^2 + \tfrac{1}{3} \cdot b^3\right)$$

if

$$F_c = \rho \cdot g \cdot \ell \cdot \frac{1}{\sin\beta} \cdot \left(h + \tfrac{1}{2} \cdot b\right) \cdot b$$

then

$$e = \frac{b}{\sin\beta} \cdot \frac{3 \cdot h + 2 \cdot b}{6 \cdot h + 3 \cdot b} = c \cdot \frac{3 \cdot h + 2 \cdot b}{6 \cdot h + 3 \cdot b}$$

It is clear that

$$e \cdot \sin\beta = d$$

and so the point of application of both F_b and F_c forces is in the depth of (h+d) under the water level.

Finally the arm p of the resulting lifting force L is:

$$L \cdot p = F_c \cdot \cos\beta \cdot (a - e \cdot \cos\beta) - F_a \cdot \tfrac{1}{2} \cdot a$$

(Notice: As shown above, the points of application of the forces F_b and F_c are in the same depth, the force F_b and horizontal component of the force F_c lies on the same line, their moment is zero and they are not taken into account in the last formula.)

The arm p of the lifting force L is then:

$$p = \frac{F_c \cdot \cos\beta \cdot (a - e \cdot \cos\beta) - F_a \cdot \tfrac{1}{2} \cdot a}{F_c \cdot \cos\beta - F_a} = \frac{\frac{1}{\tan\beta} \cdot \left(h + \tfrac{1}{2} \cdot b\right) \cdot b \cdot (a - e \cdot \cos\beta) - \tfrac{1}{2} \cdot h \cdot a^2}{\frac{1}{\tan\beta} \cdot \left(h + \tfrac{1}{2} \cdot b\right) \cdot b - h \cdot a} =$$

$$= \frac{\left(h + \tfrac{1}{2} \cdot b\right) \cdot (a - e \cdot \cos\beta) - \tfrac{1}{2} \cdot h \cdot a}{h + \tfrac{1}{2} \cdot b - h} = a \cdot \frac{\left(h + \tfrac{1}{2} \cdot b\right) \cdot \left(1 - \frac{3 \cdot h + 2 \cdot b}{6 \cdot h + 3 \cdot b}\right) - \tfrac{1}{2} \cdot h}{\tfrac{1}{2} \cdot b}$$

farther:

$$p = a \cdot \frac{(2 \cdot h + b) \cdot \frac{6 \cdot h + 3 \cdot b - 3 \cdot h - 2 \cdot b}{6 \cdot h + 3 \cdot b} - h}{b} = a \cdot \frac{(2 \cdot h + b) \cdot \frac{3 \cdot h + b}{3 \cdot (2 \cdot h + b)} - h}{b}$$

and finally:

$$p = \tfrac{1}{3} \cdot a$$

Identically the centre of gravity of triangular body is in 1/3 of the side a. The analogical solution for rectangle is trivial and will not be performed. Any more complicated shape can be assembled from triangles and rectangles (see Fig. 4). As a result it can be specified that the point of application of the lifting force is in the centre of gravity of the draught volume.

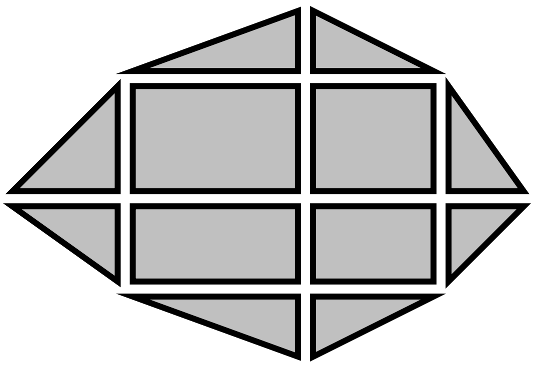

Fig. 4. The body assembled from triangles and rectangles.

