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APPLICATION OF SELECTED MULTI-AXIAL FATIGUE CRITERIA ON THE RESULTS OF PROPORTIONAL FATIGUE EXPERIMENTS

APLIKACE VYBRANÝCH KRITÉRII ÚNAVOVÉ PEVNOSTI NA VÝSLEDKY PROPORCIONÁLNÍCH ÚNAVOVÝCH EXPERIMENTŮ

Abstract

The paper describes the experimental results obtained for the combined loading of the specimens in the region of high-cycle fatigue. The specimens were manufactured from common structural steel 11523.1, melt T31052.

The following experiments were performed: The first set of the specimens was loaded by the alternating torque amplitude. The second set was loaded by the in fully reversed push-pull. The third set of specimens was loaded by the combination of the torque and of the fully reversed push-pull. The phase shift is zero in this experiment. The results were evaluated by the modified conjugated strength criterion and other generally used multiaxial fatigue criteria. The stress-strain analysis of the specimens by FEM was performed to determine parameters (constants) of particular strength criteria.

Abstrakt

Článek popisuje výsledky experimentů při kombinovaném zatěžování zkušebních vzorků v oblasti vysokocyklové únavy. Použité zkušební vzorky byly vyrobeny z běžné konstrukční oceli 11523.1, tavba T31052.

Byly provedeny následující experimenty: První sada zkušebních vzorků byla zatěžovaná střídavou amplitudou krouticího momentu. Druhá sada zkušebních vzorků byla namáhaná střídavým tahem-tlakem. Třetí sada zkušební vzorků byla zatěžována amplitudou krouticího momentu v kombinaci s amplitudou střídavého tahu-tlaku. U tohoto experimentu byl fázový posuv roven nule. Na výsledky experimentů bylo aplikováno upravené konjugované kritérium pevnosti a další běžně užívaná multiaxiální únavová kritéria pevnosti. Pro získání potřebných vstupních hodnot kritérií pevnosti byla provedena napěťově deformační analýza zkušebních vzorků metodou konečných prvků.

1 INTRODUCTION

Although the material failure phenomenon in the conditions of multiaxial fatigue is investigated for many years by world-known research institutes, a reliable mathematical description making possible to describe this boundary state was not introduced yet. Hence, it is still necessary to perform expensive prototype verification. The number of laboratories especially in aircraft and automotive industry is evidence of this fact. We bring another build-stone into the mosaic of this interesting technical field in this contribution.

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Number of fatigue experiments using both the reconstructed and new proposed devices, were performed at the VSB-TU Ostrava. The aim was to verify the quality of the conjugated strength criterion [1] proposed at our department.

Our contribution describes certain findings obtained from three different types of mechanical material loading. The experiments were performed on hollow specimens manufactured from the steel 11523.1. The experimental data obtained at the fatigue limit were evaluated primarily, i.e. for specimens which were damaged at 10^7 cycles. The below presented methodology is realized for this lifetime. The experimental data obtained for given combinations of loading even for the region of lifetime strength are mentioned in this contribution as well. The obtained data were used to determine the constants of a modified version of the conjugated strength criterion whose application can be suitable even for prediction of a limit number of cycles in the region of the lifetime strength [2].

2 EXPERIMENTAL MATERIAL

The experiments were performed on hollow specimens (Fig. 1) manufactured from low carbon steel CSN 411523.1 melt T31052. The specimens were polished on the outer surface. The chemical content and basic mechanical properties of this material are summarized in Tab. 1 and Tab. 2.

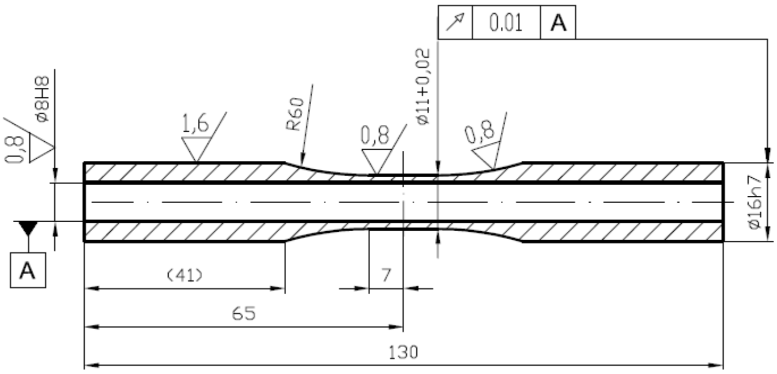

Fig. 1 Testing specimen.

