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František FOJTÍK[*], Jan FUXA[**], Zdeněk PORUBA[***]

NEW APPLICATIONS OF CONJUGATED STRENGTH CRITERION

NOVÁ APLIKACE KONJUGOVANÉHO KRITÉRIA PEVNOSTI

Abstract

This article describes experimental results under combined loading of specimens manufactured from construction steel 11523. Specimens were loaded by amplitude of the torque, then by combination of torque and tension prestress. For evaluation of the results the conjugated strength criterion was applied. Further, another approximation based on the same boundary conditions as the described conjugated strength criterion was proposed. The performed experiments and their results embody a good agreement with bellow mentioned conjugated strength criterion. The experiments were performed on modified testing machine.

Abstrakt

Tento příspěvek popisuje výsledky experimentů při kombinovaném zatěžování zkušebních vzorků vyrobených z konstrukční oceli 11523. Zkušební vzorky byly zatěžovány amplitudou krouticího momentu, dále pak kombinací amplitudy krouticího momentu s tahovým předpětím. Na výsledky experimentů bylo aplikováno konjugované kritérium pevnosti. Dále byla navržena jiná aproximace, která vychází ze stejných okrajových podmínek jako popsané konjugované kritérium pevnosti. Výsledky experimentů vykazují dobrou shodu s použitým kritériem. Zkoušky byly provedeny na upraveném zkušebním zařízení.

1 INTRODUCTION

For the verification of the multiaxial strength criteria in the area of material fatigue the testing stand SCHENCK [1-3] was reconstructed at the Department of mechanics of materials. This stand is continuously equipped by new devices which make the testing stand abilities wider. In the present time it is possible to load specimens by alternating torque amplitude in the combination with the prestress in axial tension/pressure, inner/outer overpressure and their combinations. Further it is possible to realize the tests in alternating bending, torsion and bending in combination with inner overpressure. The reached loading cases lead to the acquisition of experimental data which are essential for the verification and proposing of the strength criteria.

2 CONJUGATED STRESS CRITERION

In the following the Fuxa's conjugated strength criterion applied in the fatigue loading will be presented. This strength criterion [4] can be written for the crack initiation in N-th cycle in the form:

$$S_\sigma = A_N - B_N \cdot \sigma_R, \tag{1}$$

where S_σ marks the stress intensity and is defined as:

[*] Ing., Ph.D., Department of Mechanics of Materials, VŠB-Technical University of Ostrava, av. 17. listopadu 15, CZ-708 33 Ostrava-Poruba, (+420) 59 732 3292, frantisek.fojtik@vsb.cz

[**] prof., Ing., CSc., Department of Mechanics of Materials, VŠB-Technical University of Ostrava, av. 17. listopadu 15, CZ-708 33 Ostrava-Poruba, (+420) 59 732 4412, jan.fuxa@vsb.cz

[***] Ing., Ph.D., Department of Mechanics, VŠB-Technical University of Ostrava, av. 17. listopadu 15, CZ-708 33 Ostrava-Poruba, (+420) 59 732 3228, zdenek.poruba@vsb.cz

$$S_\sigma = 2^{-1/2} \cdot \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2}, \quad (2)$$

σ_R is the reference stress value producing the identical value as the octaedric normal stress and can be written as:

$$\sigma_R = (\sigma_1 + \sigma_2 + \sigma_3)/3, \quad (3)$$

where σ_1, σ_2, σ_3 are the principal stresses. The value A_N can be considered as dependent on the cycle number N [4], B_N is the constant equal to:

$$B_N = 3 \cdot \left(\sqrt{3} \cdot \tau_f / \sigma_f - 1 \right), \quad (4)$$

where σ_f is the value of real strength in tension and τ_f is the value of real shear strength.

The criterion described above was set according to the first experiment – alternation torsion and this constant setting was then applied in the results of another experiments.

3 ALTERNATING TORSION

Specimens manufactured from steel 11523 [5] were loaded by amplitude of torque in the condition of alternating cycle and testing frequency of 25 Hz. The amplitude of torque was gradually decreased until the limit 10^7 cycles was reached.