3 THE REED'S DIAGRAM

As shown in the previous chapter, in the inclined position the gravitational and lifting force lie on the different carrying line. The perpendicular distance between them is called "the arm of stability" - p on Fig. 5. The moment L·p can have stabilizing or destabilizing effect, depending on the position of one force with respect to another. The p-ϕ curve, the dependence of the arm of stability p on the inclination angle ϕ, is called "the Reed's diagram" (see Fig. 5).

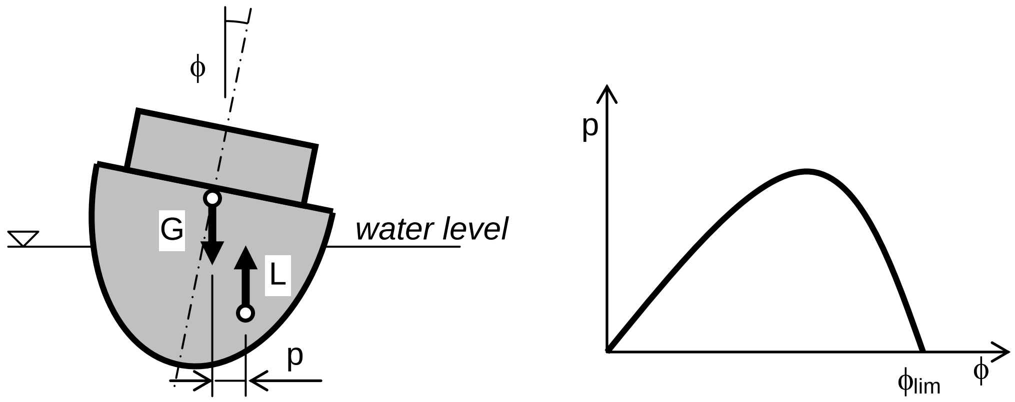

Fig. 5. The inclined vessel and the Reed's diagram.

Through inclining the arm of stability increases - the vessel tends to return to the equilibrium position. Then the arm of stability decreases and finally, in the limit position, reaches zero. Any bigger inclining leads to the loss of stability and uncontrolled turnover. The important parameter is the limit angle ϕ_{lim}, determining the point of the lost of stability.

4 THE STABILITY OF THE STRUCTURE

The supporting structure of the conveyor belt consists of two pontoons assembled into one structure (see Fig. 6).

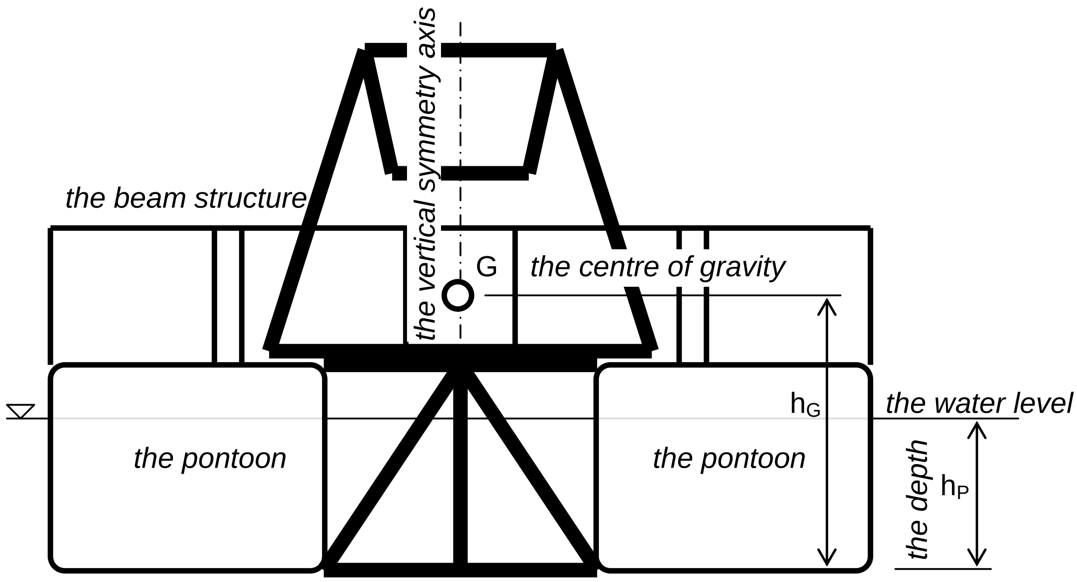

Fig. 6. The conveyor belt supporting structure.