Tab. 1 Chemical properties of the specimen material.

C [%]	Mn [%]	Si [%]	P [%]	S [%]	Cu [%]
0.18	1.38	0.4	0.018	0.006	0.05

Tab. 2 Mechanical properties of specimen material.

Ultimate tensile strength [MPa]	Tensile yield stress [MPa]	Elongation at fracture [%]	Reduction of area at fracture [%]
560	400	31.1	74.0

The following material parameters were experimentally found out so that the fatigue criteria mentioned below could be evaluated.

Tensile modulus: $E = 2.06 \cdot 10^5$ MPa.
Poisson's ratio: $\mu = 0.3$,
fatigue limit in fully reversed torsion: $t_{-1} = 160.7$ MPa,
fatigue limit in fully reversed tension: $f_{-1} = 240$ MPa,
fatigue limit in repeated tension: $f_0 = 370$ MPa,
torsion true fracture strength: $\tau_f = 516.6$ MPa.

3 USED MULTIAXIAL FATIGUE METHODS

The generally used fatigue strength criteria were used for the analysis of realized experiments. The results obtained experimentally for the given loading combinations on the fatigue limit will be evaluated by them.

3.1 Crossland method

Crossland published his results in the 50th of previous century. His criterion uses the square root from the second invariant of stress tensor. This invariant is determined from the stress amplitude. Another term added to the equation is the hydrostatic stress calculated from maximal stress values [3].

$$a_C \cdot \left(\sqrt{J_2}\right)_a + b_C \cdot \sigma_{H,\max} \leq f_{-1}, \tag{1}$$

where coefficients a_C and b_C are defined as:

$$a_C = \frac{f_{-1}}{t_{-1}}$$

$$b_C = \left(3 - \frac{f_{-1}}{t_{-1}}\right),$$

other parameters in the equation are:

J_2 second invariant of stress tensor deviator, f_{-1} fatigue limit in fully reversed axial loading (in tension, in bending or in rotating bending), $\sigma_{H,\max}$ maximum value of hydrostatic stress during load history, t_{-1} fatigue limit in fully reversed torsion.

3.2 Sines method

Sines published his results in the same period as Crossland. The formulation of both criteria are similar, they differ in the determination of hydrostatic stress. Sines calculate this stress from mean stress values [4].

$$a_S \cdot \left(\sqrt{J_2}\right)_a + b_S \cdot \sigma_{H,m} \leq f_{-1}, \tag{2}$$

where coefficients a_S and b_S are defined as:

$$a_S = \frac{f_{-1}}{t_{-1}}$$

$$b_S = 6 \cdot \frac{f_{-1}}{f_0} - \sqrt{3} \cdot \frac{f_{-1}}{t_{-1}},$$

where f_0 is fatigue limit in repeated bending, $\sigma_{H,m}$ mean value of hydrostatic stress during load history, other parameters in the equation are defined as in the case of Crossland method.

3.3 Dang Van method

This criterion belongs to the mesoscopic criteria. The mesoscopic criteria have their common point in an assumption that not the apparent macroscopic quantities, but their mesoscopic counterpart related to the least homogenous agglomerates of grains should be checked for fatigue evaluation.

Dang Van initiated the solution and presented a way of transforming the mesoscopic quantities towards macroscopic stresses [8]. Dang Van criterion can be written for the lifetime at the fatigue limit in following way:

$$a_{DV} \cdot C_a + b_{DV} \cdot \sigma_{H,\max} \leq f_{-1}, \tag{3}$$

where:

$$a_{DV} = \frac{f_{-1}}{t_{-1}}$$

$$b_{DV} = 3 - \frac{3}{2} \cdot \frac{f_{-1}}{t_{-1}},$$

where C_a is the shear stress amplitude on an examined plane, $\sigma_{H,max}$ maximum value of hydrostatic stress during the load history.

3.4 McDiarmid method (McD)

This criterion is widely used. On the base of number of experiments McDiarmid proposed the following form of the criterion:

$$\frac{f_{-1}}{t_{AB}} \cdot C_a + \frac{f_{-1}}{2 \cdot S_u} \cdot N_{\max} \leq f_{-1}, \tag{4}$$

where C_a is the shear stress amplitude on an examined plane, f_{-1} is the fatigue limit in fully reversed axial loading, $N_{\max}$ is maximum normal stress on the plane examined, S_u is tensile strength, t_{AB} is fatigue limit in fully reversed torsion with crack in A or B system. The crack parallel with the surface is typical for the type A. The crack leading inside down from the surface is typical for type B [5]. The following equivalence was used for the solution: $t_{AB} = t_{-1}$.

