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DESIGN AND NUMERICAL CALCULATION OF VARIABLE TEST SECTION FOR SMALL SUPERSONIC WIND TUNNEL

NÁVRH A NUMERICKÝ VÝPOČET MĚNITELNÉHO MĚŘICÍHO PROSTORU PRO MALÝ SUPERSONICKÝ AERODYNAMICKÝ TUNEL

Abstract

The article deals with design and numerical calculation of a variable test section for small supersonic wind tunnel. The supersonic wind tunnel is designed to be driven by a supersonic ejector. The test section, which is in focus, is considered to by placed on its suction inlet. Schlieren method will be used to investigate the flow within. The purpose of the test section is to demonstrate effects, which occur in supersonic flows, e.g. shock waves, interactions of shock waves with boundary layers etc. Proper demonstration of such phenomenon requires different conditions gained within test section. Internal parts of the device are designed to be interchangeable or variable to provide this capability. The work deals with investigation and design of construction of the variable test section. Consequently, shape of the supersonic inlet nozzles for chosen Mach numbers are carried out. Methods of characteristics and CFD are employed to manage this task. The construction of the test section and obtained numerical results are presented.

Abstrakt

Článek se zabývá návrhem a numerickým výpočtem měnitelného měřicího prostoru pro malý supersonický aerodynamický tunel. Supersonický tunel je navržen s pohonem supersonickým ejektorem, přičemž měřicí prostor bude umístěn na jeho sání. K výzkumu proudění bude použito optických metod kombinovaných s pneumatickými měřeními. Účelem měřicího prostoru bude předvádění jevů, ke kterým dochází při nadzvukovém proudění, tj. rázové vlny, jejich interakce s mezními vrstvami a podobně. Správná demonstrace těchto jevů vyžaduje rozdílnou konfiguraci průtočného kanálu měřicího prostoru. Vnitřní vestavba měřicího prostoru je tak navržena jako vyměnitelná. Práce se zabývá výzkumem a návrhem měřicího prostoru a návrhem tvaru vstupních supersonických trysek pro daná Machova čísla. Je použita metoda charakteristik a CFD metoda. Jsou prezentovány výsledky výpočtů a navržená konstrukce měřicího prostoru.

1 INTRODUCTION

Recently, there is a construction of small-scaled supersonic wind tunnel in progress at the Department of Power Engineering Equipment. It is of ejector-driven desultory conception which construction, optimization and operating had been investigated by Dvořák [1] and Kolář in conjuction with Dvořák in [2] and [3]. Simultaneously, the test section is designed for this tunnel. The reason is to obtain the device, where the effects of high-speed, viscid flow could clearly be demonstrated to

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non-expert students. Transonic expansion, Fanno process, flow pattern around bodies, weak and strong shock waves are ranked among effect under investigation. Moreover, interaction of induced shock waves with themselves, interaction with boundary and shear layer etc. is awaited to be visualizable. Schlieren method accordingly with high-speed pressure probe Kulite will be the most widely used to investigate mentioned phenomena. Test section is constructed from aluminium alloy profiles, to ensure its maximal modularity. Resulting assemble can be seen at fig. 1.

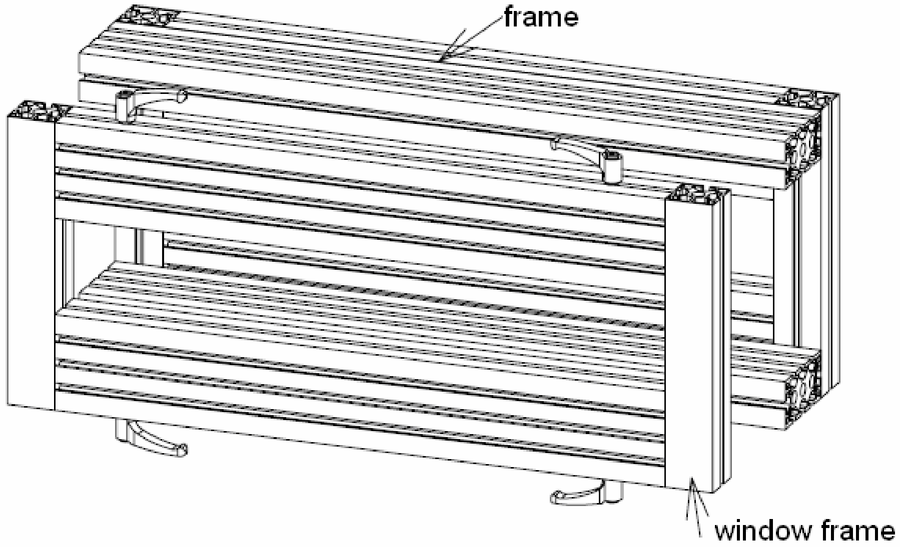

Fig. 1 Test section frame composed from aluminium alloy profiles.