The position of the centre of gravity G, given by the height h_G above bottom, is constant through inclining. But the position of the lifting force point of application changes. The arm of stability (see Fig. 7) is then:

$$p = x_L \cdot \cos\phi + y_L \cdot \sin\phi - h_G \cdot \sin\phi$$

where:

x_L is the x coordinate of the lifting force point of application,
y_L is the y coordinate of the lifting force point of application,
h_G is the y coordinate of the center of gravity (due to symmetry the x coordinate is zero),
ϕ is the inclining angle.

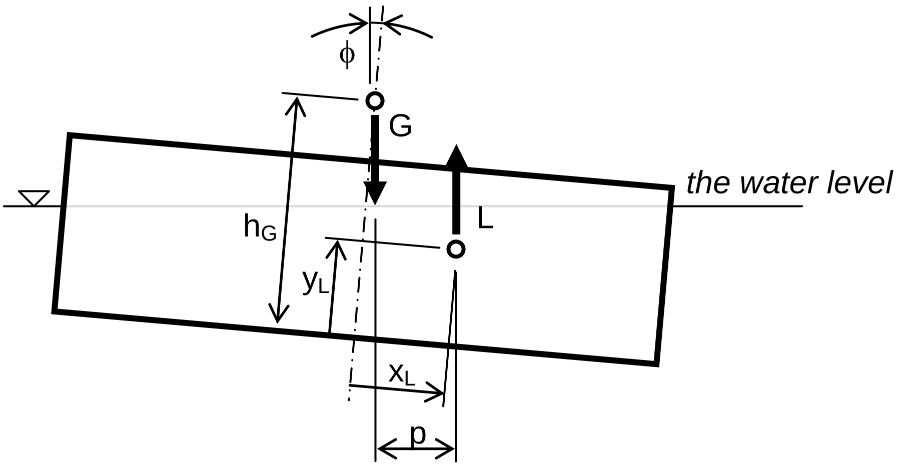

Fig. 7. The arm of stability.

In the following text the position of the lifting force point of application, expressed by the x_L and y_L coordinates, will be emphasized.

The equilibrium of gravitational and lifting force appears when the draught volume is:

$$V = \frac{m}{\rho}$$

where:

m is the mass of the whole structure,

ρ is the water density.

This volume is calculated and is constant throughout all the inclining.

If the shape of pontoon is the simple block, then the draught area (see Fig. 8) is:

$$A = \frac{V}{\ell}$$

where ℓ is the length of the block. This area is calculated and is constant throughout all the inclining.

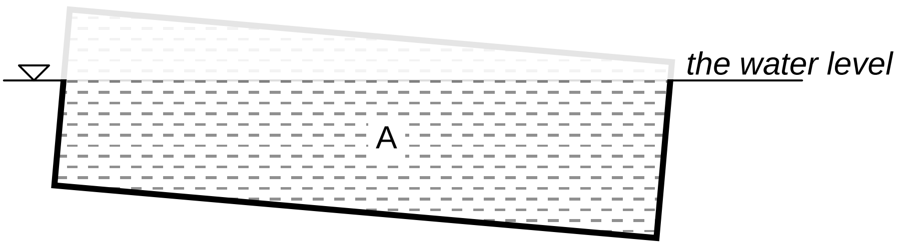

Fig. 8. The draught area.

