DECOMPOSITION METHODS FOR A PIV DATA ANALYSIS WITH APPLICATION TO A BOUNDARY LAYER SEPARATION DYNAMICS

DEKOMPOZICE DAT ZÍSKANÝCH METODOU PIV S APLIKACÍ NA VÝZKUM DYNAMIKY ODTRŽENÍ MEZNÍ VRSTVY

Abstract

Separation of the turbulent boundary layer (BL) on a flat plate under adverse pressure gradient was studied experimentally using Time-Resolved PIV technique. The results of spatio-temporal analysis of flow-field in the separation zone are presented. For this purpose, the POD (Proper Orthogonal Decomposition) and its extension BOD (Bi-Orthogonal Decomposition) techniques are applied as well as dynamical approach based on POPs (Principal Oscillation Patterns) method.

The study contributes to understanding physical mechanisms of a boundary layer separation process. The acquired information could be used to improve strategies of a boundary layer separation control.

1 INTRODUCTION

Classical concept of a BL separation starts from the Prandtl’s idea of a BL and is built for 2D cases. The necessary condition for BL separation is the increasing pressure in the streamwise direction, i.e. positive (or adverse) pressure gradient along the flow path. The necessary second determining factor is presence of viscous effects in the BL no matter being of laminar or turbulent in nature.

Above a critical Reynolds number, the flow field is shown to undergo self-sustained 2D low-frequency fluctuations in the upstream region of the separation, resulting into aperiodical vortex shedding further downstream. Low-frequency fluctuations, also called “flapping”, have been shown to be a characteristic feature of separated layers in general ([3]). It has been argued that they are due to a global instability manifested in the reattachment region ([4]), triggered by topological flow changes generating secondary recirculation zones ([2]).
2 APPROACHES

Apparently the phenomenon in question is substantially 3D and highly dynamical in nature in the same time. To analyze such a phenomenon, appropriate methods of analysis should be applied on experimental data. Actually no universal methods for such analysis are available.

The main problem is a huge number of degrees of freedom of the dynamical system, which should be reduced substantially. For this purpose the decomposition methods based on energetic (POD) and stability (POPs) analysis are to be used. Reduction of the dynamical system dimension is to be done using the BOD method. This approach has been proposed and tested in [11].

2.1 Methods Based on POD

The Proper Orthogonal Decomposition (POD) method has applications in almost any scientific field where extended dynamical systems are involved. Recently, the POD has been widely used in studies of turbulence. Historically, it was introduced in the context of turbulence by Lumley [6] as an objective definition of what was previously called big eddies and which is now widely known as coherent structures. Details on this classical method could be found in [7].

The next step is the Bi-Orthogonal Decomposition (BOD) representing itself an extension of the POD. While POD analyses data in spatial domain only, the BOD performs spatiotemporal decomposition. Aubry [1] presented the BOD as a deterministic analysis tool for complex spatiotemporal signals. First, a complete two-dimensional decomposition was performed. These decompositions were based on two-point temporal and spatial velocity correlations. A set of orthogonal spatial ("topos") and temporal ("chronos") eigenmodes are to be computed to allow the expansion of the velocity field.

2.2 Stability Analysis – POPs

The Principal Oscillation Patterns (POPs) decomposition is based on modal structures based on temporal or spatial linear evolution dynamics. Each mode is characterized by a complex frequency involving information on frequency, phase and growth/decay information. On the other hand, the POPs analysis is unable to resolve standing oscillations.

The basis of the POPs analysis was formulated by Hasselmann [5] for discrete Markov processes in linearized dynamical systems driven by white noise with application in climatology. The POPs theory is a special case of more general Principal Interaction Patterns (PIPs) method for nonlinear dynamical systems.

In the POPs approach the fluctuating part of Navier-Stokes equation is modeled by Langevin equation for the linear Markov process. The method provides the complex modes characterized with frequency and so called e-fold time $\tau_e$ which characterizes the delay for the mode amplitude reduction in ratio $1 : e$. The least stable modes are pointed out. Detailed description of this method is given in [9].

3 EXPERIMENT

The existing blow-down wind tunnel was used with one-sided diffuser angle 15.5 deg. Mean flow velocity in the diffuser inlet was 7.8 m/s, turbulent boundary layer on the bottom wall. In Fig. 1 the schema of experimental setup is shown. The coordinate system was defined with the x-axis in the input flow direction and y-axis is perpendicular to the wall. Origin of the coordinates is located in the beginning of the diffuser; the cross-section here is $100 \times 100$ mm$^2$. Downstream of this section, the upper wall is inclined, while the bottom plane-wall is used to study the boundary layer separation. To prevent separation from the upper wall this is permeable and sucked out. The bottom-wall boundary layer was of turbulent nature, about 5 mm thick. The suction velocity along the upper wall could be estimated to 3 m/s.
Standard configuration of time resolved PIV was used for measurements, particles were generated using fog SAFEX generator, introduced to the inlet of the blow-down facility. Measurements take place in the plane of symmetry of the channel, two velocity components in this plane are evaluated in grid 79 x 18 points, 5108 subsequent complete vector fields are evaluated with frequency 1545 Hz representing approx. 3.2 s. Details on used measuring techniques are given in Uruba, Knob [8].

