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BASIC FORMS OF TWO-DEGREE-OF-FREEDOM CONTROLLERS

ZÁKLADNÍ TVARY REGULÁTORŮ SE DVĚMA STUPNI VOLNOSTI

Abstract

Controllers with two-degree-of-freedom have recently been more and more frequently available. Description of their operation has not been worked out in literature up to now. The aim of the paper is to show the basic different equivalent forms of two-degree-of-freedom controllers and to explain their operation.

Abstrakt

V poslední době jsou stále častěji k dispozici regulátory se dvěma stupni volnosti. Popis jejich funkce a různých tvarů není doposud v dostupné literatuře souborně zpracován. Cílem příspěvku je ukázat základní různé ekvivalentní tvary regulátorů se dvěma stupni volnosti a objasnit jejich funkci.

1 INTRODUCTION

When standard controllers (with one-degree-of-freedom) are used, then there must be a tuning trade-off between the control performance of the servo and regulatory responses, see Fig. 1 [4 – 6], where $G_C(s)$ is the standard controller transfer function, $G_P(s)$ – the plant transfer function, $E(s)$ – the transform of the error, $W'(s)$ – the transform of the desired variable, $U(s)$ – the transform of the manipulated variable, $Y(s)$ – the transform of the controlled variable, $V_1(s)$ and $V_2(s)$ – the transforms of the disturbance variables.

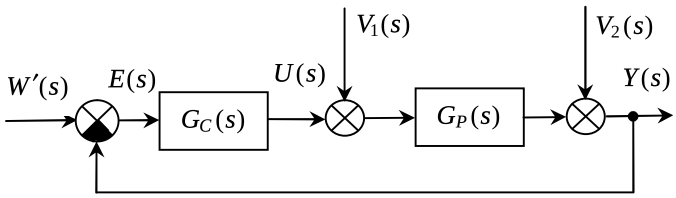

Fig. 1 Control system with standard controller

In accordance with Fig. 1 the error transfer functions have the forms

$$G_{w'e}(s) = \frac{E(s)}{W'(s)} = \frac{1}{1+G_C(s)G_P(s)} \tag{1}$$

$$G_{v_1e}(s) = \frac{E(s)}{V_1(s)} = -\frac{G_P(s)}{1+G_C(s)G_P(s)} \tag{2}$$

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$$G_{v_2e}(s)=\frac{E(s)}{V_2(s)}=-\frac{1}{1+G_C(s)G_P(s)} \tag{3}$$

From the transfer functions (1) and (3) it follows that the standard controller tuning from the point of view of the desired variable $w'(t)$ is equivalent to the tuning from the point of view of the disturbance variable $v_2(t)$, which works on the plant output (except for the sign).

From the transfer function (2) it is obvious, that the tuning from the point of view of the disturbance variable $v_1(t)$, which works on the plant input, should be different. It is given by the plant transfer function $G_P(s)$ in the numerator. Big problems arise in the cases when the plant and the controller have an integral character [8]. Therefore in the cases when there are changes of the desired variable $w'(t)$ and the disturbance variable $v_1(t)$, the standard controller must be tuned as trade-off from point of view of both variables $w'(t)$ and $v_1(t)$. It is not always possible namely in the case of integrating plants and standard controllers with the integral term [8]. In these cases the smart solution is the use of two-degree-of-freedom (2DOF) controllers [1 – 4,7,8].

2 BASIC FORMS OF 2DOF CONTROLLERS

The behaviour of the 2DOF PID controller is often described by relation [4, 8]

$$U(s)=K_P\left\{bW(s)-Y(s)+\frac{1}{T_I s}[W(s)-Y(s)]+T_D s[cW(s)-Y(s)]\right\} \tag{4}$$

where K_P is the controller gain, T_I − the integral time, T_D − the derivative time, b – the set-point weight for the proportional term, c – the set-point weight for the derivative term.

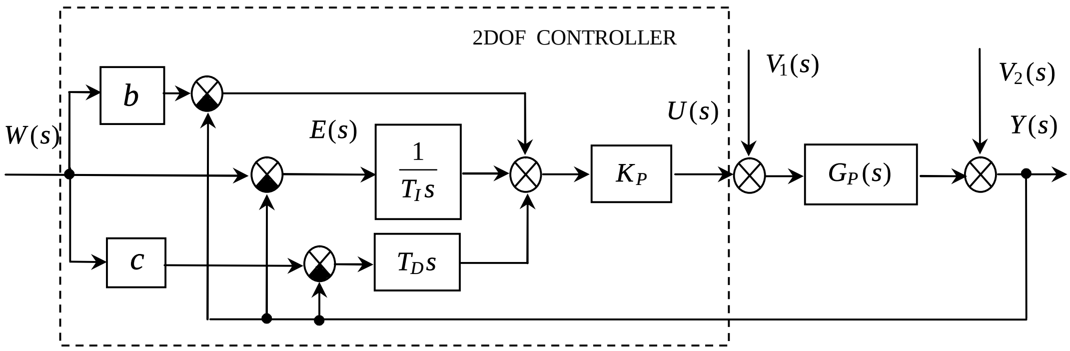

Fig. 2 Control system with 2DOF controller corresponding to relation (4)