4.1 The stability in lateral plane

The process of inclining of the pontoon can be divided into 3 phases:

1. phase (see Fig. 9) begins in horizontal position of the pontoon board, ends when the water level goes through the top corner of pontoon.

(The sketch on Fig. 9 is rotated by the angle ϕ so that the pontoon board is horizontal and the water level is sloped.)

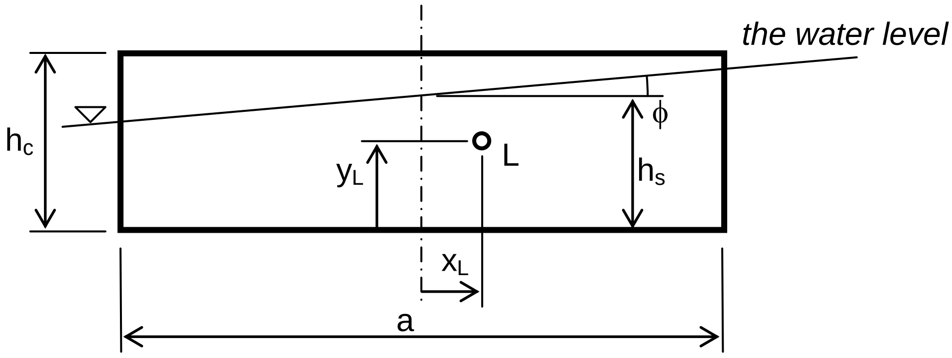

Fig. 9. The inclining, 1.phase.

Here a is the pontoon width,
h_c is the pontoon height.

2. phase (see Fig. 10) begins when the water level goes through the top corner of pontoon and end when it goes through the bottom corner.

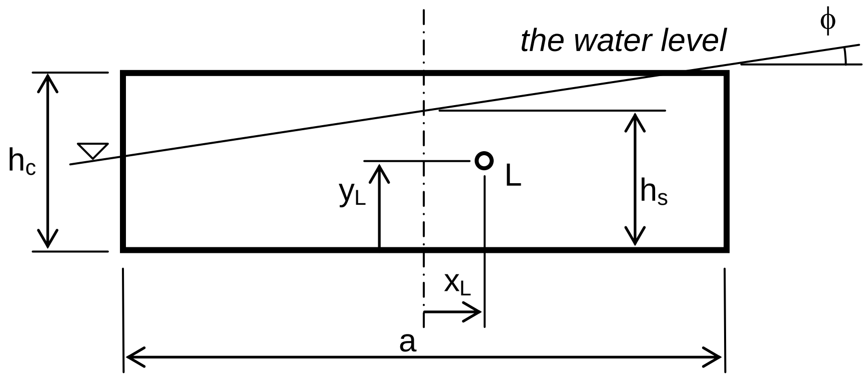

Fig. 10. The inclining, 2.phase.

3. phase (see Fig. 11) begins when the water level goes through the bottom corner and ends by the loss of stability.

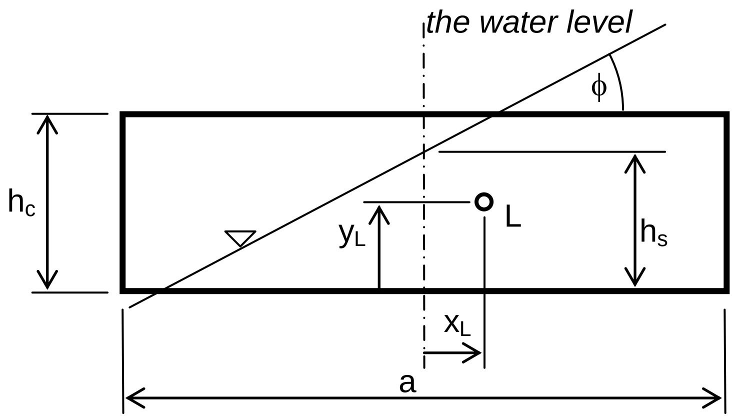

Fig. 11. The inclining, 3. phase.