The results are placed in Table 1. In the Fig. 1 can be seen measured values, Fuxa's approximation curves (5) as will be seen below [6]. The Fig. 1 presents the newly proposed approximation (6) which is based on the same conditions as the Fuxa's approximation curves. Point of crack initiation under static torsion [7] was measured by reconstructed testing machine INOVA in the institute lab [8, 9].

Tab. 1 Experimental results for alternating torsion

Specimen Nr.	Stress amplitude τ_a [MPa]	Number of cycles	Notes
1	222,5	73400	
2	202	284900	
3	190	1006400	
4	175,4	3203600	
5	169,5	10105000	No crack generated
6	513	0,25	Static test

Fuxa's approximation:

$$\tau_{aF} = (\tau_f + \tau_C)/2 + (\tau_f - \tau_C)/2 \cdot \cos\left\{ \pi \cdot \left[\log(4 \cdot N_f) / \log(4 \cdot N_C) \right]^{a_1} \right\}, \quad (5)$$

for N_f in interval [*1/4; N_C*] and τ_{aF} in interval [τ_f ; τ_C].

where:

τ_f (513 MPa) is a value of real shear strength, τ_C (171,15 MPa) is the stress at the fatigue limit, N_C (9300000) is number of cycles at the fatigue limit, a_1 (0,953) is constant, τ_{aF} is the limit stress amplitude under alternating torsion and N_f is the limit number of cycles (until crack initiation). The mentioned values were obtained by nonlinear regression methods.

New approximation:

$$\tau_a = \tau_f - (\tau_f - \tau_C) \cdot \sin\left\{\pi/2 \cdot \left[\log(4 \cdot N_f)/\log(4 \cdot N_C)\right]^{a_2}\right\}, \quad (6)$$

where:

a_2 (1,40) is constant. Other parameters, constants and conditions of the approximation are the same as in previous case.

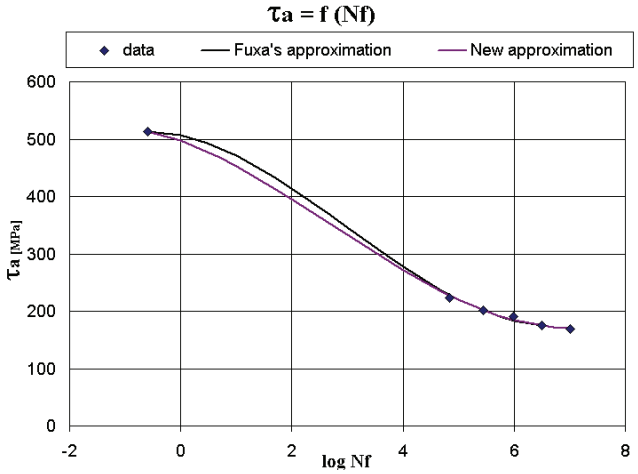

Fig. 1 S-N – curve for alternating torsion

4 ALTERNATING TORSION AND TENSION PRESTRESS

Every series of specimens [6] was loaded by different constant axial tension stress. For given tension stress value was chosen the torque amplitude which was the specimen loaded until the crack initiation by.

Tab. 2 Experimental results for alternating torsion and tension prestress

Nr.	Tension prestress [MPa]	Stress amplitude τ_a [MPa]	Number of Cycles	CH_F [%]	CH_N [%]	Notes
1	110	174	649000	2,01	1,85	
2	110	154,2	11500000	1,70	1,70	No crack generated
3	160,4	148	2320000	1,89	1,73	
4	160,4	139,5	11600000	0,3	0,30	No crack generated
5	228,4	203,7	28640	5,78	3,85	
6	228,2	170	139210	2,14	1,63	
7	228,4	120	10800000	2,89	2,89	No crack generated
8	310	116	1141000	0,78	1,08	
9	310	104,7	1130000	0,97	0,97	No crack generated

This amplitude was gradually decreased until the value when was the specimen able to endure 10^7 of cycles. The testing frequency was also 25 Hz.

