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EVALUATION OF THE COMPUTATIONAL SIMULATIONS OF NONLINEAR VIBRATION OF ROTORS SUPPORTED BY HYDRODYNAMIC BEARINGS

VYHODNOCENÍ VÝPOČTOVÝCH SIMULACÍ NELINEÁRNÍHO KMITÁNÍ ROTORŮ ULOŽENÝCH V HYDRODYNAMICKÝCH LOŽISKÁCH

Abstract

The article deals with the influence of hydrodynamic bearings on the response of rotor systems excited by their imbalance. In the developed computational model the bearings are considered as cylindrical, short and cavitated (π-film). The hydraulic forces are determined by integration of the pressure distribution in the oil film which is described by the Reynolds equation. To solve the equation of motion a Runge-Kutta method of the 4th order is applied. To evaluate character of the rotor vibration forms of orbits, phase trajectories, steady-state time series of kinematic parameters of the centres of the bearings and the discs and their frequency spectra, Poincaré maps and bifurcation diagrams are used. The results of the carried out computational simulations show that due to nonlinear stiffness and damping parameters of the hydrodynamic bearings the imbalance force can excite not only periodic but also quasi-periodic or even chaotic oscillations and that their character and stability depends on the speed of the rotor rotation. The simulations also confirm that only a combination of several tools enables to evaluate the character of vibration of complex rotor systems reliably.

Abstrakt

Článek se zabývá vlivem hydrodynamických ložisek na odezvu rotorových soustav buzených nevývahou rotujících částí. Ve vypracovaném výpočtovém modelu jsou hydrodynamická ložiska považována za válcová, krátká a kavitovaná (π-filmová). Hydraulické síly se počítají integrací tlakového rozložení popsaného Reynoldsovou rovnicí. K řešení pohybové rovnice je použita metoda Runge-Kutta 4. řádu. Charakter kmitání rotoru se posuzuje podle řady matematicko fyzikálních parametrů jako jsou tvary orbitů, fázové trajektorie, časové průběhy kinematických parametrů středů ložisek a kotoučů v ustáleném stavu, jejich frekvenční spektra, Poincarého mapy a bifurkační diagramy. Výsledky provedených počítačových simulací ukázaly, že v důsledku nelineárních tuhostí a tlumení hydrodynamických ložisek může nevývaha rotoru vybudit nejenom periodické, ale i kvaziperiodické nebo i chaotické kmitání, a že jeho charakter a stabilita závisí na rychlosti otáčení rotoru. Výpočty rovněž potvrdily, že charakter kmitání složitých rotorových soustav může být spolehlivě posouzen pouze na základě vyhodnocení více parametrů.

1 INTRODUCTION

The chaotic oscillations and bifurcations can be observed at a number of engineering systems having different physical substance (mechanical, hydromechanical, electrical, etc.) [1], [2]. Such behaviour is marked for high irregularity and difficult predictability. Formally it was associated with random phenomena but the further studies showed that the chaotic behaviour had a completely deterministic essence and that it was a characteristic feature of many nonlinear systems [3-5].

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The hydrodynamic journal bearings are often used to support rotors due to their efficient damping, high load capacity, long service life and low power losses. But also, as their damping and stiffness parameters are nonlinear, they can become a source of vibration of large amplitude or vibration that is irregular and hardly predictable. This follows from both the theoretical analyses and computational simulations and it has been confirmed by observations carried out on real technological systems. Therefore it is very important to identify the operating conditions at which such vibration can occur.