4 RESULTS

A few preliminary results have been already obtained and presented in [10]. Here classical evaluation of “separation point” position is shown together with the first information on flow dynamics in the separation zone. The evaluation of separation point as a place of zero mean wall shear stress, or zero mean streamwise velocity component close to the wall is shown in Fig. 2. The line $l_0$ represents locus of zero mean streamwise velocity.

![Fig. 2 Mean streamwise velocity distribution](image)

The velocity fluctuation could be recognized in the whole separation region, but maximum fluctuation activity have been detected in the region above the $l_0$ line - see Fig. 3.

![Fig. 3 Velocity fluctuation energy distribution](image)

The first results of POD and POPs analysis will be shown. As an example in Fig. 4 the first 4 most energetic toposes are presented. The toposes consist of vortical structures, lower modes with
high energy of one or two of them, while higher order modes 10 and 25 of 5 or even 10 small structures respectively.

![Fig. 4 Toposes No. 1, 2, 3, 4, 10 and 25](image)

Then, the POPs analysis is to be carried out after the system truncation. The system dynamics is modeled by 30 most energetic POD modes representing more than 82% of the total fluctuating energy.

![Fig. 5 The POPs eigenvalues](image)

The POPs analysis shows the less stable modes. Fig. 5 shows complex eigenvalues of the 30 evaluated POPs modes. The eigenvalues are either real or complex conjugate. The real part of a given eigenvalue $\beta$ is connected with stability of the mode and value of the e-fold time $\tau_e$, while imaginary part determines the frequency $f$:
\[ \tau_c = -\frac{1}{\text{Re}(\beta)} \quad f = \frac{\text{Im}(\beta)}{2\pi}. \]

All modes are stable, as the real parts of eigenvalues are negative and thus the e-fold time is positive. Then, the modes are ordered according to their stability, less stable first (i.e. \( \beta \) closer to 0 and \( \tau_c \) bigger).

**Tab. 1** First 10 modes

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>( f ) [Hz]</th>
<th>( \tau_c ) [ms]</th>
<th>( n = \tau_c/T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>330</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>138</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>1.8</td>
<td>132</td>
<td>0.24</td>
</tr>
<tr>
<td>4</td>
<td>3.2</td>
<td>132</td>
<td>0.42</td>
</tr>
<tr>
<td>5</td>
<td>9.1</td>
<td>65</td>
<td>0.59</td>
</tr>
<tr>
<td>6</td>
<td>10.6</td>
<td>47</td>
<td>0.50</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>12.2</td>
<td>41</td>
<td>0.50</td>
</tr>
<tr>
<td>9</td>
<td>6.3</td>
<td>39</td>
<td>0.25</td>
</tr>
<tr>
<td>10</td>
<td>2.4</td>
<td>37</td>
<td>0.12</td>
</tr>
</tbody>
</table>

In Tab. 1 there are first 10 modes with corresponding frequencies \( f \), e-fold times \( \tau_c \) and ratios \( n \). The ratio \( n \) shows relationship of e-fold time \( \tau_c \) and the period \( T = 1/f \) and could be expressed as

\[ n = \frac{\tau_c}{f} = \frac{\tau_c}{T}. \]

Its physical meaning is number of periods while the mode amplitude decays by factor \( e \). The given mode could be considered as periodic only when the decay during one period is not very big, let say not more than factor 10. This situation corresponds to \( n = 0.43 \), for \( n \) bigger then this value the mode behaves periodically, because we could positively register more than one period during the mode single appearance. For \( n < 0.43 \) we have nearly aperiodical mode behavior as it disappears before the first period is accomplished. In the case \( f = n = 0 \) we have completely aperiodical behavior with proportional decay, when \( 0 < n < 0.43 \) than the behavior is nearly aperiodical. In our case we have proportionally decaying modes 1 and 7, nearly aperiodical modes 2, 3, 4, 9, 10 and periodic modes 5, 6, 8.

**Fig. 6** Real part of the least stable POPs 1 mode

Now we show topology of some typical modes. The first mode No. 1 is proportionally decaying with the imaginary part vanishing, its real part is shown in Fig. 6, isolines map the vorticity (negative values by dashed line) showing position of vortices. The vortices are distributed more or
less equally within the whole separation region. During the mode occurrence its structure does not change, it only decays.

The nearly aperiodical mode No. 2 is shown in Fig. 7. The vortical structures are distributed along the \( l_0 \) line. They move slightly during the mode decay.

The example of periodic mode is in Fig. 8, where the mode No. 5 is shown. This mode exhibits maximal vorticity concentration along the \( l_0 \) line, where system of traveling counter-rotational vortex pairs could be identified.

The other evaluated modes manifest qualitatively the same type of behavior however the detailed topology is different, of course.

5 CONCLUSIONS

The stability modes have been studied on the reduced dynamical model of a boundary layer separation region. The pulsating and traveling structures associated with flapping process have been identified. Traveling structures are concentrated close to the zero streamwise mean velocity component line, while pulsating structures are distributed in the whole separation region.

ACKNOWLEDGEMENTS

The author gratefully acknowledges financial support of the Grant Agency of the Czech Republic, projects No. 101/08/1112 and P101/10/1230.
REFERENCES


