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EFFECT OF UNCERTAINTY OF THE SUPPORT STIFFNESS, DAMPING AND UNBALANCE EXCITATION ON DYNAMICAL PROPERTIES OF THE TURBINE ROTOR OF AN AIRCRAFT ENGINE

VLIV NEURČITOSTI TUHOSTI ULOŽENÍ, TLUMENÍ A BUZENÍ NEVÝVAHOU NA DYNAMICKÉ VLASTNOSTI TURBÍNOVÉHO ROTORU LETECKÉHO MOTORU

Abstract

The behavior of rotor systems is influenced by a number of parameters whose values are not exactly known or which depend on operating conditions and can change during the rotor running. The presented article deals with analysis of a free turbine rotor of an aircraft engine taking into account the uncertainties of its supports stiffness, damping and unbalance excitation. The investigated rotor consists of two shafts mutually connected by a claw coupling and of two bladed wheels. The rotor is mounted with the stationary part by three rolling-element bearings. The dynamic properties of the rotor are evaluated using the Campbell diagram and frequency response characteristics for the defined range of values of the rotor angular speed.

Abstrakt

Chování rotorových soustav je ovlivněno řadou parametrů, jejichž hodnoty nejsou přesně známy nebo závisejí na provozních podmínkách, které se mohou při chodu rotoru měnit. Předložený článek se zabývá analýzou rotoru volné turbíny leteckého motoru s uvážením vlivu neurčitostí tuhosti jeho uložení, tlumení a buzení nevývahou na jeho dynamické vlastnosti. Zkoumaný rotor se skládá ze dvou hřídelů vzájemně spojených zubovou spojkou a dvou olopatkovaných kol. Se stacionární částí je rotor spojen pomocí tří valivých ložisek. Dynamické vlastnosti rotoru jsou vyhodnocovány pomocí Campbellova diagramu a amplitudo frekvenčních charakteristik stanovených pro požadovaný rozsah hodnot úhlové rychlosti otáčení rotoru.

1 INTRODUCTION

The rotors of aircraft engines belong to extremely stressed machine parts. Due to manufacturing and assembling inaccuracies, they are always slightly imbalanced. This produces their lateral vibration and forces that are transmitted through the coupling elements into the rotor casing and strongly affect the service life of all rotor components.

Lateral vibration of rotors is influenced by a number of parameters, whose values are not exactly known or which depend on current operating conditions and can change during the rotor running. The coefficients of internal and external damping, position and magnitude of the rotor unbalances, stiffness of the rotor supports and loading by external forces are among them. This implies the dynamical properties of rotors have an uncertain character and are described by quantities whose magnitudes can acquire values from a certain interval.

There are several approaches developed for analysis of systems with uncertain input parameters. They include "the worst-case scenario" method, probabilistic methods, fuzzy set theory or variant approach using computational simulations.

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The fuzzy set methods use the probability distribution to represent the uncertainties of values of physical or geometrical quantities and are based on application of interval mathematics. The formulation of their principal ideas can be found e.g. in [1]. Kulpa et al. [2] focused their attention on analysis of linear systems concentrating on trusses and frames. Length of the bars and their loading were the quantities possessing uncertain values. Sága et al. applied the fuzzy set methods for investigation of demage prediction of materials [3], whose properties were described by uncertain course of the Woehler curve, or for analysis of randomly excited railway vehicles [4]. The fuzzy sets theory was extended to the field of finite elements. Some applications from this area can be found in [5], [6]. The experience shows that the range of uncertain values of the results obtained by application of interval mathematics is rather overestimated. For the correction several approaches have been suggested. One of them consists in application of a Monte Carlo method [7].

In the presented paper this problem is treated by means of repeated computational simulations for different combinations of values of uncertain input parameters. This requires to apply a simple model to minimize the computational costs. Advantage of this procedure is that sensitivity of the rotor response on individual factors that are the cause of its uncertain behaviors can be easily studied.

