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THE GEOMETRIC AND MATERIAL NON-LINEARITY ON BENDED BEAM

GEOMETRICKÁ A MATERIÁLOVÁ NELINEARITA OHYBU NOSNÍKU

**Abstract**

Large deformations of steel reinforcements can often occur in mines and underground objects. Behavior of these reinforcements is constitutively and geometrically non-linear. There are needs for operative analysis of such reinforcements. Software for this task is being developed. The software uses the beam computation model. The paper discusses computational analysis of steel arch reinforcement with P-28 type of a cross-section. There were analyzed the 3D finite element models with use of the ANSYS software. The material has been defined as elastic-plastic with multi-linear hardening. The results have been used for creation of so-called substitute bending stiffness. The substitute bending stiffness is defined in relation to the function of the normal force and the bending moment of the beam model.

**Abstrakt**

V důlních a podzemních dílech dochází mnohdy k velkým deformacím ocelových obloukových výztuží. Je při nich překročena mez kluzu a výztuž se chová fyzikálně i geometricky nelineárně. Snaha o operativní analyzování ocelových výztuží vede k zpracování softwaru vyžadujícího znalosti ohybové tuhosti profilů používaných pro konstrukci ocelových obloukových výztuží dlouhých důlních a podzemních děl zejména při velkých deformacích. Práce se zabývá analýzou profilu P-28. Na základě modelů realizovaných v programovém systému ANSYS se sestrojila charakteristika tzv. náhradní ohybové tuhosti uvedeného profilu, která je funkcí vnitřních sil, tj. ohybového momentu a normálové síly. Tato charakteristika je jedním z podkladů pro analyzování kompletu výztuží při mimořádných zatíženích

## 1 INTRODUCTION

The paper is focused to the mine steel support bending. The subject relates to the miners safety. The steel support is exposed to the similar influences and very often the plastic strain appears. The friction connection of the single parts of the support has very good behavior under sudden impact of the rock. The bending load of the support is very important feature. The bending of the beam in plastic range is the subject of experimental testing and computational modeling.

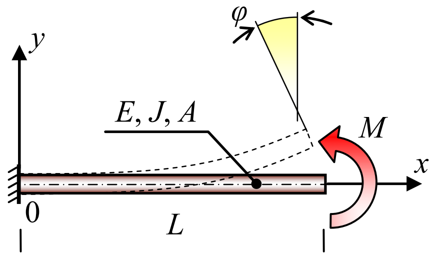


**Fig. 1** The straight beam exposed to bending moment

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## 2 THE NON-LINEAR BENDING OF THE STEEL ARCH SUPPORT

The idea of the operational analysis is based on the reduction of the 3D physical reality into the 1D beam element model with corresponding geometrical and physical properties. The large deformation of the beam cross section occurs under strong bending moment loading. This leads to the change of the cross section quadratic moment of inertia - the most important bending stiffness parameter. Besides the yield stress limit is reached and exceed and plastic material behavior must be taken into account.

Suppose the straight beam of the length L, cross sectional area A, quadratic moment of inertia J, from material with the Young modulus E, exposed to the bending moment M (see fig. 1). The rotation angle $\Delta\phi$ of one cut with respect to another in distance $\Delta L$ is

$$\Delta\phi = \frac{M \cdot \Delta L}{E \cdot J}$$

It can be derived that

$$E \cdot J = \frac{M \cdot \Delta L}{\Delta\phi}$$

The product E·J is usually called “substitutional bending stiffness EJ”. Up to certain level it is constant and then it decreases. The general idea is to express the EJ dependence on the bending moment M (the EJ-M curve). Then in the iterative cycle of solution the EJ could be updated with respect to the bending moment M. It is the alternative way how to include into the beam based model both geometrical non-linearity (large deformation) and material non-linearity (plasticity) and also the stiffness decrease due to the change of the beam cross sectional area.

The beam length L (see fig. 2) is divided into short elements. Every element is exposed to certain bending moment M. The element then is assigned by the EJ stiffness with respect to the value of the bending moment.

