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SPECIAL CASES OF TWO-DEGREE-OF-FREEDOM CONTROLLERS

SPECIÁLNI PŘÍPADY REGULÁTORŮ SE DVĚMA STUPNI VOLNOSTI

### Abstract

For a few years controllers have already been available, which enable the processing of a controlled variable in lieu of the control error in derivative and proportional terms. These controllers are special cases of controllers with two-degrees-of-freedom. With regard to that controller modifications are not summarily described. The aim of the article is to describe them in detail and to explain their functions.

### Abstrakt

Již řadu let jsou k dispozici regulátory, které umožňují v derivační a proporcionální složce zpracovávat místo regulační odchylky regulovanou veličinu. Tyto regulátory představují speciální případy regulátorů se dvěma stupni volnosti. Vzhledem k tomu, že tyto modifikace regulátorů nejsou dosud souborně zpracovány, cílem článku je přehledně je popsat a objasnit jejich funkci.

## 1 INTRODUCTION

The task of the controllers with two-degrees-of-freedom (2DOF) is to ensure demanding control quality simultaneously from the point of view of the disturbance and the desired variables [1, 2, 7, 8]. In the case of the disturbance variable it concerns its fast attenuation. While in the case of the desired variable its fast and accurate tracking is concerned [3 – 5]. The 2DOF controllers are described in detail e.g. in [1, 7, 8].

The controllers, which process the controlled variable in lieu of the control error in the derivative term enable the rejection of derivative kicks, i.e. the sharp changes of the manipulated variable and rejection or markedly attenuation of the controlled variable overshoots for sudden changes of the desired variable. If the controller contains the integral term, then the proportional term can process the controlled variable in lieu of the control error and in this case the proportional kick can be rejected. As will be shown, these controllers are the simplest cases of 2DOF controllers and they have established names (abbreviations).

## 2 SPECIAL CASES OF 2DOF CONTROLLERS

Behaviour of 2DOF PID controller is often described by relation [2]

$$U(s) = K_p \left\{ bW(s) - Y(s) + \frac{1}{T_I s} [W(s) - Y(s)] + T_D s [cW(s) - Y(s)] \right\} \quad (1)$$

where  $K_p$  is the controller gain,  $T_I$  – the integral time,  $T_D$  – the derivative time,  $b$  – the set-point weight for the proportional term,  $c$  – the set-point weight for the derivative term.  $W(s)$  – the transform

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of the desired variable,  $U(s)$  – the transform of the manipulated variable,  $Y(s)$  – the transform of the controlled variable.

Relation (1) can be arranged in the form

$$U(s) = G_F(s)G_C(s)W(s) - G_C(s)Y(s) \quad (2)$$

where

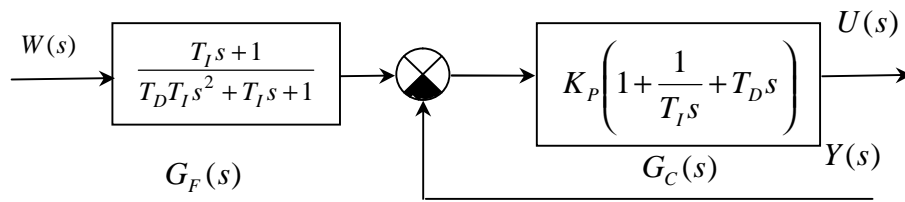
$$G_C(s) = K_p \left( 1 + \frac{1}{T_I s} + T_D s \right) = K_p \frac{T_D T_I s^2 + T_I s + 1}{T_I s} \quad (3a)$$

is the standard (1DOF) PID controller transfer function and

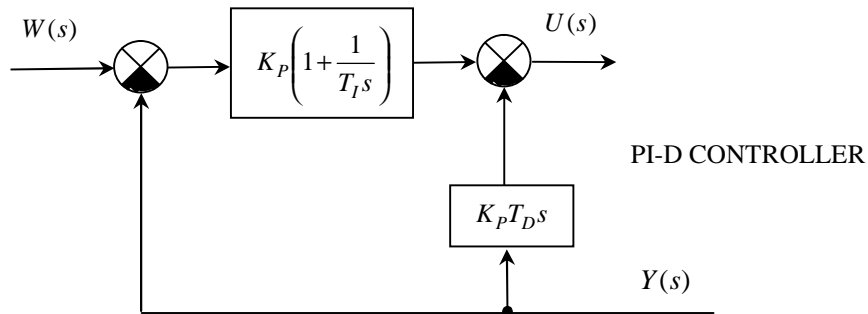
$$G_F(s) = \frac{c T_D T_I s^2 + b T_I s + 1}{T_D T_I s^2 + T_I s + 1} \quad (3b)$$

is the input filter transfer function.

a)



b)



**Fig. 1** PI-D controller

### PI-D controller

The PI-D controller ensues from the 2DOF PID controller (1) for  $b = 1$  and  $c = 0$ .