2 THE INVESTIGATED ROTOR AND FORMULATION OF THE PROBLEM

The investigated aircraft engine rotor (Fig.1) consists of a long hollow shaft and of two bladed wheels attached to its overhanging end. The shaft is composed of two parts mutually connected by a claw coupling. The rotor is mounted with its casing by three rolling-element bearings. All components of the rotor, including the wheels and their blades, are made of steel. The rotor is loaded by its weight and by centrifugal forces caused by unbalances of the wheels.

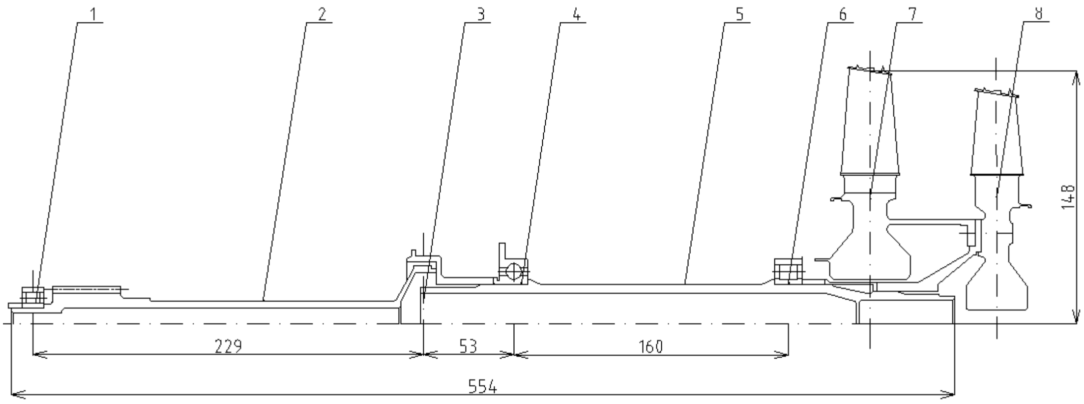

Fig. 1. Scheme of the aircraft engine turbine rotor: *1* – Bearing **A**, 2 – shaft **A**, 3 – claw coupling, 4 – bearing **B**, 5 – shaft **B**, 6 – bearing **C**, 7 - bladed-wheel **B**, 8 – bladed-wheel **A**.

The technological problem was to analyze dynamical properties of the rotor taking into account the uncertain values of the internal and external damping, position of the wheels unbalances and of the rotor supports stiffness. For its solution a computer modelling method was chosen.

In the computational model the shaft is represented by a beam-like body that is discretized into shaft finite elements based on the Euler beam theory. The stiffness of the shaft is considered as linear. The wheels are represented by axisymmetric absolutely rigid bodies attached to the shaft. The bearings and the bearing housings are implemented into the computational model by linear and isotropic springs and damping elements. The rotor is loaded by its weight and is excited by centrifugal forces and their moments produced by the wheels unbalances. The claw coupling is modelled by a spherical constraint.

Material of the rotor is considered as linear and isotropic (Table 1) and in the computational model it is represented by Kelvin-Voight rheological material. External damping is estimated to be of the Rayleigh type.

Tab. 1 Material properties

Young's modulus	Poisson ratio	Material density
210000 MPa	0.3	7800 kg/m^3

Lateral oscillation of the rotor is governed by the equation of motion that in the stationary coordinate system takes the form [8-10]

$$\mathbf{M}\,\ddot{\mathbf{x}} + \left(\mathbf{B} + \eta_V\,\mathbf{K}_{\mathrm{SH}} + \omega\,\mathbf{G}\right)\dot{\mathbf{x}} + \left(\mathbf{K} + \omega\,\mathbf{K}_{\mathrm{C}}\right)\mathbf{x} = \mathbf{f}\ . \tag{1}$$

$\mathbf{M}$, $\mathbf{B}$, $\mathbf{K}$, $\mathbf{G}$, $\mathbf{K}_\mathrm{C}$ are the mass, external damping, stiffness, gyroscopic and circulation matrices of the rotor system, $\mathbf{K}_{\mathrm{SH}}$ is the stiffness matrix of the shaft, $\mathbf{f}$ is the vector of generalized forces, $\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}$ are the vectors of generalized displacements, velocities and accelerations, η_V is the coefficient of the shaft internal damping and ω denotes the angular velocity of the rotor rotation.