The test section is composed from two side-frames, where the optical glass is fixed into. Both frames are connected one to the other by upper and lower parts, which dimension defines absolute depth of test section. Variable inner placing is to be one-sidedly fixed on selected side-frame. The other one ensures the accessibility into test section itself. Modularity of Mach number at the test section intake is requested for supersonic measuring purposes. Test section intake nozzle will not be designed adjustable in respect to the small scale of whole tunnel. Instead, wide range of single designed nozzles for selected Mach number will be made.

2 METHODS OF SUPERSONIC NOZZLE DESIGN

2.1 Method of Characteristic

Twin-sinusoidal contraction method is used to design the convergent subsonic part of nozzle. To learn more about it, see e.g.[4]. Supersonic part has been designed by the use of method of characteristic, which is described by Shapiro in publication [5]. In mathematics, this method became well known from solutions in field of partial, second-order and hyperbolic-type derivative equations. In there, characteristics have meaning of planes or curves, which are basically integrals of characteristic equations. For solution of partial differential equation, there are invariant relations valid along characteristics. There is a presumption of validity of linearized potential equation within solved region. Linearized potential equation is defined as

$$(1-M^2)\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0\cdot \tag{1}$$

Equation (1) has to be of hyperbolic type within the investigated region. This condition is met only in supersonic flows. Isentropic flow with neglected viscosity influence (including non-turbulent flow) and constant value of expansion coefficient are the additional requirements of validity of eq. (1). When all this simplifications are presumed, the characteristics are curves having been defined as

$$\frac{dy}{dx} = \frac{1}{\pm\sqrt{M^2 - 1}} \,. \tag{2}$$

By the modification of eq. (2), the characteristic-streamlines angle can be found from

$$\alpha = \pm \arcsin \frac{1}{M} \,. \tag{3}$$

As seen in eq. (3), characteristics are Mach lines along the elementary disturbance are propagated to. For infinitesimal disturbance, equation of a streamline deflection across characteristic yields

$$\vartheta - \vartheta_0 = \pm(\omega(\lambda) - \omega(\lambda_0)) \text{, where} \tag{4}$$

$$\omega(\lambda) = \sqrt{\frac{\kappa+1}{\kappa-1}}\, arctg \sqrt{\frac{\kappa-1}{\kappa+1} \frac{\lambda^2 - 1}{1 - \frac{\kappa-1}{\kappa+1}\lambda^2}} - arctg \sqrt{\frac{\lambda^2 - 1}{1 - \frac{\kappa-1}{\kappa+1}\lambda^2}} \tag{5}$$

is so-called Prandtl-Meyer function. Its meets physical sense for $1 \le \lambda \le \lambda_{\max}$, where

$$\lambda_{\max} = \sqrt{(\kappa+1)/(\kappa-1)} \,. \tag{6}$$

Value of Prandtl-Meyer function defines the turn of stream, needed to increase the velocity from $\lambda = 1$ to the value λ, during isentropic expansion. In numerical solution, there is continuous expansion substituted by the set of characteristic of defined strength. Strength of characteristics is defined by intensity $\Delta\omega$. Subsequently, the deflection of velocity vector across wave is

$$\Delta\vartheta = \mp\Delta\omega \,. \tag{7}$$

Waves have a location of median characteristic, which orientation to coordinate system is given by angle

$$\overline{\alpha} = \frac{\alpha_1 + \alpha_2 + \vartheta_1 + \vartheta_2}{2} \,, \tag{8}$$

where α_1 and α_2 are the values of Mach angle upstream and downstream of wave.

2.2 Method of numerical solution

The method of characteristic is suitable for the design of supersonic nozzles, but does not introduce viscosity effects into the solution. To verify designs of the nozzles, the software Fluent was used. Density-based solver, second order upwind and turbulence model SST k-w with transitional flow option were chosen [6]. The size of computational cells was about 0.0125 mm and wall adjacent cells satisfied $y^+ = 0.4 \div 1.8$.

3 RESULTS

Two variants of a supersonic nozzle with exit Mach number of 1.4 ($\omega = 9°$) are in fig. 2. The first design (presented on the left in fig. 2) was made for angle enlargement 2.25° and was based on 0.75°-intervals of ω. The throat of the nozzle is formed by a radius of 100mm. Three characteristics, one symmetry reflection and two wall reflections were needed to reach the exit Mach number.

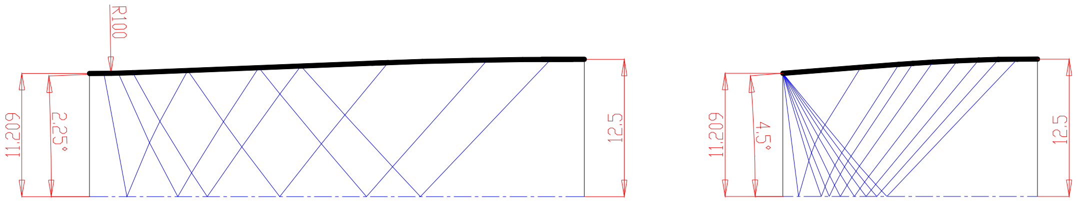

Fig. 2 Designs for supersonic nozzle with exit Mach number of 1.4.