After substitution  $b = 1$  and  $c = 0$  in (1) and (3b) there is obtained

$$U(s) = K_p \left( 1 + \frac{1}{T_I s} \right) [W(s) - Y(s)] - K_p T_D s Y(s) \quad (4a)$$

$$G_F(s) = \frac{T_I s + 1}{T_D T_I s^2 + T_I s + 1} \quad (4b)$$

The PI-D controller name and structure follow directly from relation (4a) and the transformation of the scheme in Fig. 1a on the equivalent scheme in Fig. 1b.

### I-PD controller

The I-PD controller ensues from the 2DOF PID controller (1) for  $b = c = 0$ .

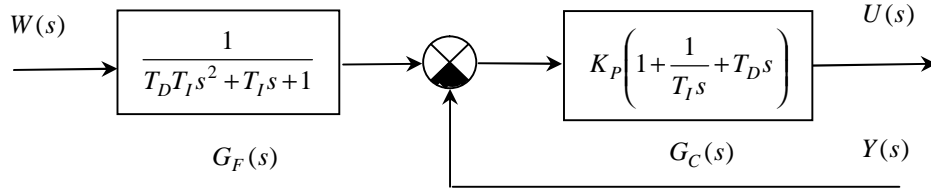
After substitution  $b = c = 0$  in (1) and (3b) there is obtained

$$U(s) = K_p \frac{1}{T_I s} [W(s) - Y(s)] - K_p (1 + T_D s) Y(s) \quad (5a)$$

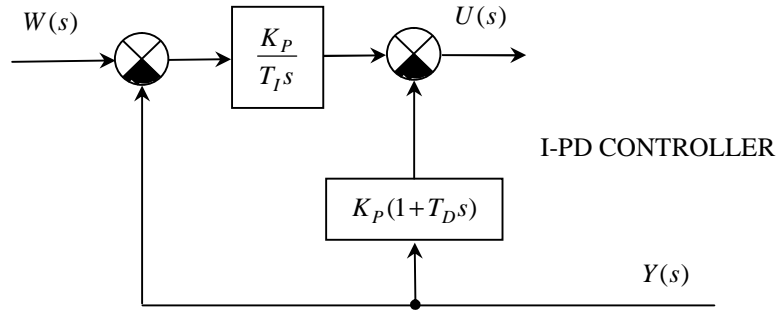
$$G_F(s) = \frac{1}{T_D T_I s^2 + T_I s + 1} \quad (5b)$$

The I-PD controller name and structure follow directly from relation (5a) and the transformation of the scheme in Fig. 2a on the equivalent scheme in Fig. 2b.

a)



b)



**Fig. 2** I-PD controller

Behaviour of the 2DOF PI controller can be obtained from (1) for is  $T_D = 0$

$$U(s) = K_p \left\{ bW(s) - Y(s) + \frac{1}{T_I s} [W(s) - Y(s)] \right\} \quad (6)$$

Relation (6) can be arranged in the form (2), where

$$G_C(s) = K_p \left( 1 + \frac{1}{T_I s} \right) = K_p \frac{T_I s + 1}{T_I s} \quad (7a)$$

is the standard PI controller transfer function and

$$G_F(s) = \frac{bT_I s + 1}{T_I s + 1} \quad (7b)$$

is the input filter transfer function.

### I-P controller

The I-P controller ensues from the 2DOF PI controller (7a) for  $b = 0$ .

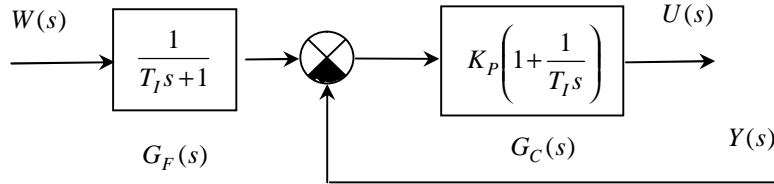
After substitution  $b = 0$  in (6) and (7b) it is obtained

