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COMPARISON OF METHODS FOR COMPUTATION OF ELECTROMAGNETIC FORCES IN THE RADIAL ACTIVE MAGNETIC BEARING

POROVNÁNÍ ODLIŠNÝCH POSTUPŮ K VÝPOČTU ELEKTROMAGNETICKÝCH SIL RADIÁLNÍHO AKTIVNÍHO MAGNETICKÉHO LOŽISKA

Abstract

Rotor systems supported by the radial active magnetic bearings (AMBs) are considered in this paper. If dynamic characteristics of the rotor systems are examined numerically by the finite element method, it is necessary to determine forces in the bearings. Considering supporting by the AMBs, a shaft levitates in an air gap between the rotor and the stator part. Forces of magnetic levitation are caused by the electromagnetic field in the bearing. Electromagnetic forces in AMB are determined by two methods. One is based upon the principle of virtual work and the other one upon the Maxwell stress tensor. The forces computations are provided for three-dimensional and two-dimensional finite element models of the radial AMB. One-dimensional approximation of the magnetic circuit is provided as well. Rates of convergence, efficiencies and accuracies of the electromagnetic forces computations are evaluated from several points of view. These are compared also with used type of material model, methods of the forces calculation and size of AMB model meshes.

Abstrakt

V tomto článku jsou uváženy rotorové soustavy uložené v radiálních aktivních magnetických ložiskách. Síly v ložiskách je třeba určit, jestliže dynamické charakteristiky rotorových soustav jsou zkoumány numericky metodou konečných prvků. U tohoto typu uložení hřídel rotorové soustavy levituje ve vzduchové mezeře mezi rotorem a statorem. Síly magnetické levitace jsou způsobeny elektromagnetickým polem v ložisku. Elektromagnetické síly v aktivním magnetickém ložisku jsou určeny pomocí dvou metod, tedy metodou Maxwellova tenzoru napětí a metodou virtuálních prací. Tyto síly jsou vypočítány pro tří rozměrné a dvou rozměrné modely radiálního aktivního magnetického ložiska a jednorozměrnou aproximaci magnetického obvodu. Rychlosti, efektivity a přesnosti výpočtů elektromagnetických sil jsou kriticky zhodnoceny z pohledů diskretizace modelu ložiska, typu magnetické materiálové charakteristiky a použitých metod k jejich výpočtu.

1 INTRODUCTION

Recently, rotating parts of rotor systems are often supported by the radial AMBs. The AMB stands out against conventional bearings as rolling element bearings and fluid film bearings. Main advantages of the AMB are non-contact operation so there are no friction losses or wear damages, a possibility of a hermetic separating of the rotor and the stator part, high operating speed, characteristics controlling of the AMB during operating, ability to operate in extreme conditions for instance high range of temperatures, radioactive and chemical aggressive environment, steam etc.

A design of the AMB is unique in every technical application, so it is necessary to know AMB characteristics already during design of a rotor system. The method of computer modeling is a

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significant tool to investigate these characteristics. By this method, the characteristics of several variants can be examined during the design of a rotor system. It is important to describe all effects originating in the radial AMB, especially those, which participate in the system response. The AMB is usually involved in the computation model by a force coupling, because the AMB posses nonlinear characteristics. The force coupling consists of electromagnetic forces. Magnitudes of the forces are derived from a distribution of the magnetic field described by the Maxwell's equations.

In general, computing of energy and forces can be very difficult in the electromagnetic fields, see in [1]. In electrical machinery systems, electromagnetic forces are commonly calculated by methods as Lorenz force, (ii) Maxwell stress tensor, (iii) virtual work and (iv) equivalent sources, see [2]. The numerical comparison of electromagnetic forces in current carrying conductors computed by first three methods is described in [3]. Accuracies and computing times for different finite element meshes are discussed. It shows the most accurate methods are those based on volume integration, so these are the virtual work method, the Maxwell stress tensor method and the Lorenz force.

