Transactions of the VŠB – Technical University of Ostrava, Mechanical Series
No. 2, 2010, vol. LVI
article No. 1790

Radek MATUŠŮ[*], Roman PROKOP[**]

ROBUST TUNING OF PI CONTROLLERS FOR INTERVAL SYSTEMS

ROBUSTNÍ LADĚNÍ PI REGULÁTORŮ PRO INTERVALOVÉ SYSTÉMY

Abstract

This paper deals with combined graphical-algebraic design of PI controllers which ensure robust stabilization of interval systems. The stability regions for parameters of these controllers are obtained via computing and plotting the stability boundary locus supplemented with the sixteen plant theorem and the final choice of the controller is based on an algebraic approach. A third order interval plant is robustly controlled in an illustrative example.

Abstrakt

Tento článek se zabývá kombinovaným graficko-algebraickým návrhem PI regulátorů, které zajišťují robustní stabilizaci intervalových soustav. Oblasti stability pro parametry těchto regulátorů jsou získány pomocí výpočtu a vykreslení umístění hranice stability, doplněné větou o šestnácti soustavách, přičemž finální volba regulátoru je založena na algebraickém přístupu. V rámci ilustrativního příkladu je robustně řízena intervalová soustava třetího řádu.

1 INTRODUCTION

A mathematical model with interval uncertainty is a standard tool for description of real technological processes. This approach helps to incorporate the simplifications made during modelling, imprecise knowledge of plant parameters or its variability into the mathematical model and subsequently into the control synthesis itself. The designed controller should ensure some desired properties of the control loop for the whole interval plant family. The essential requirement of all users is the (robust) stability of the feedback control system.

Despite existence of many modern control technologies, the present industrial practice still prefers the application of classical PI or PID compensators with fixed parameters [13], [14]. The reason of this popularity is that PI(D) controllers are cheap, reliable, easily utilizable and usually sufficient at the same time. Thus, an easy and effective way of PI/PID tuning is still very topical, especially in case that these algorithms are able to cope with various uncertain conditions comparably with other approaches [15], [16].

The main aim of this paper is to present a control design method for interval systems and to demonstrate its capabilities on an example for the third order interval plant. The computation of robustly stabilizing PI controllers with fixed parameters uses the stability boundary locus plotting in combination with the sixteen plant theorem, which is described in [10], [11]. Following choice of the final controller is based on general solutions of Diophantine equations in the ring of proper and Hurwitz stable rational functions (R_{PS}), Youla-Kučera parameterizations and conditions of divisibility in the specific ring. Besides, the selected controller can be further tuned through the only scalar tuning parameter $m > 0$. The main ideas of this technique are adopted from [4], [8], [12].

[*] Ing., Ph.D., Tomas Bata University in Zlín, Faculty of Applied Informatics, Department of Automation and Control Engineering, Nad Stráněmi 4511, Zlín, tel. (+420) 576 033 015, e-mail rmatusu@fai.utb.cz
[**] prof., Ing., CSc., Tomas Bata University in Zlín, Faculty of Applied Informatics, Department of Automation and Control Engineering, Nad Stráněmi 4511, Zlín, tel. (+420) 576 035 257, e-mail prokop@fai.utb.cz

2 DETERMINATION OF STABILIZING PI CONTROLLERS

A possible approach to calculation of stabilizing PI controllers based on plotting the stability boundary locus is proposed in [10], [11]. The method supposes the classical closed-loop control system with the controlled plant:

$$G(s) = \frac{B(s)}{A(s)} \tag{1}$$

and PI controller:

$$C(s) = k_P + \frac{k_I}{s} = \frac{k_P s + k_I}{s} \tag{2}$$

First, one needs to use the substitution $s = jw$ in the plant (1) and subsequently to decompose the numerator and denominator of this transfer function into their even and odd parts:

$$G(jw) = \frac{B_E(-w^2) + jwB_O(-w^2)}{A_E(-w^2) + jwA_O(-w^2)} \tag{3}$$

Then, the expression of closed-loop characteristic polynomial and setting the real and imaginary parts to zero lead to the equations:

$$\begin{aligned} k_P &= \frac{w^2 A_O(-w^2)B_O(-w^2) + A_E(-w^2)B_E(-w^2)}{-w^2 B_O^2(-w^2) - B_E^2(-w^2)} \\ k_I &= w^2 \frac{A_E(-w^2)B_O(-w^2) - A_O(-w^2)B_E(-w^2)}{-w^2 B_O^2(-w^2) - B_E^2(-w^2)} \end{aligned} \tag{4}$$

Simultaneous solving of these relations and plotting the obtained values into the (k_P, k_I) plane result in the stability boundary locus, which splits the (k_P, k_I) plane up to the stable and unstable regions. The determination of the stabilizing one(s) can be done via a test point within each region. Furthermore, this technique can be embellished with the Nyquist plot based approach from [9] to avoid potential problems with proper frequency gridding. In this refinement, the frequency axis can be divided into several intervals by the real values of w which fulfil:

$$\mathrm{Im}\left[G(s)\right] = 0 \tag{5}$$

Such intervals are then sufficient for testing.

