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MODELING AND SIMULATION OF FLOW OVER A WAVY SURFACE

MODELOVÁNÍ A SIMULACE OBTÉKÁNÍ ZVLNĚNÉHO POVRCHU

Abstract

This paper presents an analytical method for calculating the physical characteristics of laminar flow using the theory of Orr-Sommerfeld equation. The assembled linear model is one of the approaches applied to the problem of gas flow over a wavy surface. Assumptions and results are compared with simulations results obtained using CFD software StarCCM+.

Abstrakt

Článek prezentuje analytický způsob výpočtu fyzikálních charakteristik laminárního proudění užitím teorie Orrovy-Sommerfeldovy rovnice. Sestavený lineární model je jedním z přístupů aplikovaný na problém obtékáni zvlněného povrchu proudem plynu. Předpoklady a výsledky řešení jsou porovnány se simulacemi pomocí CFD programu StarCCM+.

1 INTRODUCTION

The origins of studies of flow over a wavy surface can be put into context with the issue of the initiation and development of waves on the water surface depending on the characteristics of the external gas flow. By analyzing the physical mechanism of hydrodynamic instability in general, the pressure and shear effects of the external flow were determined as the decisive factors of the phenomenon.

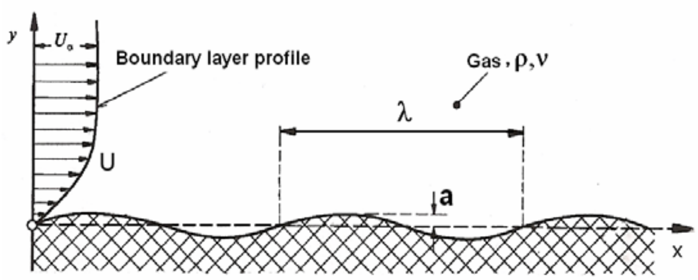

Fig. 1 Scheme of the phenomenon.

Effects of shear and pressure forces acting on the surface are defined by means of pressure and velocity field and thus one of the approaches is a description of flow using the Navier-Stokes equations. Computational complexity of these equations, however, led to a search for approaches that use simplistic assumptions based on geometrical and physical specifics of the problem. Among the number of approaches, the importance of the principle of Orr-Sommerfeld equation originally derived

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in connection with the issue of instability of laminar flow, see [1], appears. The equation is derived by linearization of equations of motion for the parallel flow assumption, which limits its use to cases of small amplitude waves in the sense of ratio of the wave amplitude to the wavelength α/λ.

2 THEORETICAL FOUNDATIONS OF THE MODEL

The boundary problem of discussed phenomenon derived on theoretical background of Orr-Sommerfeld equation gives for example paper [2]. The basic premise is the decomposition of the variables into their average values and fluctuation components

$$\begin{aligned} u &= U + a\hat{u}e^{i\alpha x}, \\ v &= a\hat{v}e^{i\alpha x}, \\ p &= P + a\hat{p}e^{i\alpha x}, \end{aligned} \tag{1}$$

where, using stream function,

$$\psi(x,y) = \int_0^y U(y)dy + a\phi(y)e^{i\alpha x} \tag{2}$$

and using velocity field definition we obtain

$$\begin{aligned} \hat{u} &= \phi' \\ \hat{v} &= -i\alpha\phi. \end{aligned} \tag{3}$$

Substituting relations (2)-(3) into the time-averaged Navier-Stokes equations, linearization and elimination of the pressure member we receive Orr-Sommerfeld equation

$$i\alpha R[U(\phi''-\alpha^2\phi) - U''\phi] = \phi^{(4)} - 2\alpha^2\phi'' + \alpha^4\phi, \tag{4}$$

where $R=U_0L/\nu$ is Reynolds number of external flow defined by characteristic velocity U_0, characteristic dimension L and kinematic viscosity ν. From relations (1)-(3) follows that velocity field of the flow over wavy surface can be obtained by solving O-S equation for unknown φ(y). Shear and pressure effects acting on the wavy surface are given by relations

$$\begin{aligned} \hat{\tau}(0) &= \phi'' + U''(0), \\ \hat{p}(0) &= -\frac{i}{\alpha}(\phi'''(0) + \alpha^2 U'(0). \end{aligned} \tag{5}$$

Given the complex nature of equation (4), the function φ can be written as the sum of real and imaginary components

$$\phi = \phi_R + i\phi_I \tag{6}$$

and velocity fluctuations are then according to (1) and (3) given by formulas

$$\begin{aligned} u'(x,y) &= real\left[a(\phi_R' + i\phi_I')e^{i\alpha x}\right] = a[\phi_R'\cos(\alpha x) - \phi_I'\sin(\alpha x)], \\ v'(x,y) &= real\left[-ai\alpha(\phi_R + i\phi_I)e^{i\alpha x}\right] = a\alpha[\phi_R\sin(\alpha x) + \phi_I\cos(\alpha x)]. \end{aligned} \tag{7}$$

