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NONLINEAR MATHEMATICAL PROGRAMMING IN ENGINEERING APPLICATIONS

NELINEÁRNÍ MATEMATICKÉ PROGRAMOVÁNÍ V INŽENÝRSKÝCH APLIKACÍCH

Abstract

This paper deals with suitable nonlinear optimization in engineering applications via mathematical programming. This tool is used for optimization and control of production of steel slabs and inverse tasks, such as determination of boundary conditions. Mathematical models contain 2D Fourier-Kirchhoff equation including boundary conditions. Presence of phase and structural changes is covered by enthalpy approach. The software implementation was executed as a link between MATLAB and modeling language GAMS.

Abstrakt

Tento článek se zabývá vhodnou nelineární optimalizací pro inženýrské aplikace za pomoci matematického programování. Tento nástroj je použit pro optimalizaci a řízení produkce ocelových bram a řešení inverzních úloh, jako je určování okrajových podmínek. Matematický model obsahuje 2D Fourierrovu-Kirchhoffovu rovnici včetně okrajových podmínek. Přítomnost fázových a strukturálních přeměn je řešena pomocí přístupu entalpie. Softwarové řešení je provedeno spojením programu MATLAB a modelovacího jazyka GAMS.

1 INTRODUCTION

Optimization is the act of obtaining the best result under given circumstances. In design, construction, and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit. Since the effort required or the benefit desired in any practical situation can be expressed as a function of certain decision variables, optimization can be defined as the process of finding the conditions that give the maximum or minimum value of a function.

There is no single method available for solving all optimization problems efficiently. Hence a number of optimization methods have been developed for solving different types of optimization problems. The optimum seeking methods are also known as mathematical programming techniques and are generally studied as a part of operations research. Operations research is a branch of mathematics concerned with the application of scientific methods and techniques to decision making problems and with establishing the best or optimal solutions. The beginnings of the subject of operations research can be traced to the early period of World War II. After the war, the ideas advanced in military operations were adapted to improve efficiency and productivity in the civilian sector [1].

The operations research technique contains many mathematical methods. For instance calculus methods, calculus of variation, mathematical programming, game theory, statistical decision theory,

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Markov process, queuing theory, simulation methods, cluster analysis etc. Modern or non-traditional optimization techniques are for instance genetic algorithms, simulated annealing, neural networks and fuzzy optimization.

Our main task is finding of appropriate tools for investigation real engineering problems via operations research. These problems contain heat transfer problems such as control of continuous casting process or inverse tasks.

2 OPTIMIZATION AND MATHEMATICAL PROGRAMMING

Mathematical programming techniques are useful in finding the minimum of a function of several variables under a prescribed set of constraints. The first step in mathematical programming is substitute real problem to a mathematical model. In practice we are very often forced to simplified real situation to more suitable way, but these simplifications cannot modify the optimization process in a significant way.

Mathematical model is basically created from the equation of objective function and the set of constraints. The goal is to find minimum or maximum of this function. Constraints can represent limitations on the behavior, performance of the system or physical limitations on design variables, such as availability, fabricability, and transportability. Any engineering system or component is defined by a set of quantities some of which are viewed as variables during the design process. In general, certain quantities are usually fixed at the outset and these are called preassigned parameters. All the other quantities are treated as variables in the design process and are called design or decision variables *x*. If an n-dimensional Cartesian space with each coordinate axis representing a design variable x_i (i = 1, 2,..., *n*) is considered, the space is called the design variable space or simply design space. If the constraints do not have significant influence in certain design problems and our model includes only objective function, we called problem as unconstrained optimization problem. Otherwise we called problem as constrained optimization problem [1, 2].

We can roughly sort mathematical models into four groups according to equations character:

- Linear programming - if the objective function and all the constraints in model are linear functions of the design variables.
- Nonlinear programming - if any of the functions among the objective and constraint functions in model is nonlinear.
- Integer programming - if some or all of the design variables $x_1, x_2, \ldots, x_n$ of an optimization problem are restricted to take on only integer (or discrete) values.
- Stochastic programming - if some or all of the parameters (design variables and/or preassigned parameters) are probabilistic (nondeterministic or stochastic).