Result of numerical computation of the first design is in fig. 3, where we can see contours of Mach number. The nozzle is relatively long, so the exit Mach number is only $M = 1.39$. The sonic line is quite curved. We can also observe a compression wave that is generated in the point where the straight part of the wall ends and the curvature part of the wall begins. Finally the velocity profile is not very aligned. To remove mentioned deficiencies, another design was made.

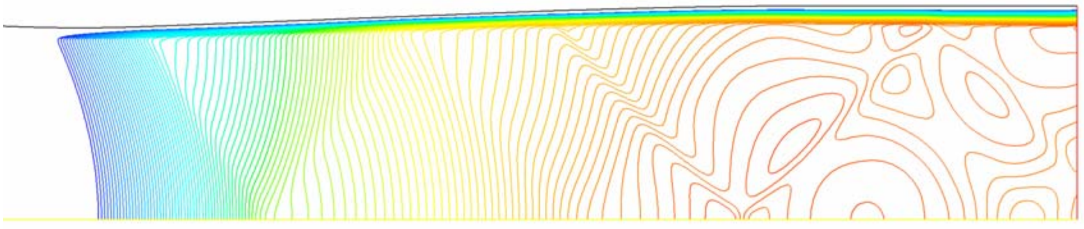

Fig. 3 Numerical computation of the first design of the nozzle for exit Mach number of 1.4. Contours of Mach number, $\Delta M = 0.004$.

For the second design (presented on the right in fig. 2), the angle of enlargement of 4.5°, sharp corner throat and Twin-sinusoidal contraction were chosen. The first three characteristics had intensity of $\Delta\omega = 0{,}25°$, other five characteristics had intensity of $\Delta\omega = 0{,}75°$. Only one symmetry reflection had to be solved to reach exit Mach number. As we can see on contours of Mach number in fig. 4, the sonic line is less curved, because the twin-sinusoidal contraction was used. The velocity profile is better aligned and reached exit Mach number is $M = 1.40$. The compression wave in the beginning of the wall curvature was removed, but another one occurred in the point of the curvature end. We also tried to refine the beginning and the end of the expansion, but the closing compression wave was not removed. Also the laminar model was in good agreement with SST k-ω model. It seems that this closing compression wave is caused by the inaccuracy of design by the help of the method of characteristics due to the viscosity effects. We can see that the expansion starts in front of the nozzle throat and the sonic line is still curved. Thus the flow over-expands and the compression wave occurs. The calculation and the design of the nozzle should be improved to prevent the closing compression wave. This wave would spread into the test section and disturb optic and pneumatic measurements.

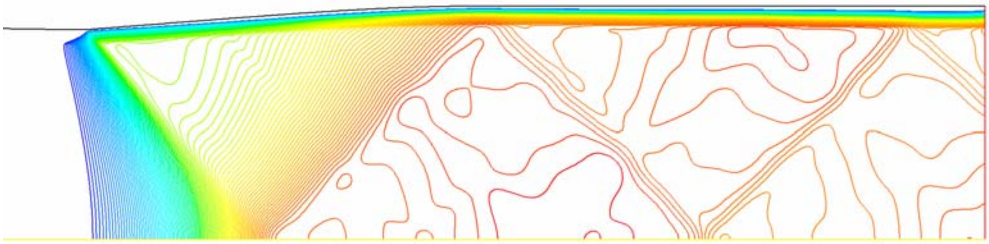

Fig. 4 Numerical computation of the second design of the nozzle for exit Mach number of 1.4. Contours of Mach number, $\Delta M = 0.004$.

4 CONCLUSION

A construction of a supersonic test section with variable geometry was designed to enable measuring and visualization of supersonic flow. The frame of the test section will be made from aluminium profiles and will allow easy alternation of inner channel profile.

The supersonic nozzle for exit Mach number of 1.4 was designed with the help of method of characteristics. It was shown that it is better to choose higher angle of enlargement to reduce the length of the nozzle and frictional loses. It was also shown that is useful to solve the beginning and the end of the expansion very in detail to prevent the creation of compression waves in wall points were the wall curvature changes takes place. However, the calculation and the design of the nozzle should be improved to prevent the closing compression wave. It seems that this closing compression wave is caused by the inaccuracy of design by the help of the method of characteristics due to the viscosity effects. The expansion starts in front of the nozzle throat and the sonic line is curved. Thus the flow over-expands and the compression wave occurs. In the next work the design will be solved even more in detailed to improve velocity profile. Also other nozzles for higher exit Mach numbers will be designed.

ACKNOWLEDGEMENT

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