3 IMPROVEMENT OF THE METHOD FOR INTERVAL PLANTS

So far, the area of stabilizing controller coefficients for a given plant with only fixed parameters can be computed. However, the paper [10] has improved the stabilization also for interval systems using the simple idea of its combination with the sixteen plant theorem [1], [2]. In compliance with this principle, a first order controller robustly stabilizes an interval plant

$$G(s,b,a) = \frac{B(s,b)}{A(s,a)} = \frac{\sum_{i=0}^{m}\left[b_i^-, b_i^+\right]s^i}{s^n + \sum_{i=0}^{n-1}\left[a_i^-, a_i^+\right]s^i}; \quad m < n \tag{6}$$

where $b_i^-, b_i^+, a_i^-, a_i^+$ are lower and upper bounds for numerator and denominator parameters, respectively, if and only if it stabilizes its 16 Kharitonov plants, which are defined as:

$$G_{i_1,i_2}(s)=\frac{B_{i_1}(s)}{A_{i_2}(s)} \tag{7}$$

where $i_1,i_2\in\{1,2,3,4\}$; and $B_1(s)$ to $B_4(s)$ and $A_1(s)$ to $A_4(s)$ are the Kharitonov polynomials for the numerator and denominator of the interval system (6), respectively.

Remind that the Kharitonov polynomials e.g. for an interval polynomial:

$$B(s,b)=\sum_{i=0}^{m}\left[b_i^-;b_i^+\right]s^i \tag{8}$$

can be constructed using the upper and lower bounds of interval parameters according to the rule [3]:

$$\begin{aligned}
B_1(s)&=b_0^-+b_1^-s+b_2^+s^2+b_3^+s^3+\ldots\\
B_2(s)&=b_0^++b_1^+s+b_2^-s^2+b_3^-s^3+\ldots\\
B_3(s)&=b_0^++b_1^-s+b_2^-s^2+b_3^+s^3+\ldots\\
B_4(s)&=b_0^-+b_1^+s+b_2^+s^2+b_3^-s^3+\ldots
\end{aligned} \tag{9}$$

The stabilization of an interval plant is grounded in the stabilization of all 16 fixed Kharitonov plants together, and so the final stability region is given by intersection of all partial regions.

4 ALGEBRAIC DESIGN OF A CONTROLLER

The methodology from the previous sections allows determining the (robustly) stabilizing combinations of proportional and integral gains in PI controller. However, the specific choice of the compensator itself is still an open question. A possible algebraic control design method is based on ideas developed in [4], [12]. It uses general solutions of Diophantine equations in R_{PS}. Moreover, it supposes the utilization of the known Youla-Kučera parameterization, which allows generating infinite amount of possible stabilizing controllers, while the choice of the final one depends on the desired properties mathematically represented by conditions of divisibility in the specific ring. Anyway, the selected controller can be further tuned. One of advantages of this approach is that behavior of regulators can be influenced by the only scalar tuning parameter $m>0$. Details of this method can be found for example in [5], [8].

From a set of developed results, this contribution takes advantage of the tuning rules for the first order plant:

$$G(s)=\frac{b_0}{s+a_0} \tag{10}$$

and feedback PI controller:

$$C(s)=\frac{k_Ps+k_I}{s} \tag{11}$$

which ensures the bounded-input bounded-output (BIBO) stability of the closed control loop and asymptotic tracking of the stepwise reference signal. The parameters of this regulator can be derived as:

$$k_P=\frac{2m-a_0}{b_0};\quad k_I=\frac{m^2}{b_0} \tag{12}$$

Obviously, these coefficients are functions of the tuning parameter $m>0$. A potential way of its selection is outlined in [5].

5 AN ILLUSTRATIVE EXAMPLE

Suppose that controlled process is described by interval transfer function [6]:

$$G(s) = \frac{[1;\,2]}{s^3 + [3;\,4]s^2 + [5;\,6]s + [7;\,8]} \tag{13}$$

and the objective is to find all possible robustly stabilizing PI controllers.