As external damping is assumed to be of the Rayleigh type, matrix $\mathbf{B}$ is expressed

$$\mathbf{B} = \alpha\,\mathbf{M} + \beta\,\mathbf{K}\ . \tag{2}$$

α and β are the Rayleigh damping coefficients.

Determination of the rotor natural frequencies and normal modes arrives at solving a quadratic eigenvalue problem [11]

$$\begin{pmatrix} -\mathbf{M} & 0 \\ 0 & \mathbf{K} + \omega\mathbf{K}_C \end{pmatrix}\begin{pmatrix} \lambda\,\mathbf{v} \\ \mathbf{v} \end{pmatrix} = \lambda\begin{pmatrix} 0 & \mathbf{M} \\ \mathbf{M} & \mathbf{B} + \eta_v\mathbf{K}_{SH} + \omega\cdot\mathbf{G} \end{pmatrix}\begin{pmatrix} \lambda\,\mathbf{v} \\ \mathbf{v} \end{pmatrix}. \tag{3}$$

λ denotes the eigenvalue and $\mathbf{v}$ is the corresponding normal mode.

As the rotor turns at constant angular speed and is loaded by its weight and unbalances of the wheels, the vector of generalized forces $\mathbf{f}$ is a harmonic function of time

$$\mathbf{f} = \mathbf{f}_0 + \mathbf{f}_C \cos\omega t + \mathbf{f}_S \sin\omega t\ . \tag{4}$$

$\mathbf{f}_0$, $\mathbf{f}_C$, $\mathbf{f}_S$ are the coefficient vectors and t is time. Then the steady state solution of the rotor equation of motion can be estimated in the form

$$\mathbf{x} = \mathbf{x}_0 + \mathbf{x}_C \cos\omega t + \mathbf{x}_S \sin\omega t \tag{5}$$

where $\mathbf{x}_0$, $\mathbf{x}_C$, $\mathbf{x}_S$ are the coefficient vectors. Their calculation leads to solving a set of linear algebraic equations [11]

$$\begin{bmatrix} \mathbf{K} & 0 & 0 \\ 0 & \mathbf{K} + \omega\mathbf{K}_C - \omega^2\mathbf{M} & \omega\left(\mathbf{B} + \eta_V\,\mathbf{K}_{\mathrm{SH}} + \omega\,\mathbf{G}\right) \\ 0 & -\omega\left(\mathbf{B} + \eta_V\,\mathbf{K}_{\mathrm{SH}} + \omega\,\mathbf{G}\right) & \mathbf{K} + \omega\,\mathbf{K}_C - \omega^2\mathbf{M} \end{bmatrix}\begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_C \\ \mathbf{x}_S \end{bmatrix} = \begin{bmatrix} \mathbf{f}_0 \\ \mathbf{f}_C \\ \mathbf{f}_S \end{bmatrix}. \tag{6}$$

3 RESULTS OF THE COMPUTATIONAL SIMULATIONS

The analysis is focused on investigation of the influence of external and material damping, stiffness of the shaft supports and position of the wheels unbalances on dynamical properties of the aircraft turbine rotor. The maximum speed of its rotation is 2500 rad/s. Eccentricity of the centre of gravity of both wheels is estimated to be 50 μm. On previous experience uncertain values of the mentioned parameters are defined by their lower and upper limits expressed by intervals given in Table 2.

Tab. 2 Uncertain values of input parameters

Rayleigh damping parameter α [s^{-1}]	5 - 10
Coefficient of viscous material damping [s]	1.2 .10^{-10} - 1.2 .10^{-6}
Stiffness of the rotor support - bearing A [MN.m^{-1}]	322.64 - 403.3
Stiffness of the rotor support - bearing B [MN.m^{-1}]	63.52 - 79.4
Stiffness of the rotor support - bearing C [MN.m^{-1}]	49.36 - 61.7
Relative position of the wheel unbalances [°]	0° - 360°

Fig. 2 shows the Campbell diagrams calculated for damping coefficients $\alpha = 5\ s^{-1}$, $\beta = 0$ s, $h_V = 1.2\ .10^{-6}$ s and for three magnitudes of the rotor supports stiffness. Figure a) is related to the reference stiffness values (support A 403.3 MN/m, support B 79.4 MN/m, support C 61.7 MN/m), figures b) and c) to the cases when the stiffness is reduced by 10% and 20% respectively.