The mathematical solution has two steps.

1. The solution of the vertical position of the pontoon in water (the vertical height h_S in Fig. 9, 10 and 11) so that the lifting force is equal to the gravitational force.

2. The solution of the x_L and y_L coordinates of the lifting force point of application.

For example for the 2. phase is:

$$h_s = h_c - \sqrt{(h_c \cdot a - A) \cdot 2 \cdot \tan\phi} + \tfrac{1}{2} \cdot a \cdot \tan\phi$$

$$x_L = \frac{\left(\tfrac{1}{2} \cdot a \cdot \tan\phi + h_c - h_s\right)^2 \cdot \left(a \cdot \tan\phi - h_c + h_s\right)}{6 \cdot A \cdot \tan^2\phi}$$

$$y_L = \frac{3 \cdot h_c^2 \cdot a \cdot \tan\phi - \left(h_c - h_s + \tfrac{1}{2} \cdot a \cdot \tan\phi\right)^2 \cdot \left(2 \cdot h_c + h_s - \tfrac{1}{2} \cdot a \cdot \tan\phi\right)}{6 \cdot A \cdot \tan\phi}$$

The total solution is complicated and will not be presented here. Finally the Reed's diagram is constructed for given dimension and masses (see Fig. 12).

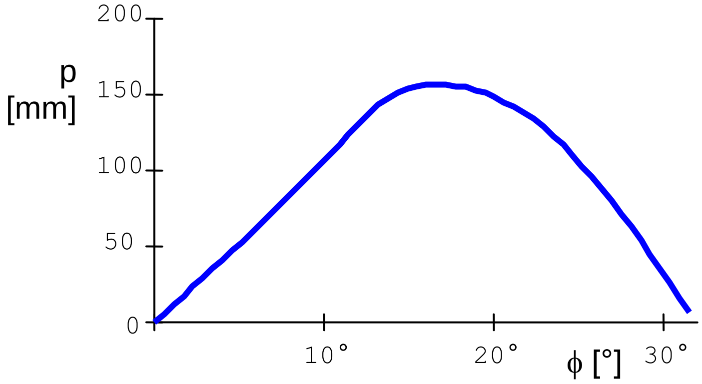

Fig. 12. The Reed's diagram for lateral stability.

It can be seen that up to the angle approx. $\phi = 13°$ the arm of stability linearly increases. In the position $\phi = 17°$ the arm is maximal. Then it decreases. Finally in the position $\phi_{lim} = 33°$ the arm is zero. After other inclining the uncontrolled turnover follows.

4.2 The stability in longitudinal plane

The sketch is on Fig. 13.

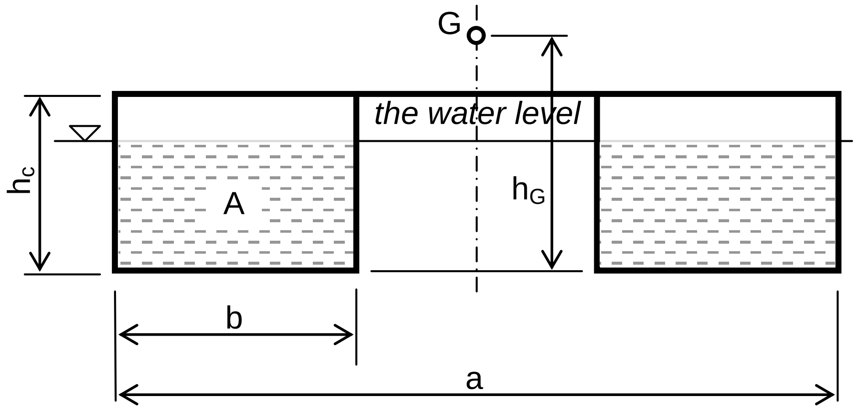

Fig. 13. The two pontoons.

Here a, b and h_c are the dimensions of the pontoons. The same as in case of lateral stability, the draught volume V and the draught area A must be granted.