First, consider e.g.:

$$G_{1,1}(s) = \frac{B_1(s)}{A_1(s)} = \frac{1}{s^3 + 4s^2 + 5s + 7} \tag{14}$$

as the first of 16 Kharitonov plants. The equation (4) here takes the concrete form:

$$\begin{aligned} k_P &= 4w^2 - 7 \\ k_I &= -w^4 + 5w^2 \end{aligned} \tag{15}$$

Using (5) and consequent stability test for two obtained intervals lead to the range of the frequency $w \in (0;\,2.236)$, which is necessary for computing/plotting the stability boundary locus. The analogical procedure has been done generally for all 16 Kharitonov plants. However, in this specific case, the locus of only 8 systems is enough to investigate, because the nominator of (13) takes only two extreme values and the construction of Kharitonov polynomials would be redundant here.

The Figure 1 provides the graphical representation of the stability boundary locus for 8 Kharitonov plants, while the Figure 2 brings closer look to the intersection, which constitutes final stability region for the interval plant (13). An arbitrary pair of (k_P, k_I) from the inside of this stability region would entail robustly stable control system.

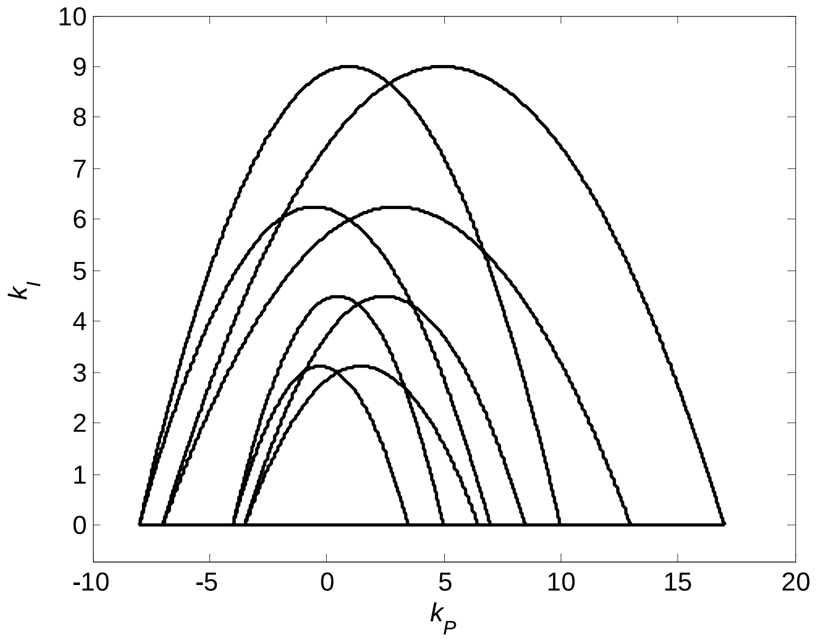

Fig. 1 Stability regions for 8 Kharitonov plants

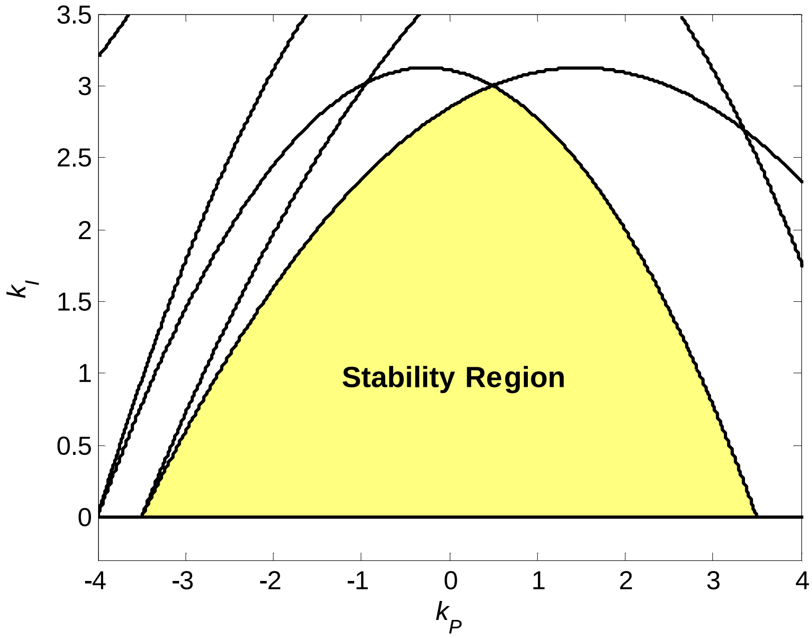

Fig. 2 Stability region for the interval plant (13)