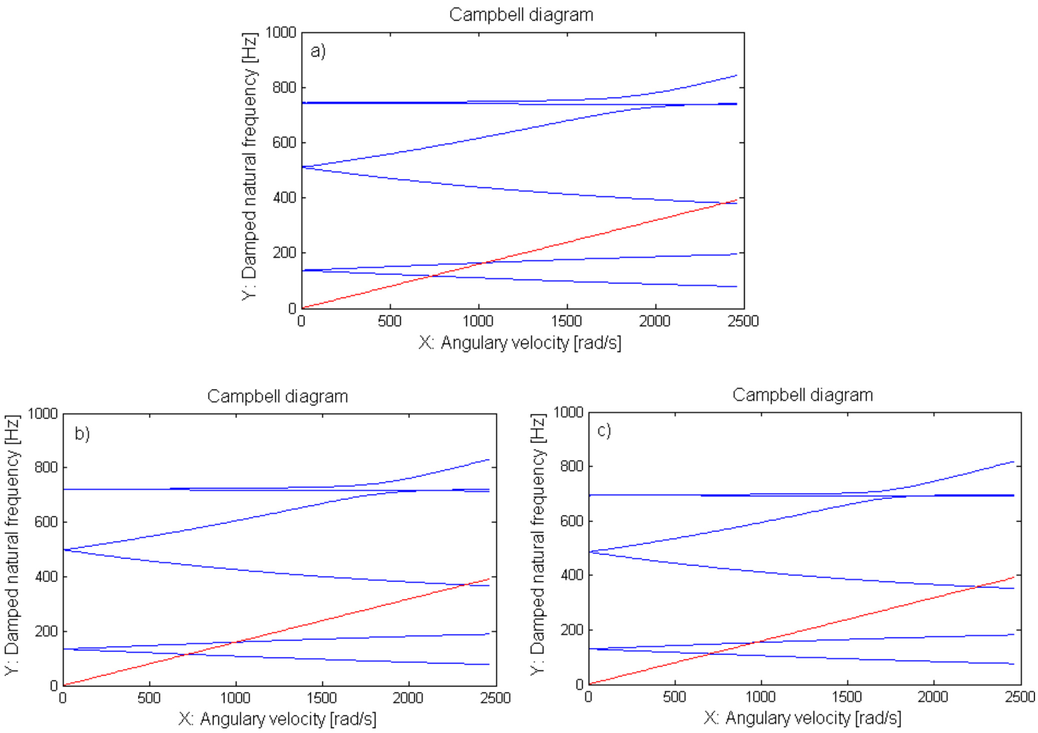

Fig. 2 Effect of stiffness support on Campbell diagrams

As evident from Fig. 3, both the internal and external damping have only little influence on the dependance of the damped natural frequencies on the speed of the rotor rotation and thus on magnitudes of the critical revolutions. The calculations were performed for reference values of the rotor supports stiffness and magnitudes of the damping coefficients given in Table 2.