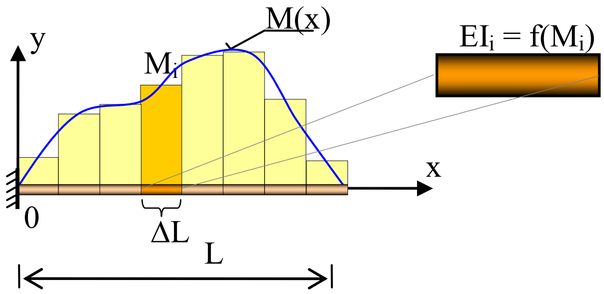


**Fig. 2** The division of the beam length into short elements

After the calculation the bending moment M and subsequently the EJ stiffness is updated with respect to the deformation results. The iteration cycle goes on until the condition

$$\| r_i - r_{i-1} \| < \varepsilon$$

is reached (here $r_i$ is the last solution results vector, $r_{i-1}$ is one before the last solution results vector and $\varepsilon$ is non-zero positive constant - the accuracy).

The steel arch support generally has a certain curvature (see fig. 3), the single pieces overlaps each other (with higher EJ stiffness). The steel arch is then exposed to the linearly distributed load (see fig. 4), both vertical (q) and horizontal (q·ε).

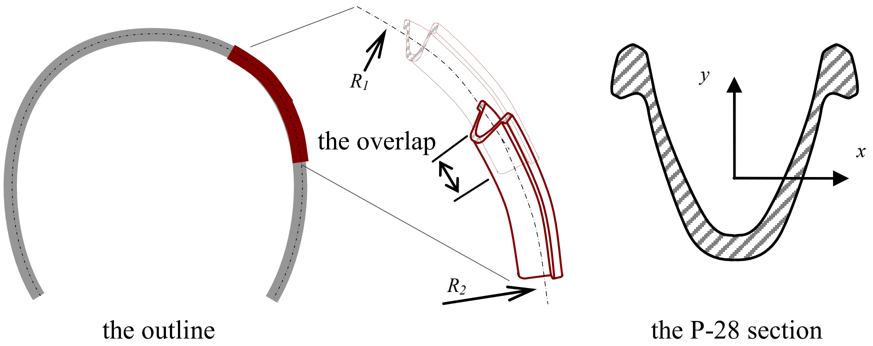


**Fig. 3** The steel arch support

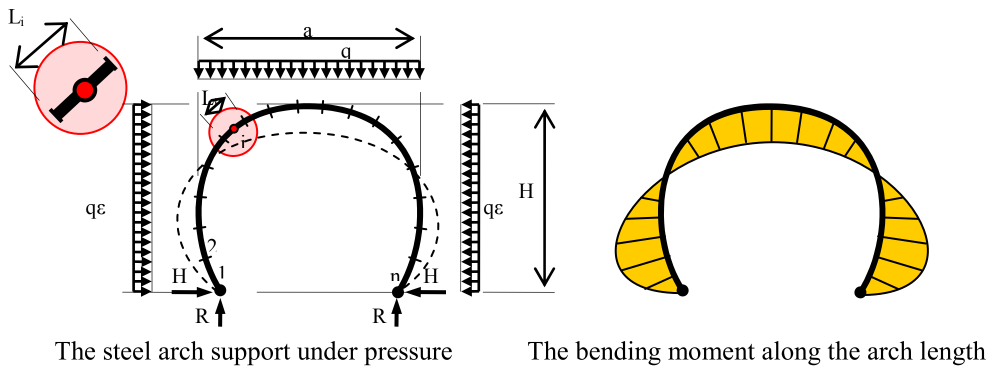


**Fig. 4** The steel arch support exposed to a pressure

The load leads to the certain distribution of the bending moment along the arch length and subsequently to the deformation of the structure. If the deformation is not very small with respect to the initial dimensions the solution in the iteration cycle has to be performed.

## 3 THE SUBSTITUTIONAL STIFFNESS CALCULATION

For the calculation of the substitutional bending stiffness EJ the straight beam of the length L=500 mm with the section profile P-28 is supposed. To simulate the bending moment in terms of rotational angle f the rigid block (body B on fig. 5) is fixed to the right and of beam (body A on fig. 5) while the left end is free in translation but fixed in rotation. The additional normal force N can also be taken into account.