The summary of methods for electromagnetic forces computation is stated and a new method for rotor performing eccentric motions with respect to the stator is proposed in the doctoral thesis [4]. The force is determined by means of the impulse method of frequency response. Values of the force computed by the impulse method and by the conventional computation are similar.

In this contribution, the field is described by the magneto-static approximation of the Maxwell's equations and the electromagnetic forces computation is based upon this approximation. These coupling forces are determined by the Maxwell stress tensor method and the virtual work method considering linear and nonlinear magnetic material characteristics of the rotor and the stator part. The calculations are carried out for three-dimensional and two-dimensional models. These models are discretized with different sizes of mesh. Obtained results are compared and discussed.

2 THE MAGNETO-STATIC APPROXIMATION OF THE ELECTROMAGNETIC FIELD

The electromagnetic forces calculation in the radial AMB is based upon the magneto-static approximation of the Maxwell's equations [1]. In the magneto-static analysis, these equations can be written in a differential form according to [5] as

$$-\nabla\cdot\left[-\sigma\,\mathbf{v}\times(\nabla\times\mathbf{A})+\sigma\nabla V-\mathbf{J}^{\mathrm{e}}\right]=0\,, \tag{1}$$

$$\nabla\times\left(\mu^{-1}\,\nabla\times\mathbf{A}\right)-\sigma\,\mathbf{v}\nabla\times(\nabla\times\mathbf{A})+\sigma\nabla V=\mathbf{J}^{\mathrm{e}}\,, \tag{2}$$

where $\mathbf{A}$ is the magnetic vector potential, $\mathbf{J}^{\mathrm{e}}$ is the externally generated current density, $\mathbf{v}$ is the velocity of a conductor, V is the electric scalar potential, μ is the permeability, σ is the electrical conductivity and ∇ is the Nabla operator.

The system of two equations (1) and (2) and boundary conditions give the unique solution of the potentials $\mathbf{A}$ and V.

3 CALCULATION OF THE ELECTROMAGNETIC FORCES IN THE AMB

The electromagnetic forces in the AMB are determined by two methods. These methods assume bodies of a finite volume. One is based upon the principle of the virtual work and the other one upon the Maxwell stress tensor.

If the virtual work method (VWM) is used the electromagnetic force $\mathbf{F}_{\mathrm{m}}$ is computed as partial derivative of the co-energy functional with respect to virtual movements

$$\mathbf{F}_{\mathrm{m}}=\left.\frac{\partial W_{\mathrm{c}}}{\partial\mathbf{p}}\right|_{\mathbf{I}_{\mathrm{c}}}=\left[\frac{\partial W_{\mathrm{c}}}{\partial y_{\mathrm{p}}}\;\frac{\partial W_{\mathrm{c}}}{\partial z_{\mathrm{p}}}\right]^{\mathrm{T}}_{\mathbf{I}_{\mathrm{c}}},\quad W_{\mathrm{c}}=\int\limits_{V_{\mathrm{body}}}\left(\int\limits_{0}^{H}\mathbf{B}\,\mathrm{d}\mathbf{H}\right)\mathrm{d}V_{\mathrm{e}}, \tag{3}$$

where **B** is the magnetic flux density, **H** is the magnetic field intensity, **p** is the vector of virtual movements, $\mathbf{I}_c$ is the vector of constant current, W_c is the co-energy functional, y_p and z_p are horizontal and vertical components of the vector of virtual movements and V_e is the volume element of a body.

Another way of computing the electromagnetic forces use the Maxwell's field concept. The method is based on the Maxwell stress tensor (MSTM). The electromagnetic forces acting on a body are calculated by the integral of the field on the surface bounding the body. So the surface integral will be

$$\mathbf{F}_m = \oint_S \mathbf{T}\,\mathrm{d}S = \oint_S \left[\frac{1}{\mu_0}(\mathbf{B}\,\mathbf{n})\mathbf{B} - \frac{1}{2\mu_0}\mathbf{B}^2\mathbf{n} \right] \mathrm{d}S\,, \tag{4}$$

where **T** is the Maxwell magnetic stress tensor in an air, **n** is the unit outward normal vector of the integration surface S and μ_0 is the permeability of vacuum $(4\pi \cdot 10^{-7}\mathrm{Hm}^{-1})$. The Maxwell magnetic stress tensor is symmetric and second rank tensor, see [1]. Considering the two-dimensional model, the surface integral is reduced to line integral along the air gap.