The Fig. 2 corresponds to relation (4). The relation (4) can be arranged in the form

$$U(s)=K_P\left(b+\frac{1}{T_I s}+cT_D s\right)W(s)-K_P\left(1+\frac{1}{T_I s}+T_D s\right)Y(s)$$

or

$$U(s)=G_{ff}(s)W(s)-G_C(s)Y(s) \tag{5}$$

where

$$G_{ff}(s)=K_P\left(b+\frac{1}{T_I s}+cT_D s\right)=K_P\frac{cT_D T_I s^2+bT_I s+1}{T_I s} \tag{6}$$

$$G_C(s)=K_P\left(1+\frac{1}{T_I s}+T_D s\right)=K_P\frac{T_D T_I s^2+T_I s+1}{T_I s} \tag{7}$$

The formula (7) expresses the standard (1DOF) PID controller transfer function.

The Fig. 3 corresponds to the relation (5) [1, 2, 4].

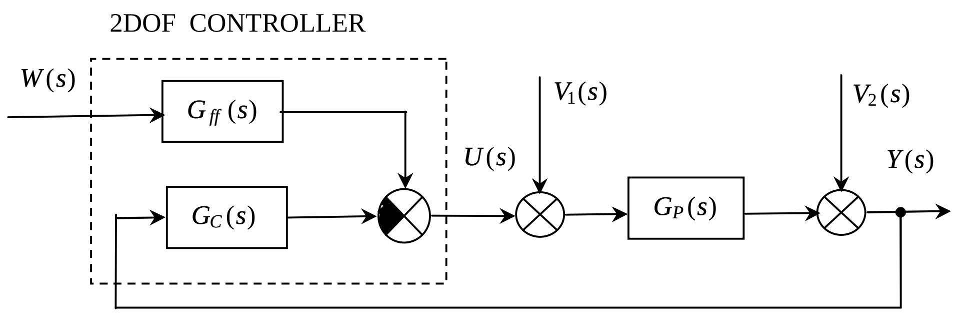

Fig. 3 Control system with 2DOF controller corresponding to relation (5)

The scheme in Fig. 3 can be arranged on the scheme in Fig. 4, for which holds [3,4,7]

$$U(s) = G_F(s)G_C(s)W(s) - G_C(s)Y(s) \tag{8}$$

where

$$G_F(s) = \frac{G_{ff}(s)}{G_C(s)} = \frac{cT_DT_Is^2 + bT_Is + 1}{T_DT_Is^2 + T_Is + 1} \tag{9}$$

is the input filter transfer function.

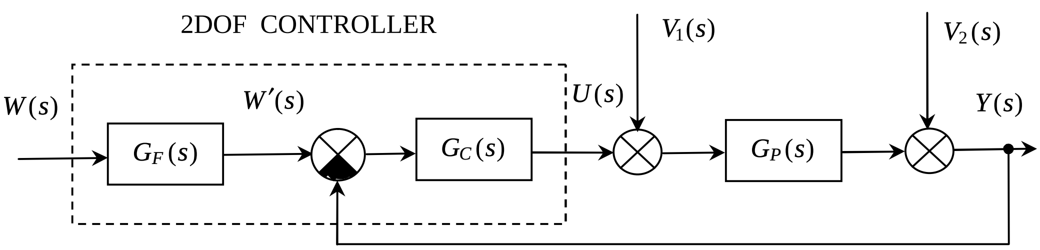

Fig. 4 Control system with 2DOF controller corresponding to relation (8)

It is often possible to meet the control system structure with the 2DOF PID controller, which is shown in Fig. 5, where $G_K(s)$ is the feedforward compensator with the transfer function [3,7]

$$G_K(s) = K_P(a + bT_Ds) \tag{10}$$

and $G_C(s)$ is the standard PID controller transfer function (7).

For manipulated variable $u(t)$ in Fig. 5 the equation

$$U(s) = [G_C(s) - G_K(s)]W(s) - G_C(s)Y(s) \tag{11}$$

holds.