Once the boundaries of robustly stabilizing PI coefficients are obtained, quite natural question emerges, which is how to find the practically suitable controller from this set. Here, the algebraic approach from the Section 4 has been applied. For the sake of appropriate order of the final controller (first order – PI type), the controlled system must be described in the form of first order plant. The nominal system (for control design) has been obtained using the middle values of interval coefficients in (13) and then via the very simple approximation, that is:

$$\frac{1.5}{s^3+3.5s^2+5.5s+7.5} \approx \frac{1.5}{5.5s+7.5} = \frac{0.\overline{27}}{s+1.\overline{36}} = G_N(s) \qquad (16)$$

For example, the tuning parameter $m = 0.7$ gives the controller parameters (12):

$$k_P = 0.1333; \quad k_I = 1.7967 \qquad (17)$$

The position of this controller in the stability region from Figure 2 is depicted in the following Figure 3. It lies on the curve which hypothetically connects the controllers tuned by various parameters *m*.

Finally, the Figure 4 shows the control responses of the loop with this PI controller and 2401 “representative” systems from the interval family (13). Each interval parameter has been divided into 6 subintervals and thus these 7 values and 4 parameters result in $7^4 = 2401$ systems for simulation. Moreover, the red curve represents the output signal for the nominal system (16). Generally, it has been assumed the step reference signal changing from 1 to 2 in 1/3 of simulation time and the step load disturbance -2 which influences the input to the controlled plant during the last third of simulation. As can be seen, the controller (11) with parameters (17) really robustly stabilizes the interval plant (13).