To be possible to specify the undesirable ranges of the rotor revolutions it is needed to develop the methods for reliable evaluation of the character of the vibration. In 1978, Holmes et al. [6] published a paper dealing with aperiodic oscillations of a rotor supported by journal bearings. In [7] Ehrich studied the dynamical behaviour of rotors mounted with the stationary part by nonlinear coupling elements. He identified the bifurcations and occurrence of the subharmonic components in their vibration. Zhao et al. [8] investigated an eccentric squeeze film damper-mounted rigid rotor and discussed its subharmonic and quasi-periodic motion produced by large magnitude of the rotor unbalance and static misalignment. The dynamical behaviour of a symmetric rigid rotor supported by plain hydrodynamic bearings was studied in [9], [10] focusing a particular attention on its nonlinear aspects. The radial and tangential components of the hydraulic bearing forces were predicted by application of a π-film, short bearing model. The vibration was induced by static and couple unbalances. The subharmonic, quasi-periodic and chaotic motion of a rigid rotor supported by short journal bearing was also observed during the theoretical and experimental investigations carried out by Adiletta et al. [11, 12].

In this work, the effect of the angular speed of the rotor rotation on the character of its vibration is studied. The investigated rotor is supported by two hydrodynamic bearings and is excited by centrifugal forces caused by imbalance of the rotating parts. The hydraulic forces are determined by application of the Reynolds equation. For solution of the equation of motion a Runge-Kutta method of the 4th order with Dormand-Prince modification is used. Forms of orbits, time series, phase trajectories, frequency spectra, Poincaré maps and bifurcation diagrams are the tools utilized for analysis of the character of the steady-state vibration of the studied rotor system. The occurrence of periodic, quasi-periodic and chaotic oscillations was found out.

2 THE MOTION EQUATION OF THE ROTOR SYSTEM

In the developed mathematical model (i) the shaft is represented by a beam-like body (Bernoulli-Euler beam theory) that is discretized into finite elements, (ii) the stationary part is considered to be absolutely rigid, (iii) inertia and gyroscopic effects of the rotating parts are taken into account, (iv) the discs are assumed to be absolutely rigid axisymmetric bodies, (v) the external and material damping of the shaft is linear, (vi) the rotor is supported by hydrodynamic bearings that are represented in the mathematical model by force couplings, (vii) the rotor turns at constant angular speed and (viii) the rotor is loaded by its weight and by forces of periodic time histories.

Lateral vibration of the rotor system is described by the equation of motion that in the fixed frame of reference takes the form

$$\mathbf{M}\,\ddot{\mathbf{q}} + (\mathbf{B} + h_{\mathrm{V}}\,\mathbf{K}_{\mathrm{SH}} + \omega\,\mathbf{G})\,\dot{\mathbf{q}} + (\mathbf{K} + \omega\,\mathbf{K}_{\mathrm{C}})\,\mathbf{q} = \mathbf{f}_{\mathrm{A}} + \mathbf{f}_{\mathrm{B}}(\mathbf{q}, \dot{\mathbf{q}}, t) + \mathbf{f}_{\mathrm{C}}, \tag{1}$$

M, **B**, **K**, **G**, $\mathbf{K}_{\mathrm{C}}$ are the mass, damping, stiffness, gyroscopic and circulation matrices of the rotor system, $\mathbf{K}_{\mathrm{SH}}$ is the stiffness matrix of the shaft, $\mathbf{f}_{\mathrm{A}}$, $\mathbf{f}_{\mathrm{C}}$, $\mathbf{f}_{\mathrm{B}}$ are the vectors of applied (including the unbalance), constraint and bearing forces, $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ are the vectors of general displacements, velocities and accelerations, h_{V} is the coefficient of viscous damping of the shaft material, ω denotes the angular speed of the rotor rotation and t is the time. Matrix **B** defines external damping of the shaft and of the stationary part (e.g. damping of a Rayleigh type).

3 THE HYDRAULIC BEARINGS FORCES

The scheme of a hydrodynamic journal bearing is illustrated in Fig. 1. Position of the rotor journal towards the bearing sleeve can be described in the stationary coordinate system O_Bxyz or in the rotor-fixed frame of reference O_Bxrt whose r-axis goes through the bearing sleeve and the shaft journal centres O_B, O_J respectively.