The process of inclining has 4 (or may be 5) phases (see Fig. 14).

The solution has the two steps again - solution of the height h_S and solution of coordinates x_L and y_L. But the full solution is very complicated and will not be presented here. The Reed's diagram is on Fig. 15, the maximum inclining angle is $\phi_{max} = 44°$.

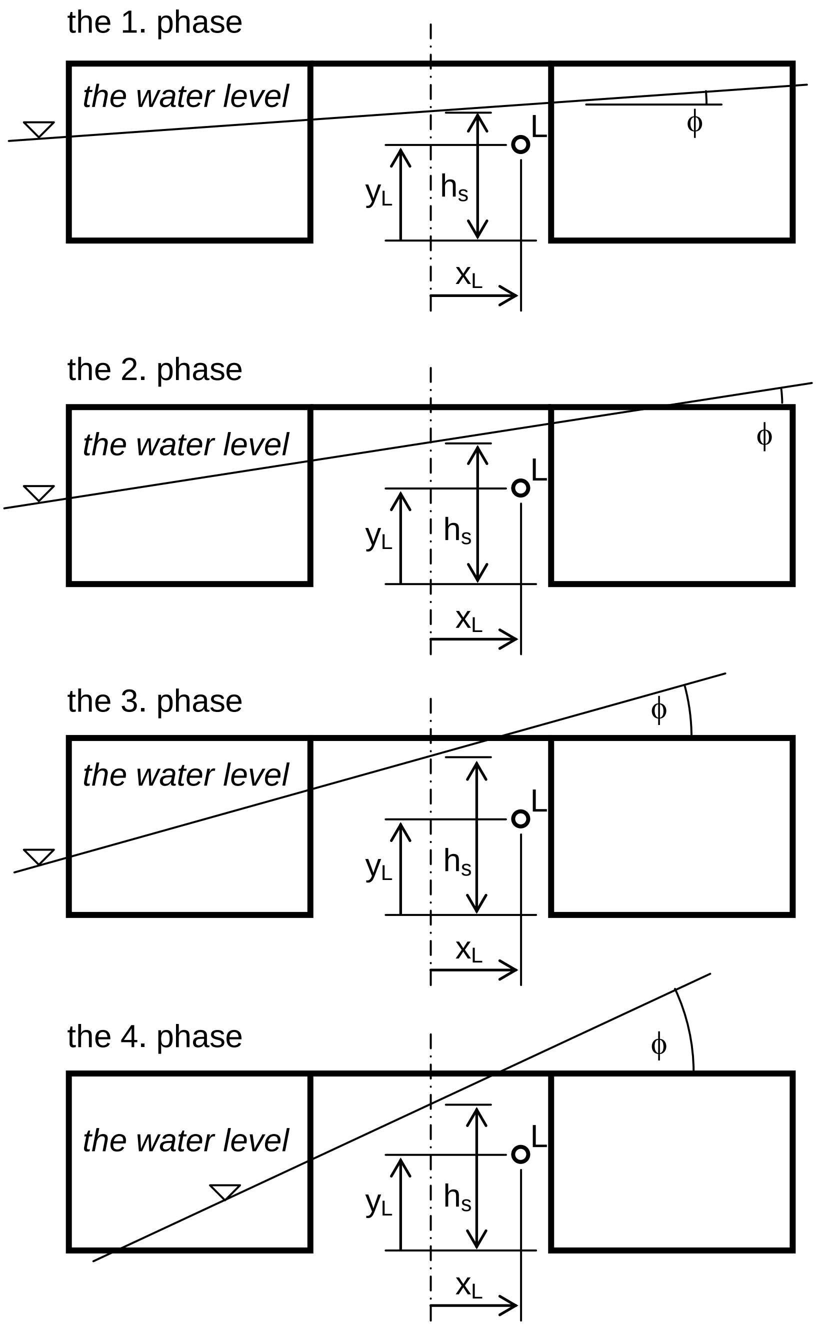

Fig. 14. The four phases of inclination.