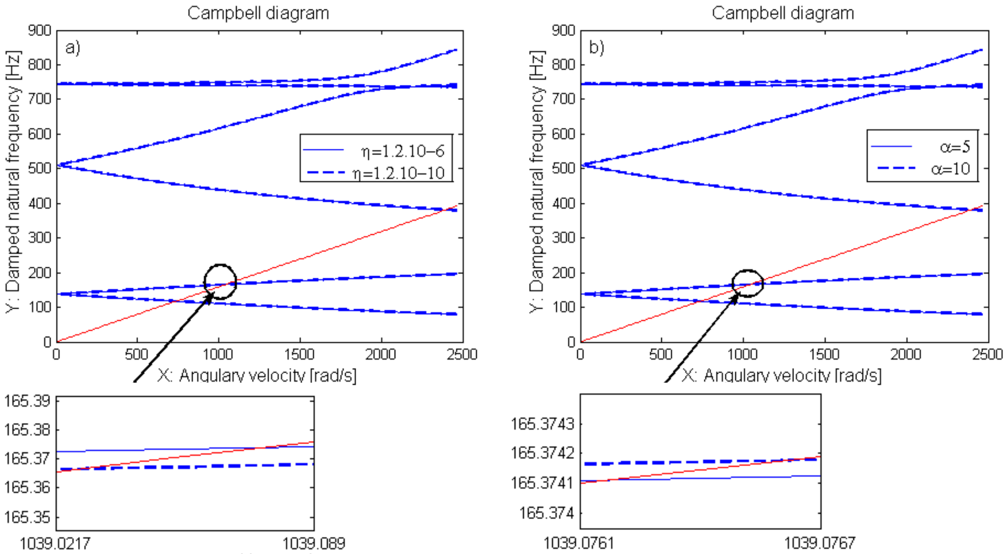

Fig. 3 Effect of external and material damping on Campbell diagrams

The results show that there are three critical speeds in the range of the operating revolutions of the rotor. One is related to forward, two critical speeds to the backward whirl modes (Table 3) As the loading caused by the unbalance is synchronous with the rotor rotation and the rotor supports are isotropic, only the forward mode can be excited. Values of the critical speeds go down with decreasing magnitude of the rotor supports stiffness.

Tab. 3 Critical speeds

	reference stiffness values	stiffness reduced by 10 %	stiffness reduced by 20 %
1. backward whirl frequency [rad/s]	738	725	711
1. forward whirl frequency [rad/s]	1039	1012	971
2. backward whirl frequency [rad/s]	2393	2324	2256

The forward normal mode related to the critical speed of 1039 rad/s is drawn in Fig. 4. It is evident that at location of the claw coupling the centre line of the shaft is continuous but is not smooth.

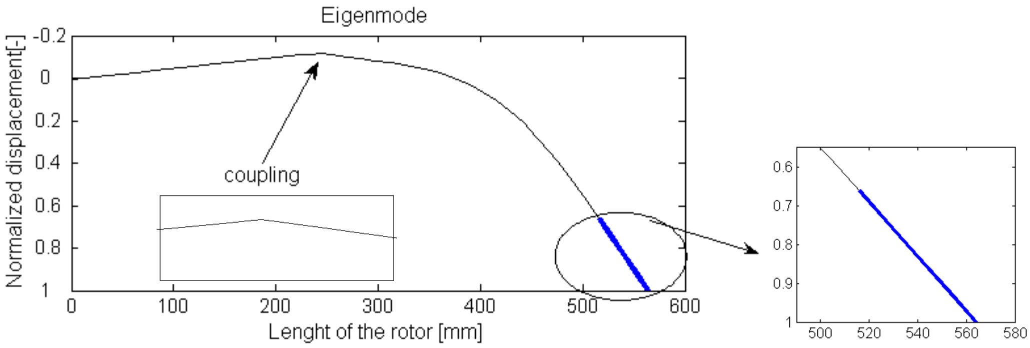

Fig. 4. The first normalized normal mode of the rotor (forward whirl)

In Fig. 5a there is drawn a frequency response characteristic referred to the centre of wheel A. The analysis was performed for the damping coefficients $\alpha = 5$ s^{-1}, $\beta = 0$ s, $h_V = 1.2 \cdot 10^{-6}$ s and for three magnitudes of the rotor supports stiffness, for the reference values (k_{REF}) and for the values reduced by 10% and 20%. The orbits in Fig. 5b correspond to the resonance speed. As evident, reduction of the supports stiffness arrives at decrease of the critical revolutions of the rotor and at reducing the resonance amplitude. The results also confirm that only the forward whirl mode is excited. Fig. 6 shows the dependence of the frequency response characteristic and of the orbits of the wheel A centre, which are related to the critical revolutions, on external damping. Rising magnitude of the Rayleigh coefficient α leads to nonlinear decrease of the resonance peak. The influence of external damping on the value of the rotor critical revolutions is negligible. In Fig. 7 there is drawn the response curve related to the centre of wheel A for three magnitudes of the mutual angular shift of unbalances of wheels A and B. The maximum amplitude of the induced vibration occurs if the shift is zero, minimum if the shift is equal to 180°.