The results of those experiments are mentioned in Table 2. The experimental results are shown in Fig. 2 and approximated by lower described Fuxa's approximation (7, 8, 9, 10) which takes the influence of mean stress into account. Particular approximations are based on measured number of cycles which is mentioned in Table 2.

Fuxa's approximation with influence of mean stress:

$$\tau_{aF2} = \left(\tau_f{}^* + \tau_C{}^*\right)/2 + \left(\tau_f{}^* - \tau_C{}^*\right)/2 \cdot \cos\left\{ \pi \cdot \left[\log\left(4 \cdot N_f\right)/\log\left(4 \cdot N_C\right)\right]^{a_1} \right\}, \tag{7}$$

$$\tau_f{}^* = 1/\sqrt{3} \cdot \left((\sqrt{3} \cdot \tau_f)^2 - 2 \cdot \sqrt{3} \cdot \tau_f \cdot B_O \cdot \sigma_t / 3 + \sigma_t^2 \cdot B_O{}^2 / 9 - \sigma_t^2\right)^{1/2}, \tag{8}$$

where (8) is the static strength condition for $N_f = 1/4$ and constant B_O is equal to:

$$B_O = 3 \cdot \left(\sqrt{3} \cdot \tau_f / \sigma_f - 1\right), \tag{9}$$

$$\tau_C{}^* = \tau_C / 2 \cdot \left\{ 1 + \cos\left[\pi \cdot \left(\sigma_t / \sigma_f\right)^{\beta}\right] \right\} \text{ is the strength condition for } N_f = N_C\,. \tag{10}$$

In the relations (7, 8, 9, 10) the σ_f is the real tension strength value, τ_f is a value of real shear strength, τ_C is the stress at the fatigue limit, N_C is number of cycles at the fatigue limit, a_1 and B are constants, τ_{aF2} is the limit amplitude of shear stress, σ_t is the constant tension stress and N_f marks the (limit) number of cycles until crack initiation.

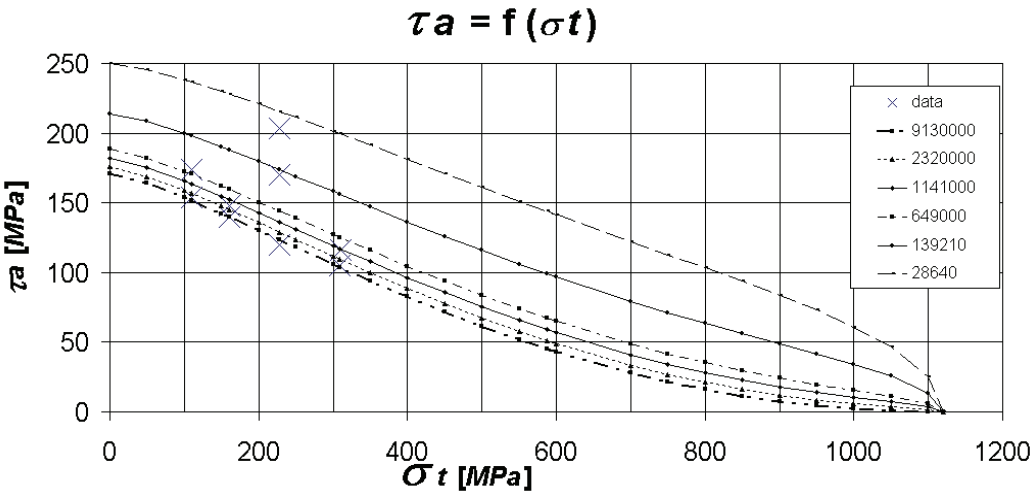

Fig. 2 Fuxa's approximation for combined torsion – tension loading

The experimental results are also shown in Fig. 3 and approximated by lower described "New approximation" (11) which takes the influence of mean stress into account. Particular approximations are based on measured number of cycles which is mentioned in Table 2.

New approximation with influence of mean stress:

$$\tau_{a2} = \tau_f{}^* - \left(\tau_f{}^* - \tau_C{}^*\right) \cdot \sin\left\{\pi/2 \cdot \left[\log\left(4 \cdot N_f\right)/\log\left(4 \cdot N_C\right)\right]^{a_2}\right\}, \tag{11}$$

where:

a_2 is constant. Other parameters, constants and conditions of the approximation (8, 9, 10) are the same as in previous case.