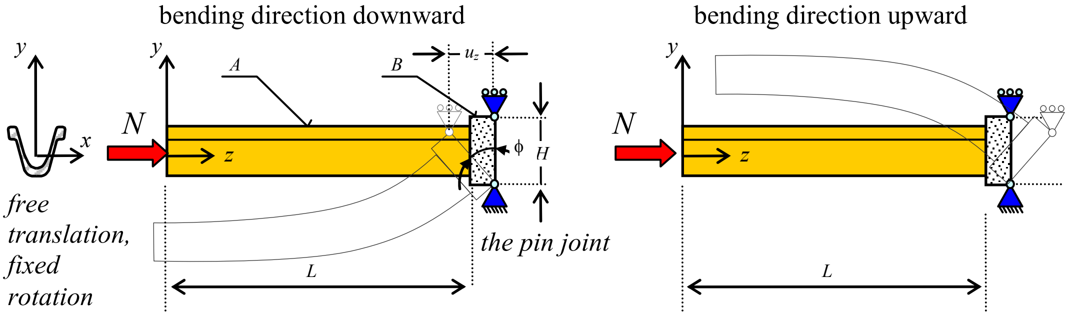


**Fig. 5** The bending of the straight beam

Note : The special boundary condition on the left end (free translation and fixed rotation) is realized in the model by an additional equations $uz_i=uz_j$ - the z-translation of all nodes on the left end are unknowns but equal one to another.

After the non-linear static analysis, including large deformation and plasticity, both deformation and bending moment of equidistant cuts is evaluated (see fig. 6). The substitutional bending stiffness is then calculated as :

$$E \cdot J = \frac{M \cdot \Delta L}{\Delta \phi}$$

where M is the bending moment, ΔL is the distance between two cuts and Δϕ is the difference between the rotational angel on two cuts.

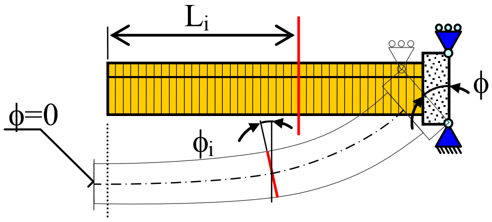


**Fig. 6** The series of the equidistant cuts

The rotation of the beam cut (see fig. 7) is represented by the displacement of the cut nodes. The nodal displacement is approximated by the plane $A_{(x,y)}=a_0+a_1\cdot y$ or by line $l_{(y)}=a_0+a_1\cdot y$. The slope $a_1$ represents the rotational angle ϕ. (The slope in the x direction is negligible.)

The bending moment M on the cut is calculated as the sum of the moments of the nodal forces with respect to the neutral axis. The neutral axis is the location where positive stress changes into negative. The stress distribution on the beam cut is replaced by the field of nodal forces (see fig. 8). This is correct if the area of the elements is rather same on the cut. The value of nodal forces on the cut area is approximated by the polynomial of second up to fifteenth order (see fig. 9). The neutral axis is supposed to be in the intersection of the polynomial, approximating forces, with the cut.

The note : The nodal forces have the general direction with respect to the beam cut (see fig. 10). Only the normal components of the nodal forces are taken into account to calculate the bending moment.

The multi-linear material model is used to define the elasto-plastic behavior (see fig. 11).