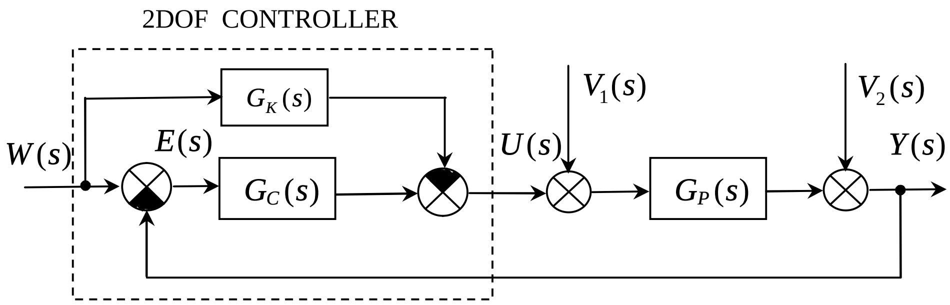

Fig. 5 Control system with 2DOF controller corresponding to relation (11)

After substitution (6) and (7) in (5) and (11) and following the comparison of the first terms on their right sides the relations

$$K_P \frac{cT_D T_I s^2 + bT_I s + 1}{T_I s} = K_P \frac{T_D T_I s^2 + T_I s + 1}{T_I s} - K_P (a + bT_D s) \Rightarrow$$

$$\Rightarrow \; b = 1 - a \,,\; c = 1 - b \tag{12}$$

can be obtained.

It is obvious, that all control system structures with the 2DOF controller in Figs 2 – 5 are mutually equivalent on the assumption that the relations (12) hold.

For $b = c = 1$ ($\alpha = \beta = 0$) formulas, which expresses behaviour of the 2DOF PID controller (4), (5), (8) and (11), describe the standard PID controller (7) and all control system schemes in Figs 2 – 5 are reduced in the scheme in Fig. 1, i.e. the relations

$$W'(s) = W(s),\; G_{ff}(s) = G_C(s),\; G_F(s) = 1,\; G_K(s) = 0 \tag{13}$$

hold.

For $T_D = 0$ the formulas (4), (5), (8) and (11) describe the 2DOF PI controller, i.e.

$$U(s) = K_P \left\{ bW(s) - Y(s) + \frac{1}{T_I s}[W(s) - Y(s)] \right\} \tag{14}$$

The formulas (6), (7), (9) and (10) will have the corresponding forms

$$G_{ff}(s) = K_P \left(b + \frac{1}{T_I s} \right) \tag{15}$$

$$G_C(s) = K_P \left(1 + \frac{1}{T_I s} \right) \tag{16}$$

$$G_F(s) = \frac{bT_I s + 1}{T_I s + 1} \tag{17}$$

$$G_K(s) = aK_P \tag{18}$$

The formula (14), which expresses the behaviour of the 2DOF PI controller, for $b = 1$ ($\alpha = 0$) describes the standard PI controller with the transfer function (16) and simultaneously for formulas (15) – (18) the relations (13) hold.

Similarly for $T_I \to \infty$ and $b = 1$ ($\alpha = 0$) the formulas (4), (5), (8) and (11) describe the 2DOF PD controller, i.e.

$$U(s) = K_P \{ W(s) - Y(s) + T_D s [cW(s) - Y(s)] \} \tag{19}$$

The formulas (6), (7), (9) and (10) will have the forms

$$G_{ff}(s) = K_P(1 + cT_D s) \tag{20}$$

$$G_C(s) = K_P(1 + T_D s) \tag{21}$$

$$G_F(s) = \frac{cT_D s + 1}{T_D s + 1} \tag{22}$$

$$G_K(s) = bK_P T_D s \tag{23}$$

The formula (19) for c = 1 (β = 0) describes the behaviour of the standard PD controller with the transfer function (21) and for the formulas (20) – (23) the relations (13) hold.

If the filtering in the derivative term

$$D(s) = \frac{T_D s}{\frac{T_D}{N}s + 1} \tag{24}$$

is used, where N = 5 ÷ 20 (usually in industrial controllers the value N = 10 is preset), then in all above given relations the term (24) must be substituted in lieu of the term $T_D s$.

For correct operation of any controller it is necessary to acquaint it with the true control error $e(t)$. If the controller contains the integral term, the control error $e(t)$ is processed foremost by it. By reason of it the set-point weight for integral term must be always equal 1, see e.g. formula (4) and Fig. 2. If the controller of the integral term is not contained, then the control error is processed foremost by the proportional term and therefore its set-point weight b = 1 (α = 0), see e.g. the formula (19).

3 OPERATION OF 2DOF CONTROLLERS

The 2DOF controllers enable tuning from the point of view of the desired variable $w(t)$, and from one of the disturbance variables $v_1(t)$ or $v_2(t)$. Most often it is the disturbance variable $v_1(t)$, which works on the plant input but the trade-off tuning from the point of view of both disturbance variables $v_1(t)$ or $v_2(t)$ is possible too.