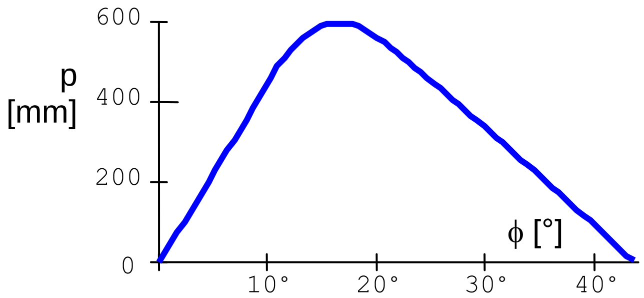

Fig. 15. The Reed's diagram for longitudinal stability.

5 THE NUMERICAL SOLUTION

The pontoons sometimes have the different shape (see Fig. 16), for example the shape of the half cylinder.

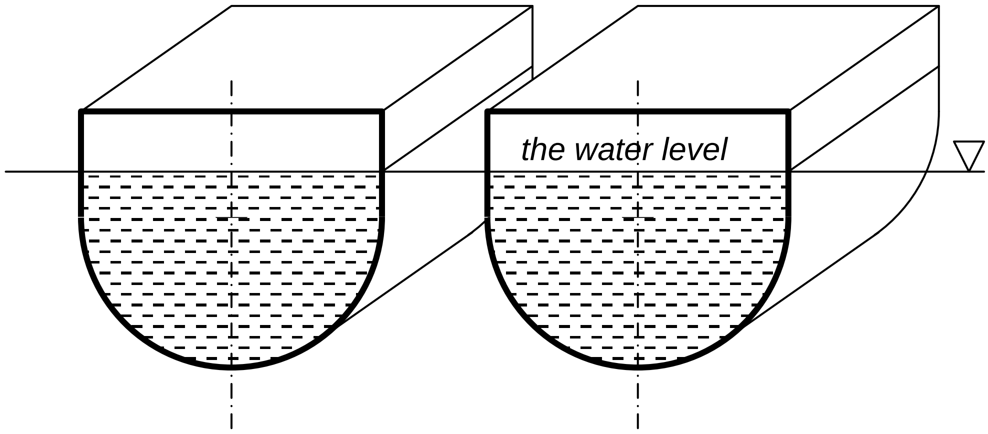

Fig. 16. The half-cylinder pontoons.

The algebraic solution is then very complicated, almost impossible. In this case the computer modeling was performed.

In the program Ansys (or any CAD software) is very easy to build the 2D or 3D model of the pontoons (see Fig. 17). It can be divided by a line, representing the water level. The draught area of the underwater part and its centroid coordinates are then calculated by software.

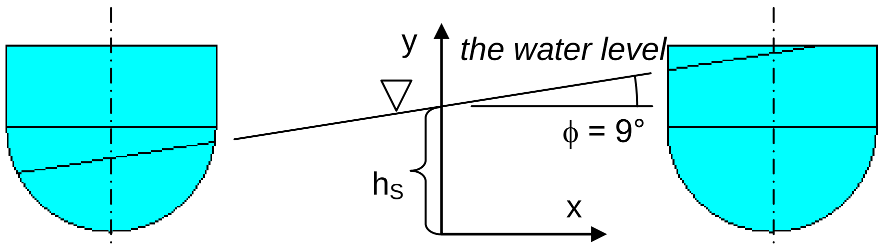

Fig. 17. The 2D model and its cut by the water level.

The solution contains 2 cycles. In the first cycle the angle ϕ (the vessel inclination) increases with a certain step. Every loop then contains the second cycle, in witch the position of the water level, determined by the height h_S, changes so that the draught area has just the value, needed to obtain the equilibrium between the gravitational and lifting force.