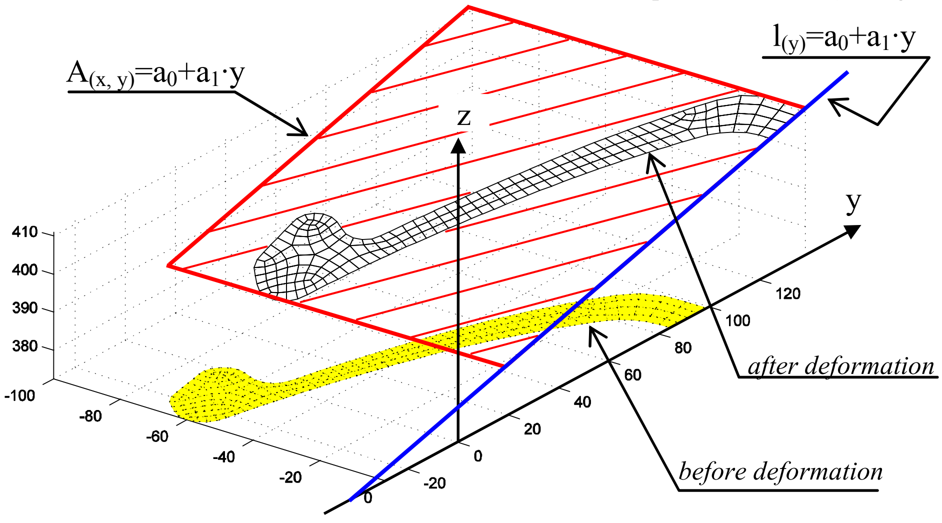


**Fig. 7** The rotated beam cut

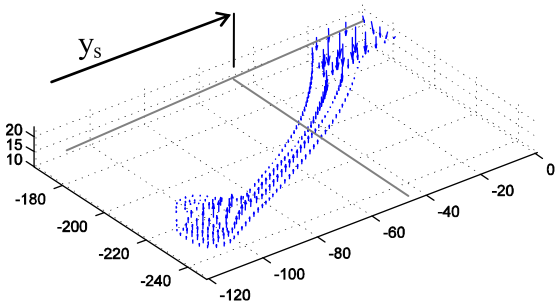


**Fig. 8** The field of nodal forces

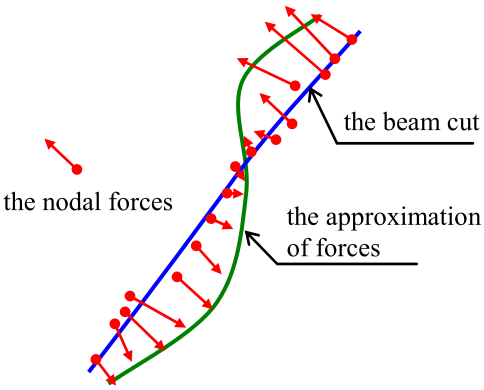


**Fig. 9** The nodal forces approximation

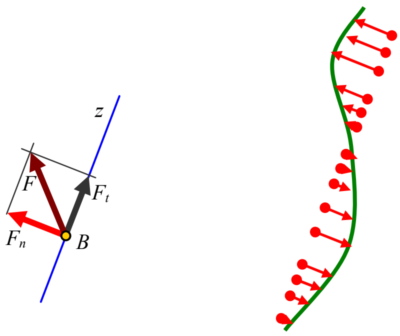


**Fig. 10** The normal and tangential components of the nodal forces

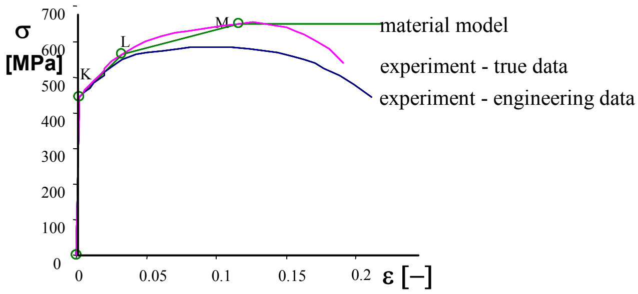


**Fig. 11** The material model - the stress-strain diagram

## 4 RESULTS

The non-linear analysis contains the large bending of the beam both downward and upward until the iterative calculation does not converge. The bending includes plasticity and large deformation (see fig. 12).