3.5 Papadopoulos method (Papad)

The Papadopoulos method is based on the Dang Van criterion. However this method integrates the input variables in all planes. The method can be found in following form [6].

$$\sqrt{a_p \cdot \left(T^2{}_a\right)} + b_p \cdot \sigma_{H,\max} \leq f_{-1}, \tag{5}$$

where:

$$a_p = 5 \cdot \kappa^2,\ b_p = 3 - \sqrt{3} \cdot \kappa,\ \kappa = \frac{f_{-1}}{t_{-1}},$$

where T_a is resolved shear stress (*a* projection of shear stress into a given direction), K is the ratio of fatigue limits.

3.6 Papuga PCr method

Papuga proposed the criterion on the base of long-term studies of multiaxial fatigue criteria in the following form (6) [7]. According to his research embodies this criterion the most accurate results for a wide range of materials.

$$\sqrt{a_C \cdot C^2{}_a + b_C \cdot \left(N_a + \frac{t_{-1}}{f_0} \cdot N_m\right)} \leq f_{-1}. \tag{6}$$

It is valid for following ratio of fatigue limits:

$\kappa < \sqrt{\frac{4}{3}} \cong 1.155$, is:

$$a_C = \frac{\kappa^2}{2} + \frac{\sqrt{\kappa^4 - \kappa^2}}{2}, \; b_C = f_{-1}$$

$\kappa \geq \sqrt{\frac{4}{3}} \cong 1.155$, is:

$$a_C = \left(\frac{4 \cdot \kappa^2}{4 + \kappa^2} \right), \; b_C = \frac{8 \cdot f_{-1} \cdot \kappa^2 \cdot \left(4 - \kappa^2\right)}{\left(4 + \kappa^2\right)^2}.$$

3.7 Matake method

The Matake criterion is the critical plane criterion of the following form [6, 8].

$$a_M \cdot C_a + b_M \cdot N_{\max} \leq f_{-1}, \tag{7}$$

where:

$$a_M = \frac{f_{-1}}{t_{-1}}$$

$$b_M = 2 - \frac{f_{-1}}{t_{-1}}.$$

Here C_a is shear stress amplitude on the plane experiencing maximum shear stress range, N_{max} maximum normal stress on the same plane.

3.8 Conjugated strength criterion (Csc)

This criterion was designed at the VSB-TU Ostrava [1]. Its modified version is provided hereafter. For the crack initiation in N-th cycle it can be written in the following form:

$$\frac{\left(A_N - B_N \cdot \sigma_R\right)}{S_\sigma} \leq f_{-1}, \tag{8}$$

where S_σ marks the stress intensity and it is defined as:

$$S_\sigma = \frac{1}{\sqrt{2}} \cdot \left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2 \right]^{\frac{1}{2}}, \tag{9}$$

σ_R is the reference stress value producing the identical value as the octahedral normal stress and can be written as:

$$\sigma_R = \left(\sigma_1 + \sigma_2 + \sigma_3\right)/3, \tag{10}$$

where σ_1, σ_2, σ_3 are the principal stresses. The value A_N can be considered as dependent on the cycle number N and it is written as:

$$A_N = \left\langle \left(A_O + A_C\right)/2 + \left(A_O - A_C\right)/2 \cdot \cos\{\pi \cdot \left[\log(4 \cdot N)/\log(4 \cdot N_C)\right]^a\} \right\rangle \cdot f_{-1} \tag{11}$$

A_O is the constant of the static reference strength criterion and can be determined based on the static torsion test:

$$A_O = 3^{1/2} \cdot \tau_f. \tag{12}$$

A_C is the stress intensity at the fatigue limit in torsion, N_C number of cycles at the fatigue limit, a material constant, τ_f is the value of true fracture strength in torsion, B_N is the constant equal to:

$$B_N = 3 \cdot f_{-1} \cdot \left(\sqrt{3} \cdot t_{-1} / f_{-1} - 1\right). \tag{13}$$

3.9 Fatigue index error

All mentioned criteria according to the results from (1 – 8) evaluate if the component is able to transfer the infinity of loading cycles. The fatigue index error ΔFI is used for evaluating those criteria. It shows the measure of a deviation from the ideal equilibrium of the left and right hand sides of mentioned criterion relations [8].

$$\Delta FI = \frac{LHS(load) - f_{-1}}{f_{-1}} \cdot 100\%, \tag{14}$$

where LHS is the left hand side of the equation. The relation $LHS(load) \leq f_{-1}$ has to be fulfilled. If LHS is greater, the component may fail.