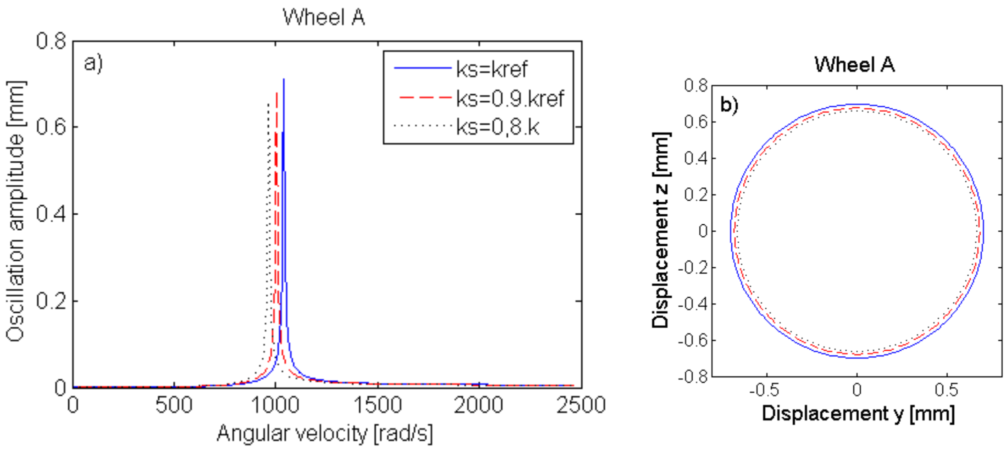

Fig. 5. Effect of uncertainties of the support stiffness
Frequency response characteristic, b) Orbits referred to the critical speed

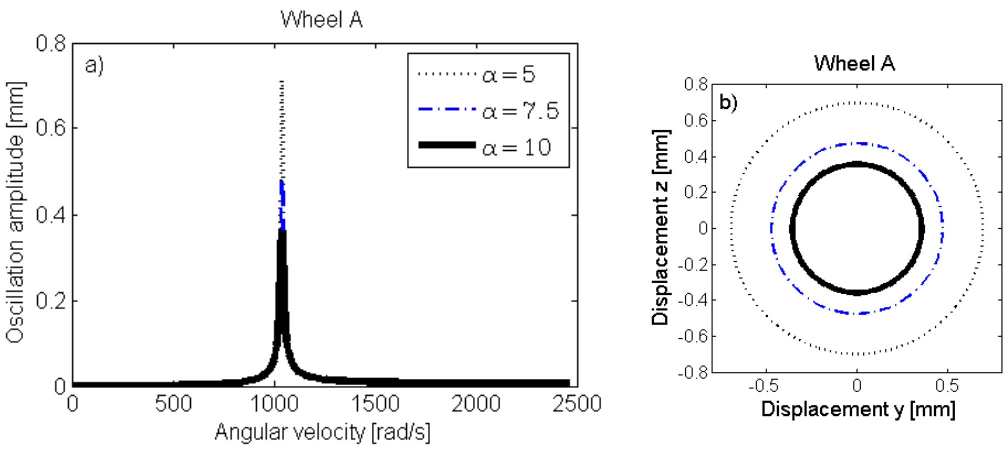

Fig. 6. Effect of uncertainties of external damping
a) Frequency response characteristic, b) Orbits referred to the critical speed