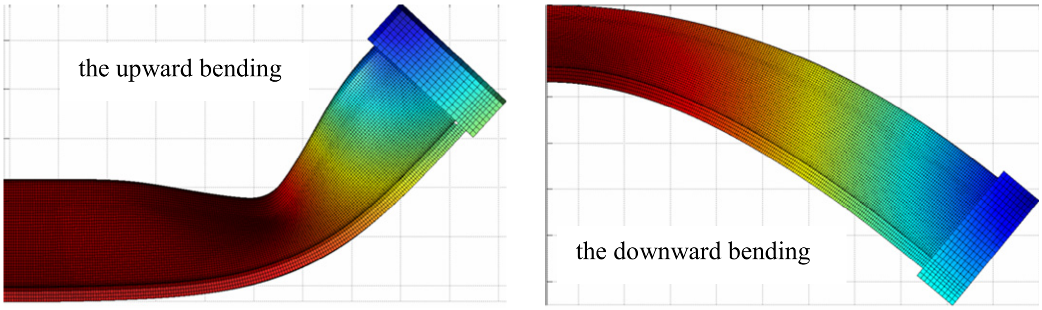


**Fig. 12** The non-linear bending of the beam

The result of the non-linear static analysis is the table of pair values the bending moment and corresponding rotational angle. This gives the bending characteristic. The calculated results were compared with these from experiment with good parity. The substitutional bending stiffness EJ (see above) is calculated and expressed in relation to the bending moment M (the EJ-M curve, see fig. 13).

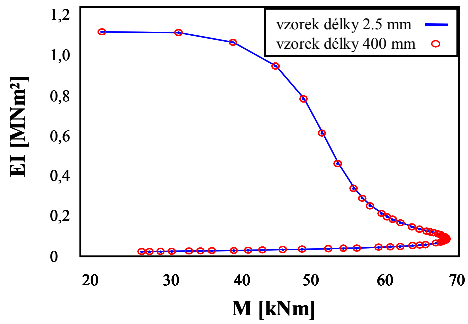


**Fig. 13** The substitutional bending stiffness - bending moment relation

In linear part (both geometric and material) the EJ stiffness is constant. Due to plasticity and due to change of the cross section area the EJ stiffness then decreases with increasing bending moment M. Under certain moment the stability limit is reached. The maximum bending moment $M_{max}$ subsequently decreases with still increasing angle. This represents the loss of stability, buckling. Under persistent bending moment the beam uncontrollably crashes. This phenomenon is followed by the large change of the cross section profile (good seen on fig. 12 left). In the EJ-M diagram (see fig. 13) this is the top right point of the curve.

The note : The above described features of the EJ-M relation can of course be observed on both downward and upward bending. But the quantitative expression is different.

The resulting data - the nodal displacements - are used to calculate the changes in the cross section parameters, such as the square area and quadratic moment of inertia. The finite element mesh of the beam (see fig. 14) is used for numerical integrating of mentioned parameters. The tetragonal elements are divided into two triangles.

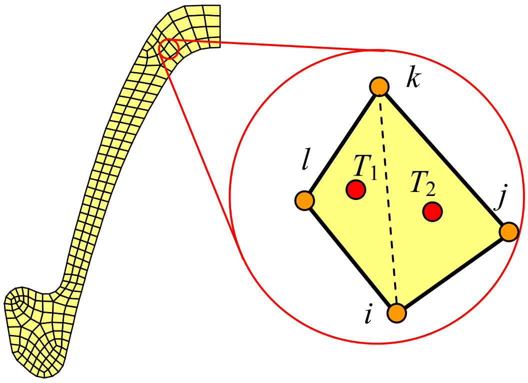


**Fig. 14** The meshed beam cross sectional area

If the square area and quadratic moment of inertia are defined as

$$S = \int_S dS \qquad \text{and} \qquad J_x = \int_S y^2 \cdot dS$$

then the numerical integration is expressed by sums

$$S = \sum_{i=1}^{n} S_i \qquad \text{and} \qquad J_x = \sum_{i=1}^{n} {y_{Ti}}^2 \cdot S_i$$

where $S_i$ is the square area of one triangular element and $y_{Ti}$ is the y coordinate of the triangular element centroid. In the diagrams below (see fig. 15 and fig. 16) the normalized square area s and normalized quadratic moment of inertia j are represented along the beam length.

$$s = \frac{S}{S_0} \qquad \text{and} \qquad j = \frac{J_x}{J_{x\_0}}$$

Here S is the square area of deformed profile, $S_0$ is the initial square area, $J_x$ is the quadratic moment of inertia of deformed profile, $J_{x\,0}$ is the initial quadratic moment of inertia.