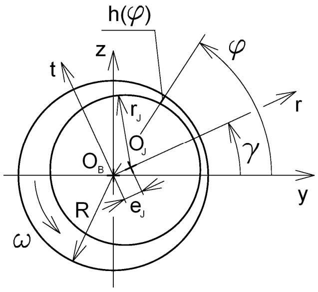


Fig. 1 Coordinate frames of the journal bearing

The thickness of the oil-film h is defined by the following relations

$$h(\varphi) = \delta - e_J \cos(\varphi - \gamma)\,, \quad \delta = R - r_J\,, \quad e_J = \sqrt{(z_J - z_B)^2 + (y_J - y_B)^2}\,, \tag{2}$$

where e_J is the shaft eccentricity, R is the radius of the bearing sleeve, r_J is the radius of the shaft journal, y_B, z_B and y_J, z_J are displacements of the bearing sleeve and shaft journal centres in the y and z directions respectively, γ denotes the position angle of the line of centres, δ is the bearing radial clearance and φ is the circumferential coordinate.

The radial and tangential components of the hydraulic bearing force (f_r, f_t respectively) are obtained by integration of the pressure distribution

$$f_r = -R\int_0^L \int_\pi^{2\pi} p_d \cos\varphi \; \mathrm{d}\varphi \; \mathrm{d}x\,, \quad f_t = -R\int_0^L \int_\pi^{2\pi} p_d \sin\varphi \; \mathrm{d}\varphi \; \mathrm{d}x\,. \tag{3}$$

L is the length of the bearing, p_d is the pressure distribution in the layer of lubricant and x is the axial coordinate.

The pressure distribution in the oil-film is determined by solving the Reynolds equation [13]. In cavitated areas the pressure of the medium is assumed to be constant and equal to the pressure in the ambient space. For the case of a short cavitated π-film bearing the horizontal and vertical components of the bearing force f_y, f_z expressed in the stationary coordinate system can be obtained in a close form

$$f_y = f_r \cos(\gamma) - f_t \sin(\gamma)\,, \quad f_z = f_r \sin(\gamma) + f_t \cos(\gamma)\,, \tag{4}$$

where [13]

$$f_r = \eta\, R\, L \left(\frac{L}{\delta}\right)^2 \left[(\omega - 2\dot{\gamma}) \frac{\varepsilon_J^2}{(1-\varepsilon_J^2)^2} + \frac{\pi\,(1+2\,\varepsilon_J^2)\,\dot{\varepsilon}_J}{2\,(1-\varepsilon_J^2)^{5/2}} \right],$$

$$f_t = -\eta\, R\, L \left(\frac{L}{\delta}\right)^2 \left[(\omega - 2\dot{\gamma}) \frac{\pi\,\varepsilon_J}{4\,(1-\varepsilon_J^2)^{3/2}} + \frac{2\,\varepsilon_J\,\dot{\varepsilon}_J}{(1-\varepsilon_J^2)^2} \right], \tag{5}$$

$$\cos(\gamma) = \frac{y_J - y_B}{e_J}\,, \quad \sin(\gamma) = \frac{z_J - z_B}{e_J}\,.$$

η in (5) means the dynamical viscosity of the lubricating oil. The relative eccentricity of the rotor journal ε_J and further kinematic parameters are defined by the following relationships

$$\varepsilon_J = \frac{e_J}{\delta}\,, \quad \dot{\varepsilon}_J = \frac{(\dot{y}_J - \dot{y}_B)\cos(\gamma) + (\dot{z}_J - \dot{z}_B)\sin(\gamma)}{\delta}\,, \quad \dot{\gamma} = \frac{-(\dot{y}_J - \dot{y}_B)\sin(\gamma) + (\dot{z}_J - \dot{z}_B)\cos(\gamma)}{\varepsilon_J}\,. \tag{6}$$

Dot (˙) denotes the first differentiation with respect to time.

4 THE METHODS FOR VIBRATION ANALYSIS OF ROTOR SYSTEMS

Character of the vibration of analyzed rotors is evaluated by the tools applicable for investigation of motion of nonlinear systems as forms of orbits, phase trajectories, steady-state time series of displacements and velocities components of the chosen points and of their frequency spectra, Poincaré maps, bifurcation diagrams, dimensions of the attractor, Lyapunov exponents and of other parameters, as given in [4], [14].