3 MODELING

The model just mentioned above should be supplemented by boundary conditions

$$\phi(0) = 0, \\ \phi'(0) = -U'(0), \tag{8}$$

treating attenuation of disturbances on the surface and next by conditions

$$\phi'(\eta) = -\alpha\phi(\eta), \\ \phi''(\eta) = -\alpha\phi'(\eta), \tag{9}$$

which provide attenuation of disturbances at large distances from the wall and are based on the fundamental solution of the OS equation in infinite point

$$\phi(y) = e^{\pm i\alpha y}. \tag{10}$$

From relations (5) and (7) it is clear the importance of function φ(y) to determine the velocity fluctuations as well as shear and pressure effects on the surface. Its real and imaginary component is shown in Figure 2

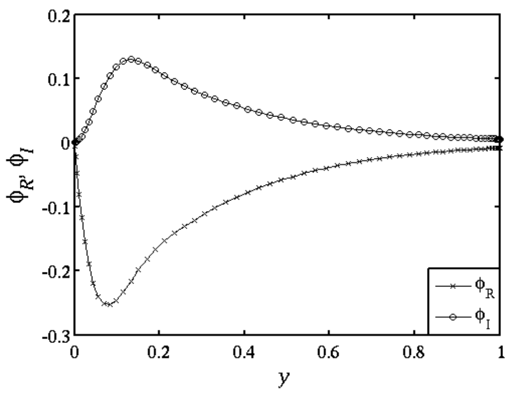

Fig. 2 Amplitude function of fluctuations.

The relations (7) show that by the choosing the x-coordinates deviation of velocity profile can be determined in the corresponding sections over the surrounding wall. Figure 3 shows the symmetry of fluctuations in the y direction if x-coordinate is shifted by Δx = λ/2 for a wave described by

$$y(x) = a\cos\left(\frac{2\pi}{\lambda}x\right). \tag{11}$$

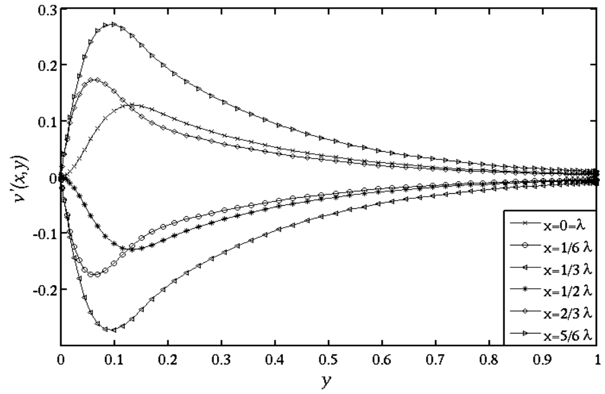

Fig. 3 Fluctuations in the transverse direction of flow.

Fluctuations become extreme near x=1/3λ and x=5/6λ. The symmetry implies the overlap of the fluctuation function for x =0 and x=λ.

4 SIMULATION

Experimental evaluation of the problem of flow over the wavy surface in comparison with the above linear model gives for example the article [2]. The technical complexity and limitations of these experiments on selected wavelengths yield the idea of numerical simulations using CFD tools. For our purposes, we carried out calculations for λ /a = 10 and 15 by software STAR-CCM+.

The primary point for evaluation of the results of the model is to assess the adequacy of the assumption of linearity that is an approximation of fluctuations of physical quantities by harmonic functions, see equation (2). Figure 4 shows normalized curves of pressure and shear forces acting on the surface for the air velocity U=0.5m/s and U=2.5m/s. From simulations showed in Figure 5, it is clear that, for low values of U, the fluctuations are of harmonic waveform, however the function course is deformed if the velocity is increased. These deformations are more pronounced for shorter wavelengths. Nevertheless, if the condition of laminar flow is satisfied, the periodic character of fluctuations correlated with the sinusoidal shape of the surface can be distinguished. Evaluation of pressure and shear effects is then given by the amplitude of fluctuations and phase shifting towards the surface. As documented in Figure 4 in contrast to the inviscid Kelvin-Helmholtz theory, the extremes and sinks of pressure are shifted to the surface in the direction of airflow. Maximum shear stress is located near the crest upstream. This displacement is smaller for shorter wavelength.