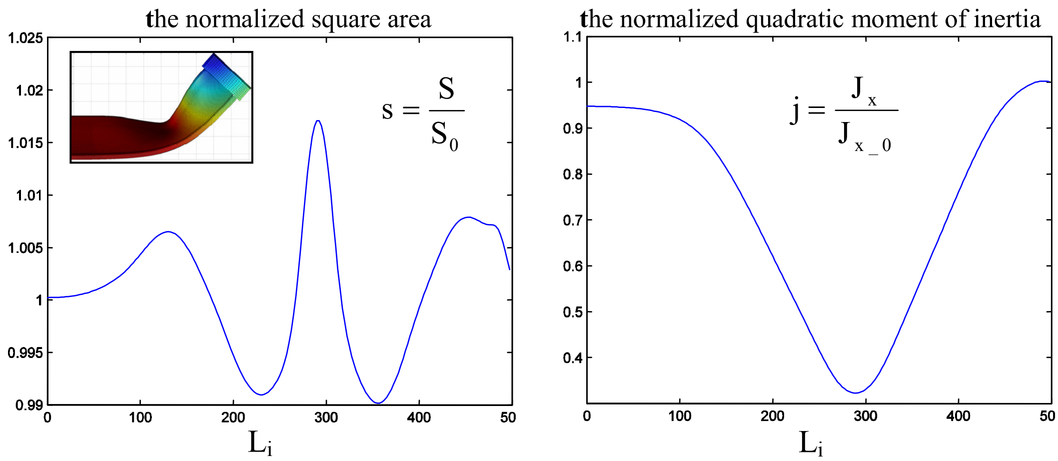


**Fig. 15** The normalized cross sectional parameters, upward bending

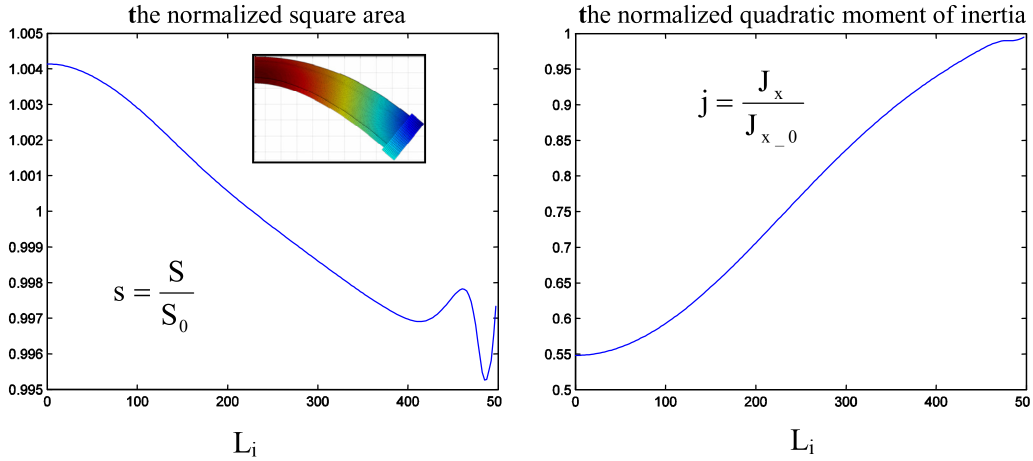


**Fig. 15** The normalized cross sectional parameters, downward bending

## 5 CONCLUSIONS

The non-linear static analysis of the bended beam was performed. The analysis includes both geometric non-linearity (large deformation) and material non-linearity (plasticity). The results (except of deformation and stress) are the moment-angle diagrams (bending characteristic) for upward and downward bending.

The methodology of the substitutional bending stiffness EJ calculation was proposed. The EJ stiffness relates to bending moment M. The EJ-M diagrams were constructed.

The changes of the beam cross section area, such as square area and quadratic moment of inertia, due to large deformation were calculated.

**Acknowledgement**
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