$$U(s) = K_p \frac{1}{T_I s} [W(s) - Y(s)] - K_p Y(s) \quad (8a)$$

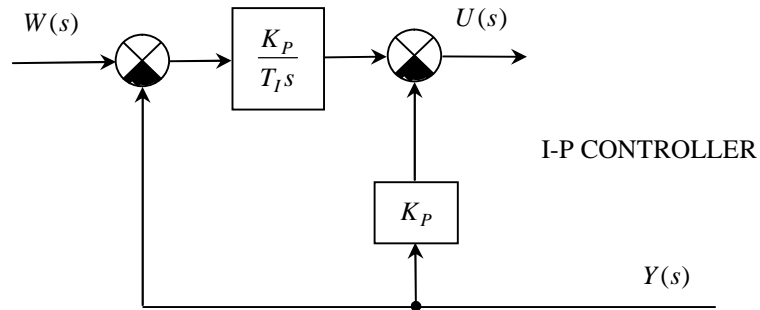
$$G_F(s) = \frac{1}{T_I s + 1} \quad (8b)$$

The I-P controller name and structure follow directly from relation (8a) and the transformation of the scheme in Fig. 3a on the equivalent scheme in Fig. 3b.

a)



b)



**Fig. 3** I-P controller

Behaviour of the 2DOF PD controller can be obtained from (1) for is  $b = 1$  and  $T_I \rightarrow \infty$

$$U(s) = K_p \{W(s) - Y(s) + T_D s [cW(s) - Y(s)]\} \quad (9)$$

Relation (9) can be arranged in the form (2), where

$$G_C(s) = K_p (1 + T_D s) \quad (10a)$$

is the standard PD controller transfer function and

$$G_F(s) = \frac{cT_D s + 1}{T_D s + 1} \quad (10b)$$

is the input filter transfer function.

## P-D controller

The P-D controller ensues from the 2DOF PD controller (9) for  $c = 0$ .

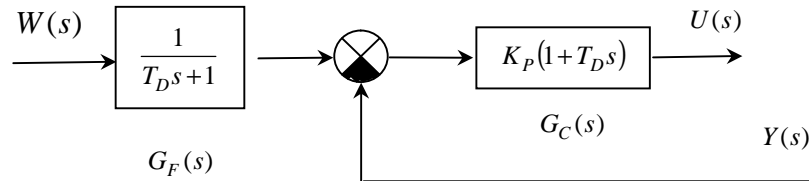
After substitution  $c = 0$  in (9) and (10b) there is obtained

$$U(s) = K_p[W(s) - Y(s)] - K_p T_D s Y(s) \quad (11a)$$

$$G_F(s) = \frac{1}{T_D s + 1} \quad (11b)$$

The P-D controller name and structure follow directly from relation (11a) and the transformation of the scheme in Fig. 4a on the equivalent scheme in Fig. 4b.

a)



b)

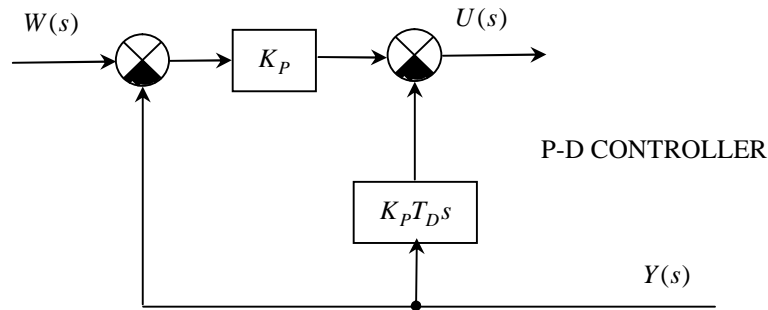


Fig. 4 P-D controller

## 3 BEHAVIOUR OF MODIFIED CONTROLLERS

The behaviour of these modified controllers, i.e. these special cases of the 2DOF controllers, directly follows from Figs 1a, 2a, 3a and 4a. The desired variable is filtered by the input filter with the transfer function  $G_F(s)$ , and therefore the velocity of its changes is restricted. The other explication of their behaviour follows for  $b = 0$  or  $c = 0$  from corresponding relations (4a) – (5a), (8a), (11a) and Figs 1b – 4b, from which it is obvious, that the proportional term ( $b = 0$ ) or derivative term ( $c = 0$ ) do not react directly to the desired variable change but these terms react on the controlled variable response (i.e. after its passage through controller and plant). In all these cases it is obvious, that these modified controllers enable the rejection of set-point kicks and the rejection or attenuation of big overshoots of the controlled variable for the step changes of the desired variable [1, 7, 8].

## 4 CONCLUSIONS

In the article special kinds of the modified controllers are described in detail. These controllers have been used in practice for long time, they have established their names and they aren't mostly associated with 2DOF controllers even if there are special cases. It is shown in which way they ensue from the 2DOF controllers, and why they have special names and how their behaviour is explained.

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