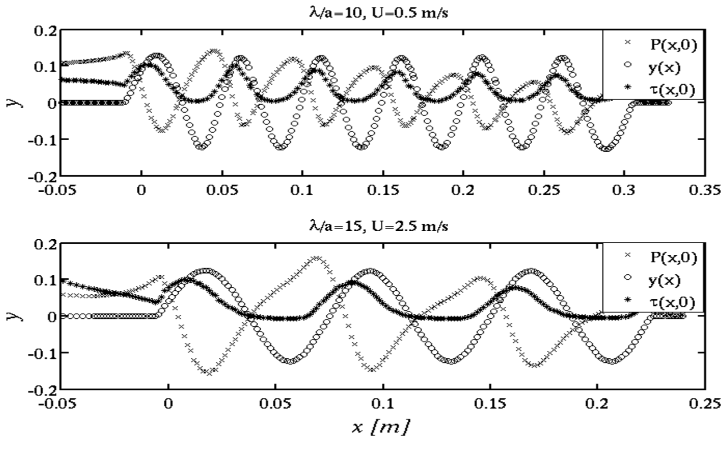

Fig. 4 Results of simulation of shear and pressure forces in StarCCM+.

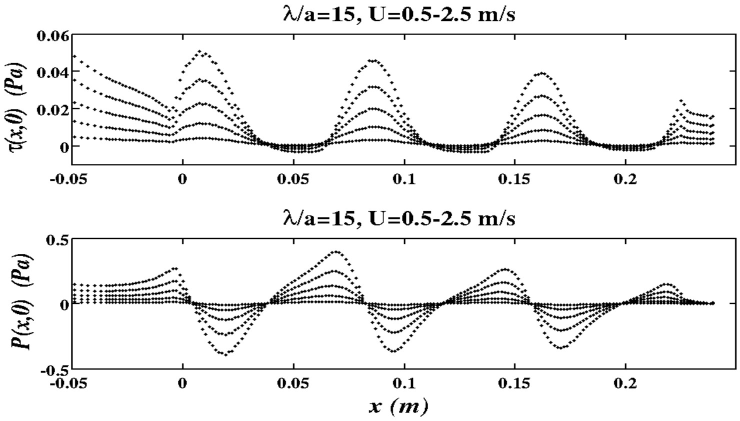

Fig. 5 Simulations of shear and pressure forces for U=0.5-2.5 m/s.

The graph of fluctuations in the transverse direction in Figure 6 shows that the amplitude of disturbances corresponds qualitatively with the fluctuations in the Figure 3. Amplitude distortions, in particular for x=5/6, are due to the gas boundary layer development. This development is not included in the linear model and hence prevents the validation of the amplitude function depending on the downstream coordinate x. Thus the importance of the amplitude function is primarily in their derivatives to calculate the shear and pressure effects, see relations (6).

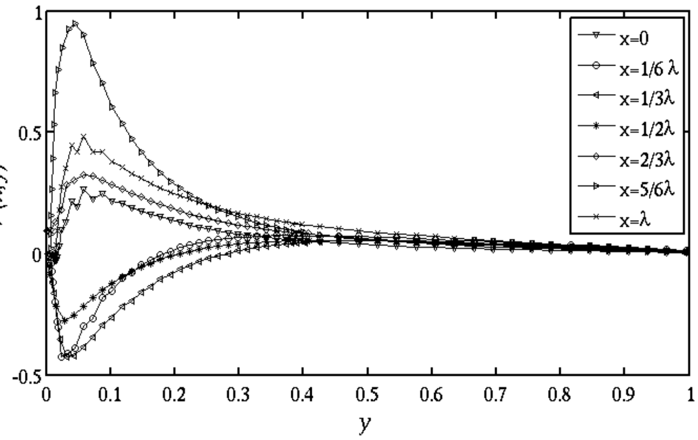

Fig. 6 Simulation of velocity fluctuations.

5 CONCLUSIONS

This paper presents a linear method for calculating the physical characteristics of parallel laminar flow. The assembled model is based on the Orr-Sommerfeld equation originally derived for solving the hydrodynamic instability. Shear and pressure forces acting on the wavy surface are calculated from the second and third derivative of the amplitude function respectively. This model predicts, inter alia, the velocity fluctuations of the laminar flow for the given x-coordinate in the direction of flow. Because of the boundary layer development amplitude of these fluctuations should be appropriately normalized.

The simulation of the laminar flow over a wavy surface has been performed for two ratios of wavelength to the wave amplitude and for several air flow velocities. For low values of air velocity, the shear and pressure fluctuations are correlated with the sinusoidal waveform, what is the assumption of the linear model. However, for greater air velocity, harmonic waveforms are distorted and strictly non sinusoidal. Hence simulations for more wavelengths and greater velocities are demanded for comparison of the amplitudes and phase shifting with the linear model and experiments to verify the possibility of improving the models of waves initiation, which were mentioned in the introduction part. This is the objective of the current research.

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