The periodic, quasi-periodic and chaotic vibration is characterized by several typical features:

- periodic motion - (i) the orbits of the points on the rotor centre line are closed curves, (ii) the frequency spectra of the time series of displacements and velocity components of the chosen points have a line character, (iii) the phase trajectories are closed curves, (iv) dimension of the attractor is equal to 1 and (v) the largest Lyapunov exponent is negative,
- quasi-periodic motion - (i) the orbits of the points on the rotor centre line are unclosed curves, (ii) the frequency spectra of the time series of displacements and velocity components of the chosen points have a line character with sidebands, (iii) the phase trajectories are unclosed and tend to fulfil the phase space between the two envelopes, (iv) dimension of the attractor is a real number greater than 1 and less or equal to 2 and (v) the largest Lyapunov exponent is equal to zero,
- chaotic motion - (i) the orbits of the points on the rotor centre line are unclosed curves, (ii) the phase trajectories are unclosed and tend to fulfil a certain part of the phase space, (iii) frequency spectra of the time series of displacements and velocity components of the chosen points have a broadband character, (iv) dimension of the attractor is greater than 2 and if it is of a non-integer value it indicates a fractal structure and (v) the largest Lyapunov exponent is positive.

5 RESULTS OF THE COMPUTATIONAL SIMULATIONS

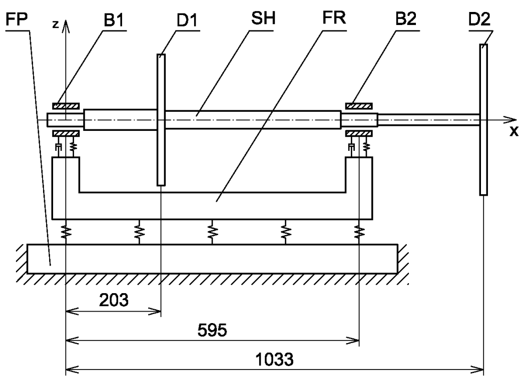


Fig. 2 Scheme of the investigated rotor system

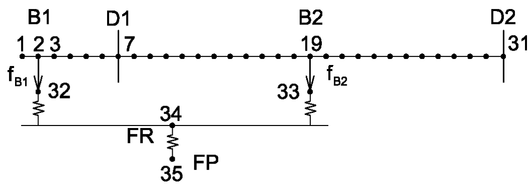


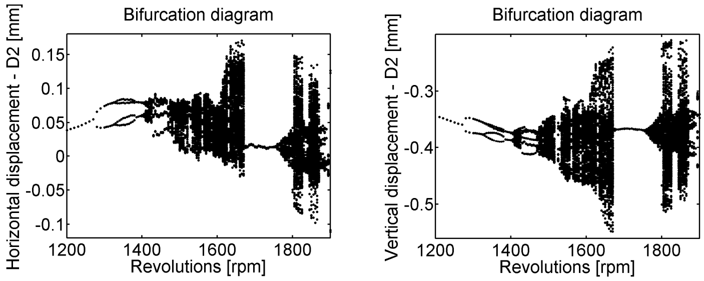
Fig. 3 Discretization of the rotor system

Rotor of the investigated rotor system (Fig. 2) consists of a shaft (SH) and of two discs (D1, D2) and is coupled with a rigid frame (FR) by two hydrodynamic journal bearings (B1, B2 – oil dynamical viscosity 0.0011 Pa·s, radius of the bearing sleeve 110.22 mm, radius of the shaft journal 110 mm and length of the bearing 35 mm). Disc D1 is located between the bearings and disc D2 is situated on the shaft overhanging end. The frame is coupled with a rigid foundation plate (FP) by a set of flexible elements.

The rotor system is loaded by its weight and is excited by unbalances of both discs (the discs D1, D2 eccentricities are 0.22 mm and 0.13 mm). The Rayleigh damping coefficients α, β and the coefficient of material viscous damping η_V are equal to 6 s^{-1}, 0 s and 2.0 10^{-7} s^{-1} respectively.