2.1 Nonlinear programming model

Optimization problems presented in this paper were created as sets of equations, where design variables are in scalar product form. Thus the use of nonlinear programming is appropriate [2]. Nonlinear model can be formulated as follows:

$$\min\left\{ f(\mathbf{x}); \mathbf{g}(\mathbf{x}) \le 0, \mathbf{h}(\mathbf{x}) = 0, \mathbf{x} \in \mathbf{X} \right\}, \tag{1}$$

where:

$\mathbf{x} = x_1, \ldots, x_n$ - is design variables vector,
$f(\mathbf{x})$ - is objective function,
$\boldsymbol{g}(\mathbf{x}) = (g_1(\mathbf{x}), \ldots, g_m(\mathbf{x}))^T$ - is constraints vector,
$\mathbf{h}(\mathbf{x}) = (h_1(\mathbf{x}), \ldots, h_p(\mathbf{x}))^T$ - is constraints vector.
m = 0 and p = 0 for unconstrained optimization case.

To find an optimum solution in large scale problems without a computer approach is not possible. Computer codes to solve mathematical programming problems are called as solvers. There is no single solver available for solving all optimization problems efficiently. Thus modeler should decide which specific solver for specific problem is appropriate. This problem can be particularly solved by using modeling languages, such as AMPL, GAMS, XPRESS-MP etc. These modeling languages work as an interface between users and solvers.

2.2 Modeling language GAMS

The optimization in this paper is solved by modeling language GAMS (General Algebraic Modeling System). GAMS is specifically designed for modeling linear, nonlinear and mixed integer optimization problems. The system is especially useful with large, complex problems. More information can be found at http://www.gams.com/. For nonlinear optimization we use a nonlinear solver CONOPT. Furthermore, we use mathematical software MATLAB for calculating the starting optimization values and for drawing (animating) the final results.

3 OPTIMIZATION IN ENGINEERING APPLICATIONS

Mathematical programming approach in this paper is used for two engineering problems. We can characterize problems as heat transfer problems. The mathematical models that were employed are based on mathematical model of temperature field. Modeling of temperature field in continuous casting process and inverse task problems is a numerically difficult problem, which contains a large system of nonlinear equations. The main task is finding appropriate mathematical tools for these type of problems, thus to examine a simplified 2D model is satisfactory. From three basic heat transfers we will consider only conduction, which plays a dominant role in the process. Convection and radiation will be considered only as boundary conditions.

3.1 Optimal control for continuous casting problem

Our main interest is in the water spray control in secondary cooling zone which effect final product quality. Simplified 2D model which assumes symmetry on cross section is show in **Fig. 1**. Another simplification is considering four cooling circuits instead of thirteen, which can occur in real machine.

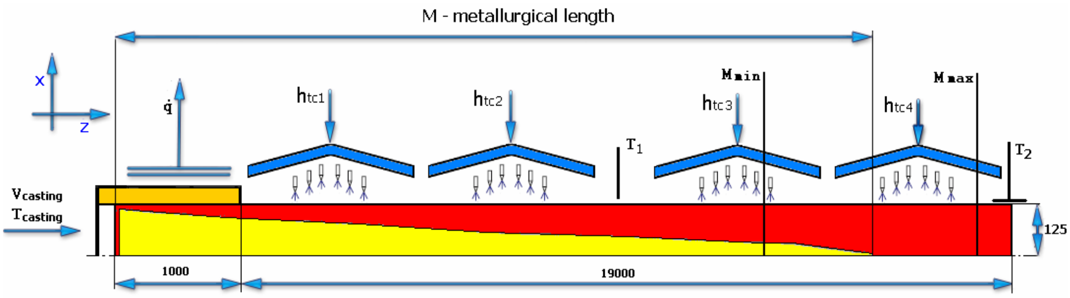

Fig. 1 Continuous casting scheme

We consider casting speed and four heat transfer coefficients, which represent water cooling circuits, as control parameters. Optimization should answer the question, how to set these parameters for temperature constraints on surface and allowed metallurgical length in optimal way.

The temperature field of the slab can be described by Fourier-Kirchhoff equation. There are phase and structural changes, thus the use of enthalpy approach with thermo-dynamical function of volume enthalpy H [J/m^3] is appropriate [3, 4, 6].

$$\frac{\partial H}{\partial \tau} = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) + v_z \frac{\partial H}{\partial z}, \tag{2}$$

where:

λ - is thermal conductivity [W/mK],

T - is temperature [K],

τ - is real time [s],
x, z - are coordinates,
v_z - is velocity ingredient [m/s].