The magnetic flux density is determined by the B-H characteristic in formulas (3) and (4). The B-H characteristic is stated for linear or nonlinear ferromagnetic environment by following formulas

$$\mathbf{B} = \mu\mathbf{H}\,,\ \text{or}\ \mathbf{B} = \mu(\mathbf{H})\mathbf{H}\,. \tag{5}$$

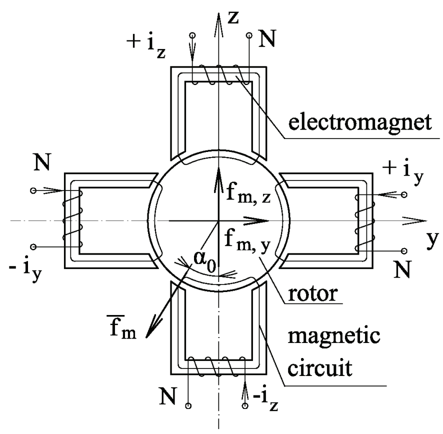

Fig. 1 Scheme of the radial AMB with electromagnetic forces

The results obtained by the three-dimensional and the two-dimensional finite element calculations are compared with one-dimensional method. One-dimensional approximation assumes the uniform size of the air gap, therefore it is presumed the current excites the homogeneous magnetic flux density. The electromagnetic force can be computed by the principle of the VWM. The energy of the magnetic field is determined by the amount of the energy accumulated in the air gap, because the magnetic circuit consists of the high relative permeability ferromagnetic material.

Considering these assumptions and according to [6], the electromagnetic force components $\mathbf{F}_m$ acting on the rotor in horizontal $f_{m,y}$ and vertical $f_{m,z}$ directions (Fig. 1) are given by the following formulas

$$\begin{aligned} f_{m,y} &= \frac{1}{4}\cos(\alpha_0)\mu_0 N^2 S_m \left[\left(\frac{I_0 + i_y}{c_0 - y} \right)^2 - \left(\frac{I_0 - i_y}{c_0 + y} \right)^2 \right], \\ f_{m,z} &= \frac{1}{4}\cos(\alpha_0)\mu_0 N^2 S_m \left[\left(\frac{I_0 + i_z}{c_0 - z} \right)^2 - \left(\frac{I_0 - i_z}{c_0 + z} \right)^2 \right], \end{aligned} \tag{6}$$

where y, z are the generalized components of displacements in horizontal and vertical direction respectively, c_0 is the size of the air gap between the rotor and stator, i_y, i_z are the controlling currents in horizontal and vertical directions respectively, I_0 is the bias current, N is the number of electromagnet coil turns, S_m is the cross-section area located in the air gap and α_0 is the angle between the electromagnetic force and vertical axis of the bearing, see Fig. 1.

4 THE COMPUTATIONAL MODEL OF the RADIAL AMB

The investigated AMB consists of eight poles equally circumferentially placed, shown on Fig. 1. The stator and outer rotor parts are made from stamped high relative permeability steel plates. The AMB is powered by the pole coils. Each coil pair is putted on series and powered by direct-current. This coil configuration is useful because of easier projection of a feedback control. The rotor position is controlled by a change of current magnitudes in two independent and perpendicular directions.

The finite element analysis of the radial AMB is based on the magneto-static approximation (1) and (2) of the Maxwell's equations. In the computational, the model is assumed as two-dimensional or three-dimensional and stationary. Further it is considered no current is generated by an external static electric and magnetic field and also currents in the electromagnets are of constant values and there is no hysteresis of the nonlinear magnetization curve of the used ferromagnetic material.