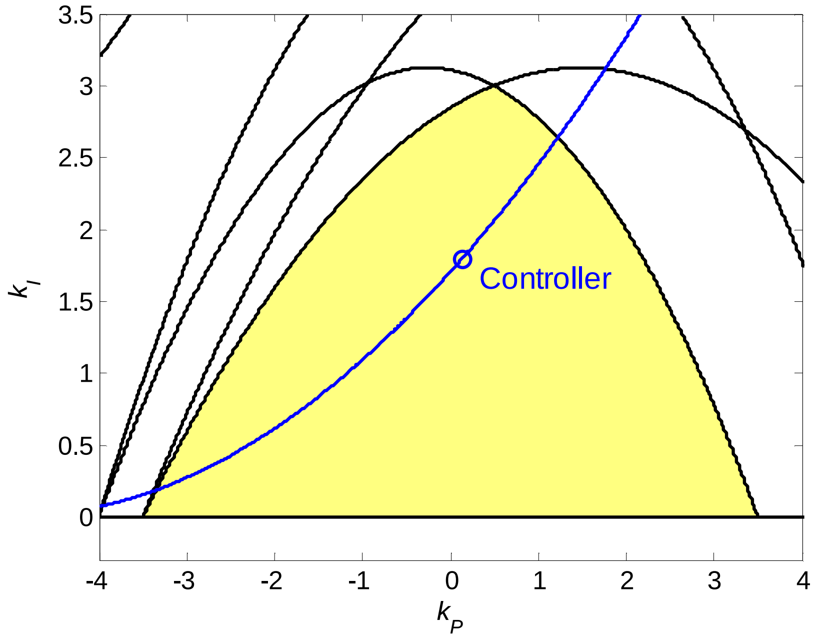

Fig. 3 The position of controller (17) in stability region

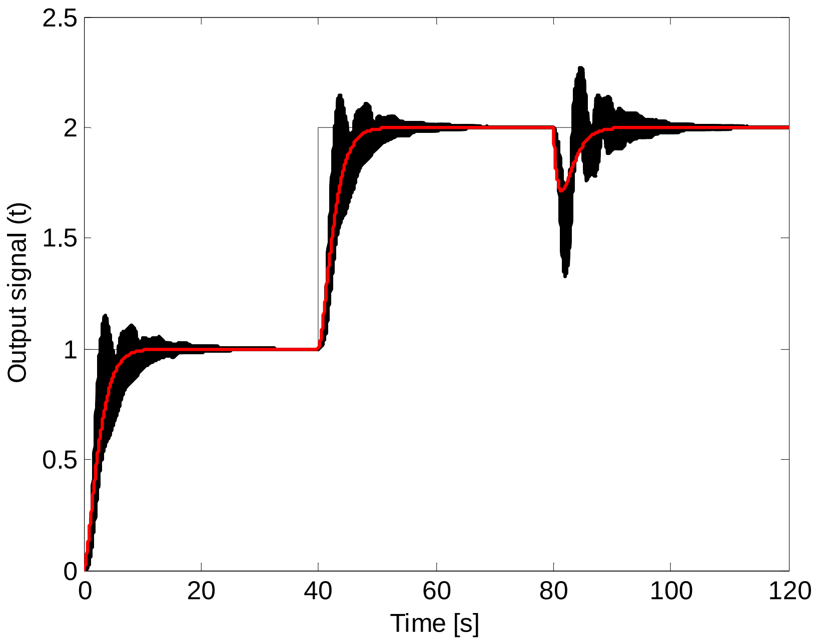

Fig. 4 The output signals of 2401 “representative” plants and the nominal system

6 CONCLUSION

The contribution has presented a robust PI controller design technique based on combination of plotting the stability boundary locus for robust stabilization and the algebraic methodology for the final choice of a regulator. The third order interval plant has been successfully controlled in the illustrative example.

Acknowledgment

The work was supported by the Ministry of Education, Youth and Sports of the Czech Republic under Research Plan No. MSM 7088352102. This assistance is very gratefully acknowledged. A previous version of this paper has been presented at the XXXII. Seminar ASR '2009 "Instruments and Control" [7].

REFERENCES

[1] BARMISH, B. R. *New tools for robustness of linear systems*. New York, USA: Macmillan, 1994.

[2] BARMISH, B. R., HOLLOT, C. V., KRAUS, F. J. & TEMPO, R. Extreme point results for robust stabilization of interval plants with first order compensators. *IEEE Transactions on Automatic Control*. 1992, Vol. 37, No. 6, pp. 707-714.

[3] KHARITONOV, V. L. Asymptotic stability of an equilibrium position of a family of systems of linear differential equations. *Differentsial'nye Uravneniya*. 1978, Vol. 14, pp. 2086-2088.

[4] KUČERA, V. Diophantine equations in control - a survey. *Automatica*. 1993, Vol. 29, No. 6, pp. 1361-1375.

[5] MATUŠŮ, R. & PROKOP, R. Single-parameter tuning of PI controllers: from theory to practice. In *Proceedings of the 17th IFAC World Congress*. Seoul, Korea: 2008.

[6] MATUŠŮ, R. Robust stabilization of interval plants using PI controllers. In *Annals of DAAAM for 2008 & Proceedings of the 19th International DAAAM Symposium*. Trnava, Slovakia: 2008.

[7] MATUŠŮ, R. Robust tuning of PI controllers for interval plants. In *Proceedings of the XXXII. Seminar ASR '2009 "Instruments and Control"*. Ostrava, Czech Republic: 2009.

[8] PROKOP, R. & CORRIOU, J. P. Design and analysis of simple robust controllers. *International Journal of Control*. 1997, Vol. 66, No. 6, pp. 905-921.

[9] SÖYLEMEZ, M. T., MUNRO, N. & BAKI, H. Fast calculation of stabilizing PID controllers. *Automatica*. 2003, Vol. 39, No. 1, pp. 121-126.

[10] TAN, N. & KAYA, I. Computation of stabilizing PI controllers for interval systems. In *Proceedings of the 11th Mediterranean Conference on Control and Automation*. Rhodes, Greece: 2003.

[11] TAN, N., KAYA, I., YEROGLU, C. & ATHERTON, D. P. Computation of stabilizing PI and PID controllers using the stability boundary locus. *Energy Conversion and Management*. 2006, Vol. 47, No. 18-19, pp. 3045-3058.

[12] VIDYASAGAR, M. *Control system synthesis: a factorization approach*. Cambridge, M.A.,USA: MIT Press, 1985.

[13] VÍTEČKOVÁ, M. Simple PI and PID controller tuning. *Transactions of the VŠB – Technical University of Ostrava, Mechanical Series*. 2006, Vol. 52, No. 2, pp. 225-230.

[14] VÍTEČKOVÁ, M. & VÍTEČEK, A. Conventional controller design. In *Proceedings of 8th International Carpathian Control Conference*. Štrbské Pleso, Slovakia: 2007.

[15] WAGNEROVÁ, R. Robust control design for hot-air aggregate. In *Proceedings of 6th International Carpathian Control Conference*. Miskolc-Lillafüred, Hungary: 2005.

[16] WAGNEROVÁ, R. Robust control verification. *Transactions of the VŠB – Technical University of Ostrava, Mechanical Series*. 2006, Vol. 52, No. 2, pp. 231-236.