The example of the result listing is:
TOTAL SURFACE AREA OF ALL SELECTED AREAS = **0.61892E+06**
CENTER OF MASS: **XC= 144.32 YC= 272.88** ZC= 0.0000

For more complicated shape the 3D modeling is performed (see Fig. 18).

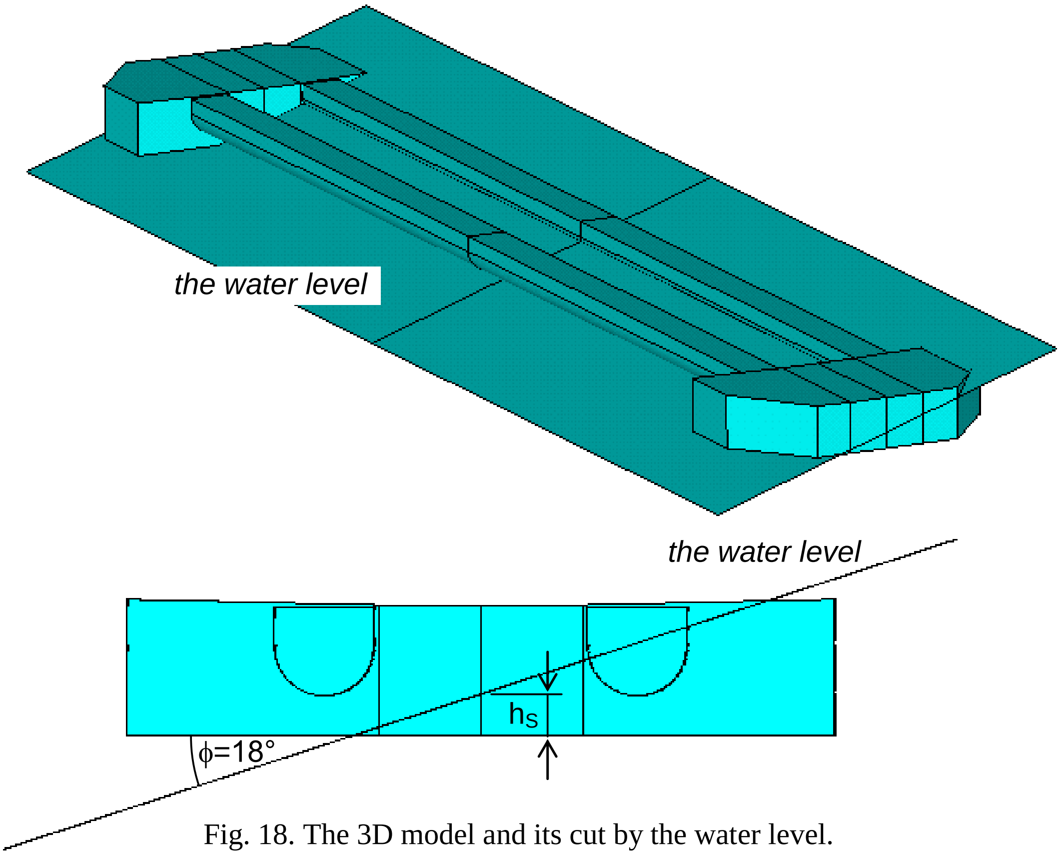

Fig. 18. The 3D model and its cut by the water level.

The example of the result listing is:
TOTAL VOLUME = 18.570
CENTER OF MASS: XC= 1.3508 YC= 0.44782 ZC= 0.0000

6 CONCLUSION

The solution of the pontoon stability is a very important problem in the process of cross-water transport. For the rectangular pontoon shape it can be solved analytically. The advantage is that such solution can be algorithmized and changed into program, for example in the MS Excel environment. For the more complicated pontoon shape the numerical solution can be performed and the data calculated by a certain software.

In both cases the result is the Reed's diagram and most important the maximum inclining angle ϕ_{max}, determining the break point - the point of the stability lost.

ACKNOWLEDGEMENT

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