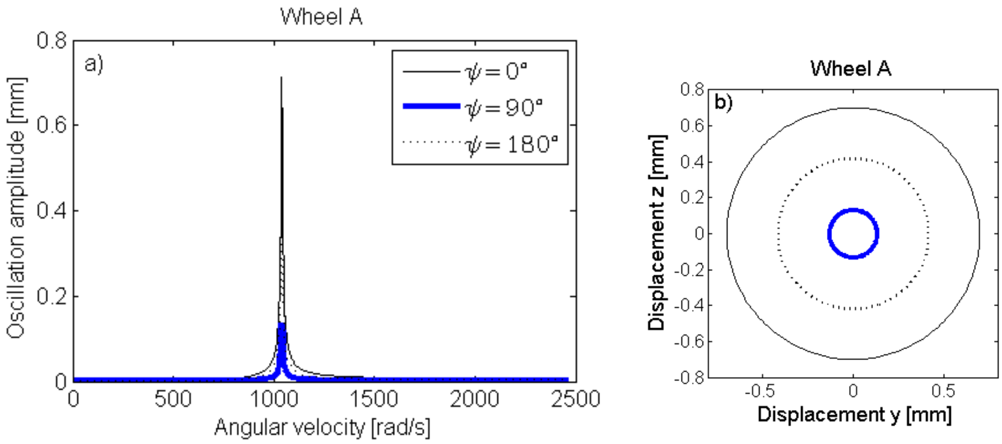

Fig. 7. Effect of uncertainties of the relative angular positions of unbalances of wheels A and B
a) Frequency response characteristic b) Orbits referred to the critical speed

The resonance characteristics related to the centre of wheel A have been calculated for all combinations of the parameters, which have uncertain values. Their lower and upper envelopes (Fig. 8) determine the range of probable amplitudes of the wheel steady state vibration. This enables to specify more accurately the speed interval, in which amplitude of the oscillations exceeds the allowable value.

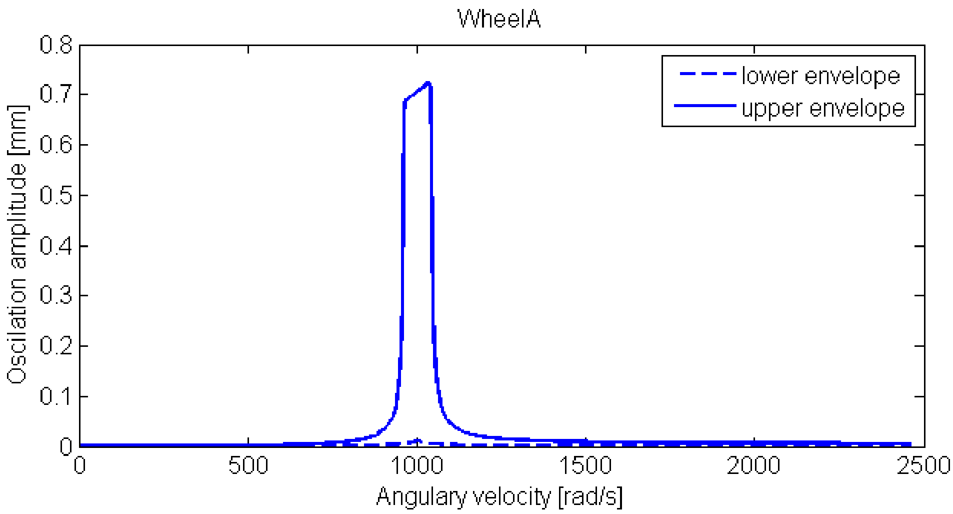

Fig. 8. Lower and upper envelopes of the frequency response characteristic

4 CONCLUSION

The objective of the carried out analysis was to investigate dependence of dynamical properties of the aircraft engine turbine rotor on uncertain values of its supports stiffness, amount of external and material damping and on mutual position of unbalances of its bladed wheels. To solve this problem, the variant approach based on computer simulations was applied.

From the Campbell diagrams it is evident that the critical speeds are affected especially by the rotor supports stiffness. The influence of external and internal damping on the rotor natural frequencies is negligible.

The frequency response characteristic is influenced by all investigated parameters. Position of the resonance peak is shifted to lower values with decreasing stiffness of the supports. As the rotor exhibits a whirling vibration, the material damping of the shaft does not affect its amplitude but rising external damping leads to reducing magnitude of the rotor oscillations. Results of the simulations show that the mutual position of unbalances of both bladed wheels affects amplitude of the vibration especially for the rotor velocities close to the resonance.

Combination of influences of all investigated quantities gives the interval of probable values of the rotor critical speed (970 and 1040 rad/s) and provides the lower and upper limits of the course of its frequency response characteristic.

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