4 FIRST EXPERIMENT - ALTERNATING TORSION

The specimens were loaded by the amplitude of the torque in the conditions of alternating cycling with the frequency of 25 Hz. The amplitude of the torque was stepwise reduced until the number of cycles 10^7 was achieved. The experimental results are summarized in Tab. 3 where τ_a is the stress amplitude in torsion. The resulting stress was obtained via stress/strain analysis using FEM in ANSYS software.

Tab. 3 Experimental results for alternating torsion.

Nr.	τ_a [MPa]	N_f [-]	Notes
1	174.	37610	
2	165.32	2423150	
3	160.7	10520000	No crack generated

5 SECOND EXPERIMENT - ALTERNATING TENSION

The second set of specimens was loaded by the simple alternate push-pull. In the case of first specimen the proper amplitude was set and the number of cycles until failure was registered. In case of other specimens, the amplitude was stepwise reduced until the fatigue limit - 10^7 cycles - was reached. The experiments were performed at the frequency 25 Hz. The experimental results are summarized in Tab. 4, where σ_a is the stress amplitude in push-pull loading. The resulting stress was obtained via stress/strain analysis using FEM in the software ANSYS.

Tab. 4 Experimental results for alternating tension.

Nr.	σ_a [MPa]	N_f [-]	Notes
1	296.2	28562	
2	265.1	235493	
3	245.5	801000	
4	240.0	10089000	No crack generated

6 THIRD EXPERIMENT - ALTERNATING TORSION AND TENSION IN PHASE

The third set of specimens was loaded in every series by fully reversed push-pull loading and in-phase fully reversed torsion loading until the crack initiation. The amplitude of tension was constant in each series. The amplitude of the torsion was gradually decreased until the value when was the specimen able to endure 10^7 of cycles. The experiments were performed at the frequency 10 Hz again. The experiments were performed in phase – phase shift of both the amplitude loading was equal to zero. The experimental results are summarized in Tab. 5, where σ_a is the stress amplitude in tension and τ_a is the stress amplitude in torsion. The resulting stress was obtained via stress/strain analysis using FEM in the software ANSYS.

Tab. 5 Experimental results for alternating tension and alternating torsion.

Nr.	τ_a [MPa]	σ_a [MPa]	N_f [-]	Notes
1	185.24	99.6	59663	
2	163.45	99.6	639273	
3	144.4	99.6	1483187	
4	136.2	99.6	11000000	No crack generated
5	133.74	195.7	101102	
6	111.42	194.6	530067	
7	95.55	195.2	1185152	
8	87.17	194.6	10120000	No crack generated

7 EXPERIMENTAL RESULTS ANALYSIS

The obtained experimental results from all described experiments were used for the analysis of above mentioned fatigue stress criteria. Only those experimental results were analyzed where the failure was not reached until 10^7 cycles. The software Pragtic was used for the analysis. This software is freely accessible at www.pragtic.com [8]. It contains all mentioned criteria with the exception of the conjugated stress criterion which was proposed at the authors' laboratory. The program in Microsoft Office Excel was created for the analysis of this criterion. The results of this study are depicted in Tab. 6.

Tab. 6 Experimental analysis results.

Nr.	τ_a [MPa]	σ_a [MPa]	*ΔFI* (%)							
			Sines	Crossland	Dang Van	McD	Papad	Papuga PCr	Matake	Csc
1	160.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.7
2	0.0	240.0	-13.5	0.0	0.0	-13.5	0.0	0.0	0.0	1.1
3	136.2	99.6	-7.72	-2.06	1.01	-14.7	-2.06	0.13	1.14	2.67
4	87.17	194.6	-11.6	-0.57	1.44	-19.0	-0.57	0.66	1.51	0.97

7 CONCLUSIONS

The common used multi-axial strength criteria (see above) and the modified conjugated strength criterion [1] with the aim of coupling the static and fatigue multi-axial criterion, have been described in this contribution.

The three sets of experiments have been performed on the hollow, thin-walled specimen made of steel 11523.1. The different stress states were generated in the specimens during the loading: alternating torsion, alternating tension, alternating tension and alternating torsion in phase with two levels. The results of the experiments have been applied to verify the mentioned strength criteria.

According to the values of the fatigue index error ΔFI stated in Tab. 6 the best results are achieved by using Papuga PCr method, Dang Van and Matake criteria. The good results have been reached using Papadopoulos, Crossland method and modified Conjugated strength criterion as well. The Sines criterion and McDiarmid method have not provided so good results.

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