In order to be exhaustive, we have to add initial condition and boundary conditions. The initial condition describes initial temperature distribution in the slab. There are four different boundary conditions. Relationship (2) can be approximated by finite-difference terms [4], on an explicit formula.

In equations (2) is enthalpy present together with temperature, so we need to recalculate enthalpy to temperature for each node and each time. The function of enthalpy is not known as a function, but as a tabular form. Various search techniques are often used in practice. This approach is not appropriate for optimization. The relation between enthalpy and temperature can be fitted by a curve, which describes the relation in the best way. For instance, we can fit the data by interpolating polynomial of the twentieth degree, Fourier series, etc [5].

In the objective function, the information maximal possible casting speed and surface temperature smoothness is required, which describes compromise between final slab quality and productivity time. Thus:

$$z = \min\left\{ \max_{k=1..N} \left\{ T_{M,k} - T_{M,k+1} \right\} - v_z \right\}. \tag{3}$$

Surface temperature smoothness is created by minimizing of maximal temperature difference between two neighbouring nodes. As constraints we have kept the metallurgical length, temperature T_1 and T_2, and heat transfer coefficients between minimal and maximal values. Next constraints are relationship between enthalpy and temperature, equations for temperature field (2) and initial and boundary conditions. In this way, we have an objective function (3) and set of constraints for all nodes including all time. For instance when we consider rough mesh 10x50x1000, we have approximately 1,000,000 constraints and variables, which is a very complex optimization problem in nonlinear programming [2]. This model was solved by modeling language GAMS. Optimal control parameters as casting speed and heat transfer coefficients were found. Final temperature field, surface and core temperatures are shows in **Fig 2**.

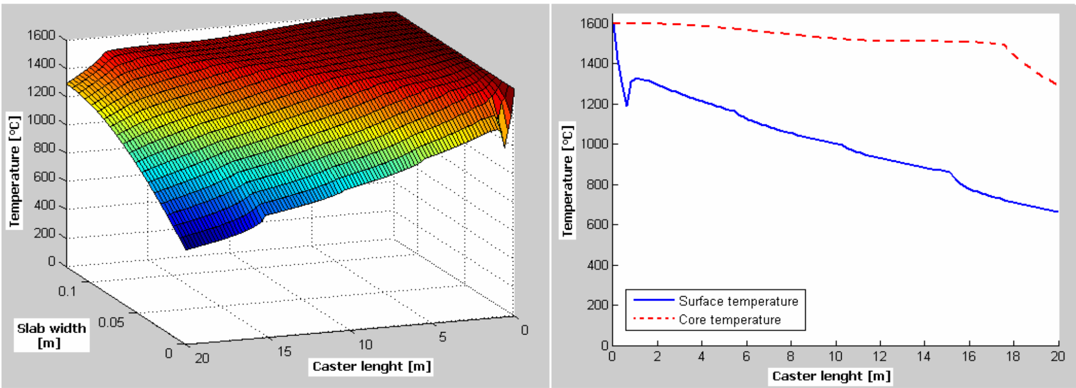

Fig. 2 Temperature field, surface and core temperatures

3.2 Optimization for inverse task problem

In this problem, attention is focused on the search for boundary conditions describing the heat transfer in engineering applications of spray cooling of metal surfaces during casting [4]. Concretely parameters which describing heat transfer coefficient. Slab temperature field is created by equation (2) with last term absence, describing steel flow. We assume normal density function for shape approximation of heat transfer coefficient:

$$h_{tc} = h_{tc,\max} \sqrt{2\pi\sigma^2} \, \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, \tag{4}$$

where:
μ - is mean,
σ - is variance.