In the computational model the shaft was represented by a beam-like body that was discretized into thirty finite elements (Fig. 3) of equal length (35 mm). The bearings sleeves, the frame and the foundation plate were considered as absolutely rigid bodies having two degrees of freedom each. The hydrodynamic bearings are represented by nonlinear force couplings. The equation of motion of the discretized rotor system has 132 degrees of freedom.

Due to high nonlinearity of the bearing forces, the numerical analysis is carried out using the Runge-Kutta method of the 4th order with Dormand-Prince modification. In this study, the time step for the direct numerical integration is adaptive and the error tolerance is less than 1.10^{-6}. The dates of the time series corresponding to the first about 100 revolutions of the rotor are omitted for the following vibration analysis. The zero initial conditions were used in all simulations.

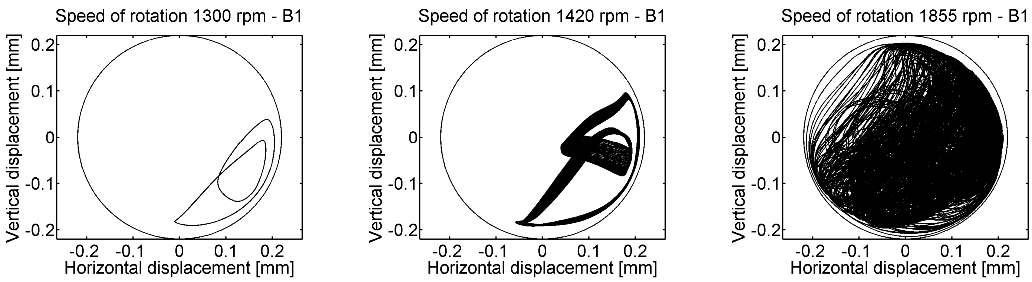


4(a) 4(b)

Fig. 4 Bifurcation diagram of the disc D2 centre at horizontal and vertical direction (Figures 4a-4b)

The speed of the rotor rotation is utilized as a control parameter to construct the bifurcation diagrams drawn in Fig. 4. Its magnitude changes in the range between 1200 rpm and 1895 rpm. It can be seen that at low rotational velocities (1200 rpm to 1270 rpm) the induced vibration is periodic and synchronous with the speed of the rotor rotation (period-one). For higher speeds the synchronous vibration bifurcates to period-two and period-four motions. In the interval of rotational velocities between 1410 rpm and 1670 rpm the vibration response changes between periodic, quasi-periodic and chaotic motions. After exceeding revolutions of about 1675 rpm the irregular (quasi-periodic or chaotic) vibration disappears and the motion becomes periodic and synchronous with the angular speed of the rotor rotation. The increase of the revolutions above 1755 rpm arrives at quasi-periodic and chaotic vibration. After exceeding the revolutions of 1875 rpm the vibration becomes again quasi-periodic and periodic synchronous with the rotor revolutions (one-periodic).

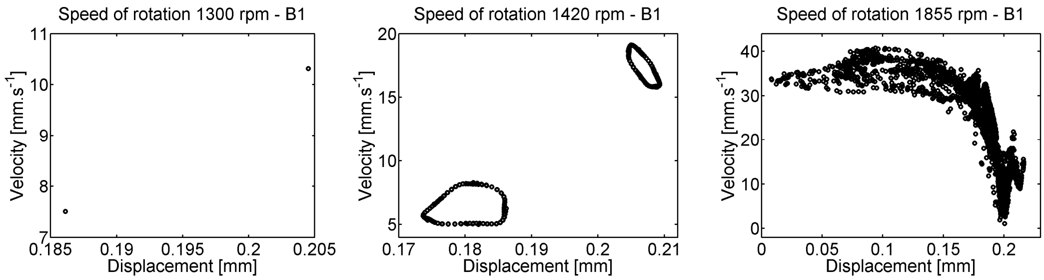
In Fig. 5 to 10 there are drawn the orbits, Poincaré maps, phase trajectories, frequency spectra and steady-state time series of displacements of the bearing B1 and disc D2 centres for several speeds of the rotor rotation. For the revolutions of 1300 rpm, 1420 rpm and 1855 rpm the motion is characterized by periodic, quasi-periodic and irregular (likely chaotic) vibration response. The periodic orbit is a closed curve (Fig. 5a) unlike the ones corresponding to the quasi-periodic (Fig. 5b) and chaotic (Fig. 5c) rotor vibration.