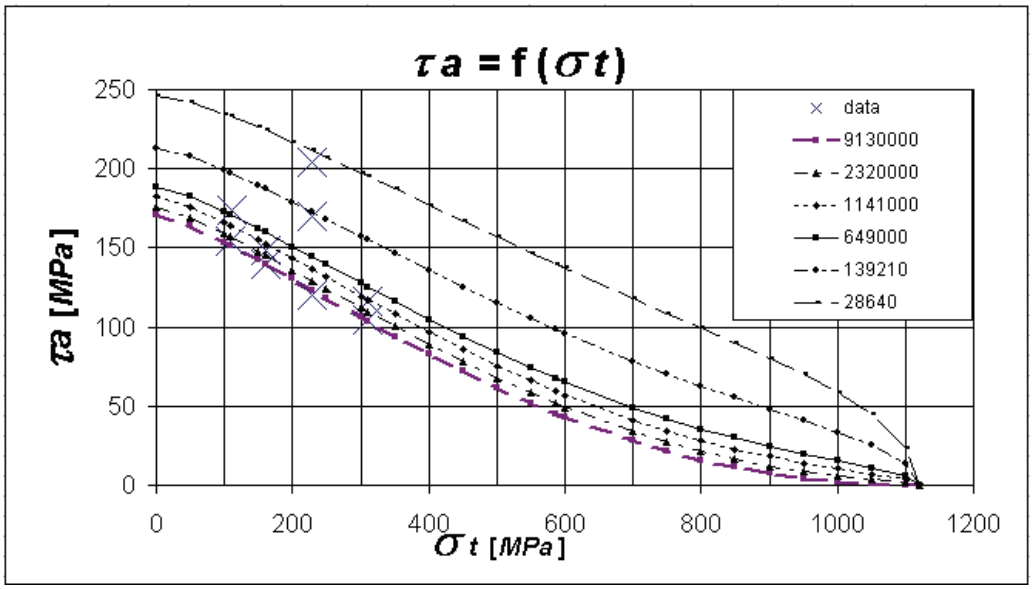

Fig. 3 New approximation for combined torsion – tension loading

The absolute mean relative error value of used approximation is mentioned in Table 2 and can be determined according to following formula:

$$CH_F = ABS(\tau_{ai} - \tau_{aFi})/\tau_{ai} \cdot 100\%, \tag{12}$$

τ_{ai} are the measured stress amplitude values (see Table 2),

τ_{aFi} are the values calculated according to the Fuxa's approximation (7).

For the error calculation of the "New approximation" the same formula (12) as in case of Fuxa's approximation was used with the necessity of indexes switch. Results are depicted in the Table 2.

6 CONCLUSIONS

The Fuxa's conjugated strength criterion and the "New approximation" based on the same boundary conditions are presented in this article or paper. Both approximations were tested on two experiment types with the results mentioned bellow. The specimen were made of the construction steel 11523.

First experiment – alternating torsion. Obtained results are approximated by Fuxa's approximation. The Fuxa's approximation embodies a good agreement with experiment – see Fig. 1. The obtained results were approximated by the newly proposed approximation. Even this approximation embodies the good agreement with the experimental results. The difference between particular approximations lies only in the low cycle area which has to be even investigated and complemented by the experimental data.

Second experiment – combined loading by amplitude of torque and by constant axial tension force. The results are successfully approximated by Fuxa's approximation with the influence of mean stress (7, 8, 9, 10). The obtained results (see – Table 2) were approximated by the newly proposed approximation. This approximation takes into the account the influence of the mean stress and uses the same boundary conditions as the Fuxa's approximation. The calculated approximation error (see Table 2) according (12) shows deviations of both approximations in the area of low cycle fatigue.

The complementation of this area by experimental data will lead to the more accurate revision of both approximations.

The significant influence on the approximation accuracy has the setting up of its constants.

Described Fuxa's approximation is a part of conjugated stress criterion which is based on the conception of reference normal and reference shear stresses, see [3] for more details.

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