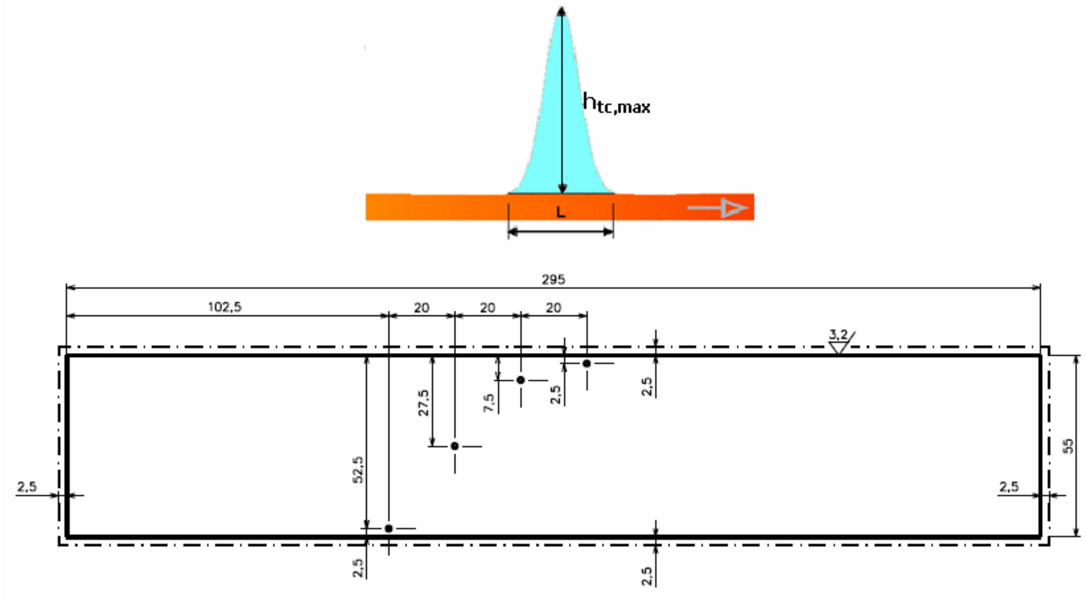

Fig. 3 Up – cooling scheme (h_{tc} distribution), Down – thermocouples position

Objective is to find unknown h_{tc} via known temperature characteristics from experiment. Thermocouples positions are shown in **Fig. 3** as well as h_{tc} distribution. The objective function is created as minimization differences between measured and computed temperatures in thermocouples positions. Thus:

$$z = \min\left\{\sum_{i=1}^{4}\left|T_i - T_i^*\right|\right\}, \tag{5}$$

where:
T_i - is calculated temperature in position i [K],
T_i^* - is measured temperature in position i [K].
In **Fig. 4** we can see experimental measurement and computer calculation comparison.

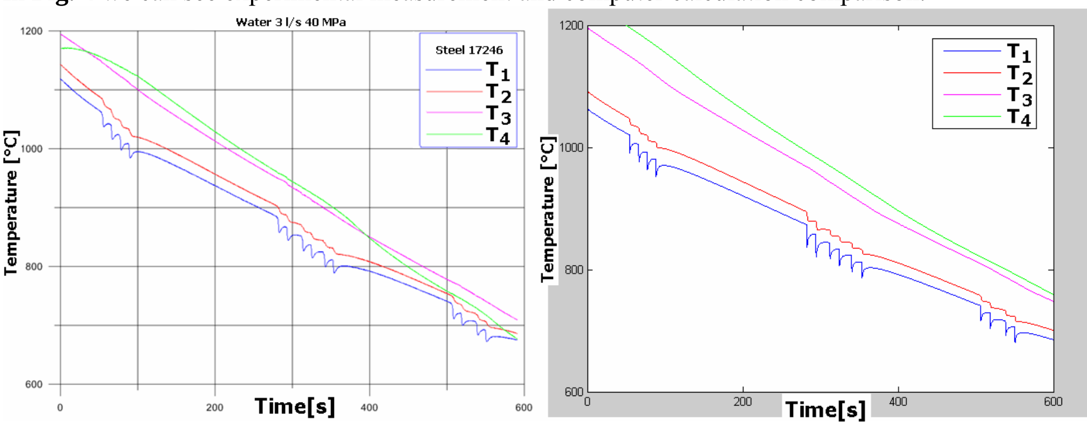

Fig. 4 Left – measured data from thermocouples, Right – calculated temperatures

4 CONCLUSION

The paper deals with suitable mathematical tools for optimizing control of continuous slab casting process and inverse tasks. We created the original mathematical models. Apart from that, nonlinear mathematical programming approach seems to be effective method for the continuous casting simulations and inverse tasks problems. The future development of our work is improvement of this model to 3D model and considering more factors, which affect the continuous casting process and cooling systems.

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