5(a) 5(b) 5(c)

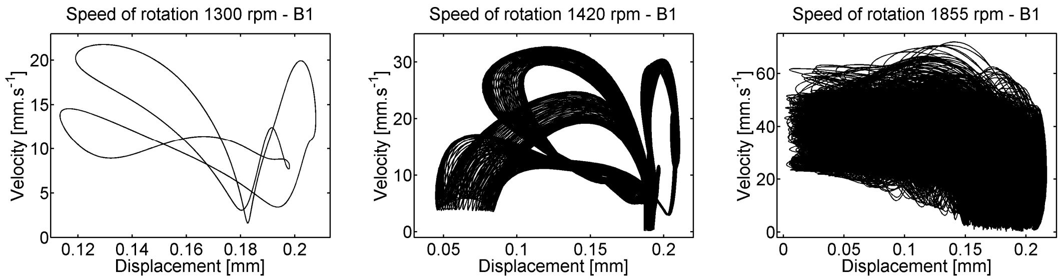
Fig. 5 The orbits of bearing B1 centre at different speed of rotation (Figures 5a-5c)

The Poincaré map related to regular vibration of the rotor turning at speed of 1300 rpm is formed by two separated points (Fig. 6a). At the speed of rotation of 1420 rpm the Poincaré map changes into two closed continuous curves (Fig. 6b) which correspond to the quasi-periodic oscillations. The Poincaré map related to the chaotic vibration that occurs at the speed of rotation of 1855 rpm is formed by a set of scattered points (Fig. 6c).



6(a) 6(b) 6(c)

Fig. 6 The Poincaré maps of bearing B1 centre at different speed of rotation (Figures 6a-6c)

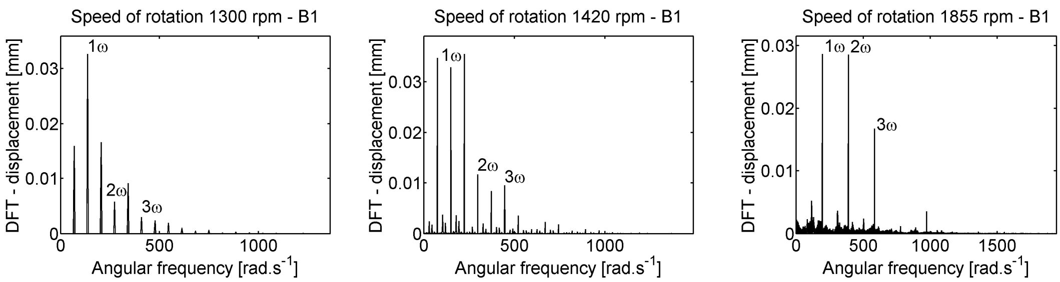


7(a) 7(b) 7(c)

Fig. 7 The steady-state phase trajectories of bearing B1 at different speed of rotation (Figures 7a-7c)

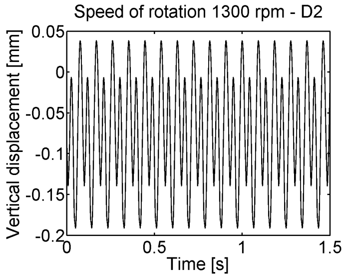
It is evident from Fig. 7 that the phase trajectories correspond well to the orbits drawn in Fig. 5. The phase trajectory (Fig. 7c) of the chaotic motion is unclosed and tends to fulfil a certain part of the phase space.