The standard PID controller is tuned by common approaches and methods from the point of view of the selected disturbance variable and sequentially the set-point weights b and c (α and β) must be suitably chosen.

The tuning from point of view of the chosen disturbance variable mostly causes big overshoots in the servo response. In case of integrating plants these overshoots cannot be removed by any standard controller (with the integral term) tuning [8]. In these cases the use of the 2DOF controllers is efficient.

From Figs 2 – 6 and the error transfer functions (2) and (3) it follows, that the 2DOF controllers have no influence on the regulatory responses. If the 2DOF controller is used then the error transfer functions (2) and (3) remain the same, only the error transfer function (1) is changed. E.g. in accordance with Fig. 4 it can be obtained

$$G_{we}(s) = \frac{E(s)}{W(s)} = \frac{G_F(s)}{1 + G_C(s)G_P(s)} \tag{25}$$

From the relation (25) it is obvious that the overshoots can be attenuated by a suitable choice of the input filter $G_F(s)$, i.e. by the suitable choice of the set-point weights b and c (α and β).

The simplest interpretation of the 2DOF PID controller operation follows directly from relation (4) and the corresponding Fig. 2. For the set-point weight values $0 \le b < 1$ and $0 \le c < 1$ the decrease of the desired variable $w(t)$ step values on the proportional and derivative inputs happen and thereby the overshoots must be decreased too. By the suitable choice of the set-point weight values b

and c it is possible to attenuate big overshoots and simultaneously it is possible to obtain the sufficient fast servo response [3,4,7,8]. In Fig. 6 the influence of the different values of the set-point weight b on the servo responses for the 2DOF PI controller and the integrating plants is shown.

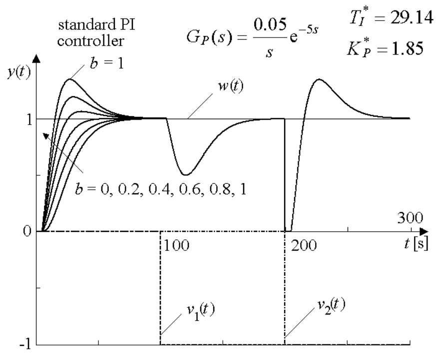

Fig. 6 Servo and regulatory responses for 2DOF PI controller and different values of set-point weight *b*

4 CONCLUSIONS

In the article different equivalent 2DOF controller structures are described and their operation is explained. The 2DOF controllers came to be widespread and therefore it is important to know their different forms and behaviour. Knowledge of the 2DOF controllers is necessary for their effective exploitation in industrial practice.

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REFERENCES

[1] ALFARO, V. M. *Analytical Robust Tuning of Two-Degree-of-Freedom PI and PID Controllers*. Report, September 20, 2007, pp. 1-42

[2] ALFARO, V. M., VILANOVA, R. & ARRIETA, O. Robust of Two-Degree-of-Freedom (2-DOF) PI/PID Based Cascade Control Systems. *Journal of Process Control*, 19 (2009), pp. 1658-1670

[3] ARAKI, M. & TAGUCHI, H. Two-Degree-of-Freedom PID Controllers. International *Journal of Control, Automation, and Systems*, Vol. 1, No. 4, December 2003, pp. 401-411

[4] ÅSTRÖM, K. J. & HÄGGLUND, T. *Advanced PID Control*. Research Triangle Park: NC, ISA-Instrumentation, Systems, and Automation Society, 2006, 460 p.

[5] BAKOŠOVÁ, M., DRAVEC, J. & ČIRKA, Ľ. Software for PID Controller Tuning. In: *Sborník příspěvků 17. ročníku konference „Technical Computing Prague 2009“*, 8 str. ISBN 978-80-7080-773-0

[6] FILASOVÁ, A. Robust Control Design. An optimal control approach. In: *Proceedings of the 3rd IEEE International Conference on InteligentEngineering Systems INES'99*, Stara Lesna, Slovakia, 1999, s. 515-518

[7] TAGUCHI, H. & ARAKI, M. Two-Degree-of-Freedom PID Controllers-their Functions and Optimal Tuning. In *Preprints of IFAC Workshop on Digital Control. Past, present and future of PID Control.* Terrassa (Spain), 5-7 April 2000, pp. 95-100

[8] VÍTEČKOVÁ, M. & VÍTEČEK, A. Two-Degree of Freedom Controller Tuning for Integral Plus Time Delay Plants. *ICIC Express Letters,* Volume 2, Number 3, September 2008a, pp. 225-229