The discrete Fourier transforms (DFT) of the time series of displacements of the bearing B1 centre drawn in Fig. 8 show that the motion at the speed of 1300 rpm (Fig. 8a) is periodic with dominant harmonic components whose angular frequencies are multiples of the speed of the rotor rotation. In this case, the motion is period-two, because there are side bands around the rotational speed (1/2, 3/2, 5/2, …). The motion at 1420 rpm (Fig. 8b) is clearly quasi-periodic as there are many natural side bands occurring around the multiplies of the angular speed of the rotor rotation. The irregular chaotic motion occurring at the speed of 1855 rpm is characterized by a broadband frequency spectrum (Fig. 8c).

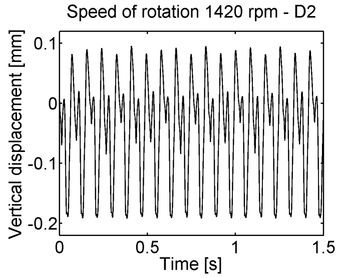


8(a) 8(b) 8(c)

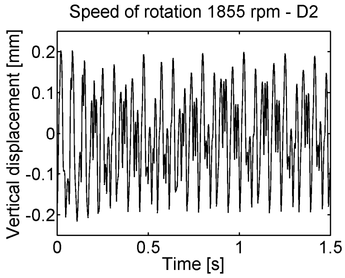
Fig. 8 Frequency spectrum of bearing B1 centre at different speed of rotation (Figures 8a-8c)



9(a)

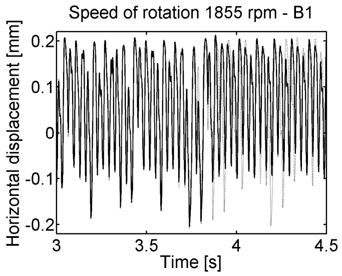


9(b)

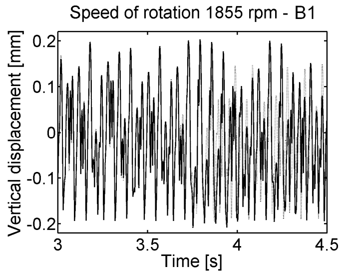


9(c)

Fig. 9 Time series in the steady-state of the disc D2 at different speed of rotation (Figures 9a-9c)



10(a)



10(b)

Fig. 10 Detail of the time series of the bearing B1 centre (Figures 10a-10b)

Fig. 9 shows the time series of the vertical displacements of disc D2 centre. Their time series for three speeds of the rotor revolutions correspond to the periodic (Fig. 9a), quasi-periodic (Fig. 9b) and chaotic (Fig. 9c) vibration of the rotor system.

Sensitivity of the chaotic motion to the change of the initial conditions can be seen in Fig. 10. At time of about 3.7 s the difference between two time series of the same horizontal and vertical displacement corresponding to close but different initial conditions can be observed.

6 CONCLUSIONS

In this article, there are presented results of investigations of a rotor coupled by two hydrodynamic bearings with a flexibly supported stationary part by means of computational simulations. The pressure distribution in the oil-film in the bearings was determined by means of the Reynolds equation. In cavitated areas it was assumed that the pressure remained constant and was equal to the pressure in the ambient space. To solve the equation of motion of the whole rotor system a Runge-Kutta method was applied. The forms of orbits, Poincaré maps, steady-state time series, phase trajectories, frequency spectra and bifurcation diagrams were the tools used for evaluation of the character of the rotor vibration in dependence on the speed of its rotation. The results of the carried out computer simulations show that in the investigated range of revolutions periodic, quasi-periodic and chaotic vibration of the rotor appears. Rising speed of the rotor rotation arrives at bifurcations of the rotor oscillations. The intervals of synchronous regular vibration change with the ones whose period increases (twice, four times) or with the ones when the oscillations become irregular. The obtained results enable to determine the ranges of the operation speed at which the work of the rotors is undesirable because of occurrence of their unpredictable motion (chaotic, quasi-periodic).

ACKNOWLEDGEMENTS

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