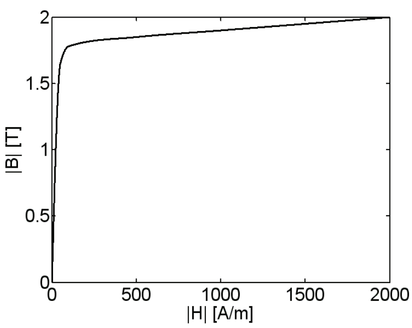

Fig. 2 The norm of the magnetic flux density versus the norm of the magnetic field intensity

Material of the stator and outer rotor parts is considered as nonlinear because the material is made from stamped plates. Relation between the magnetic flux density and the magnetic field intensity is shown in Fig. 2. The other parts are supposed to be made from linear materials, so the relative permeability is of constant value. There is relative permeability of value 1 in the coils. The inner rotor part is made of common structural steel with relative permeability of value 5000.

The AMB model has been made. There is an air in the surrounding of the bearing model. Two and three-dimensional model is shown in Fig. 3. Both the models, are discretized by the linear Lagrange elements. The meshes are refined in the places where a high concentration of the magnetic flux density is supposed. The electromagnetic force computations have been tested for different sizes of the bearing meshes, so number of degrees of freedom (DOF) was 100 000 – 400 000 for two-dimensional models and 250 000 – 410 000 for three-dimensional models.

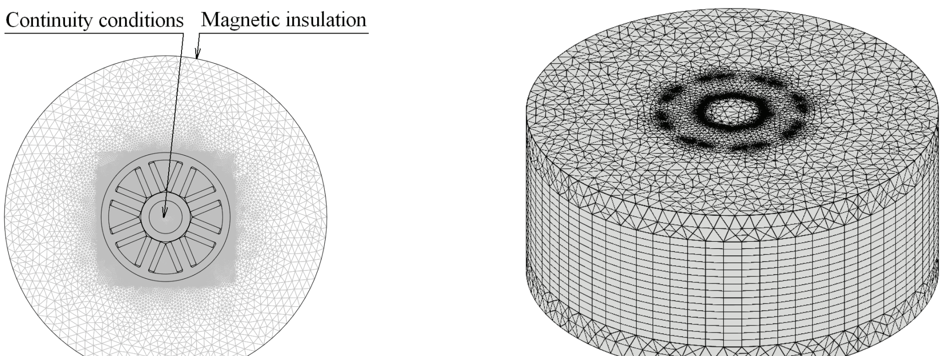

Fig. 3 The generated two-dimensional model of the radial AMB (left) and generated three-dimensional model of the radial AMB (right)

Power supply of the coils by direct-current is given by the current density

$$J = \frac{N_c\, I}{S_p}, \tag{7}$$

where I is the current in the coil, N_c is the number of coil turns and S_p is the area of the pole cross-section. The sign of the current density states a direction of the current flow.

The Dirichlet boundary condition in the form of magnetic insulation is defined on the outer edge, respective on the outer face as shown in Fig. 3. The boundary condition of continuity is defined on the all internal edges respective faces.

5 THE RESULTS OF THE NUMERICAL SIMULATIONS

The magneto-static finite element analysis of the radial AMB is provided in the COMSOL Multiphysics software. Magneto-static mode of the AC/DC module of the software COMSOL Multiphysics is used for the numerical simulations.

The task is to compute the electromagnetic forces by using two methods in three-dimensional, two-dimensional and one-dimensional model of the magnetic bearing circuit considering linear and nonlinear material properties. The calculation is carried out for the central position of the rotor (y= 0 mm, z = 0 mm) and for the current supply of the bearing magnets (1 A – upper electromagnet, 2 A – left electromagnet, 3 A – lower electromagnet and 4 A – right electromagnet).

The investigated radial AMB is designed to support the rotor system consisting of the cantilever rotor with two fixed discs (described in detail in [6]). The radial AMB has the following dimensions and parameters: the shaft diameter – 76.6 mm, the rotor diameter – 105 mm, the stator diameter – 273 mm, the size of the air gap – 0.5 mm, the thickness of stator frame – 15.9 mm, the bearing width – 200 mm, the number of coil turns – 64 and the current range 0÷8 A.

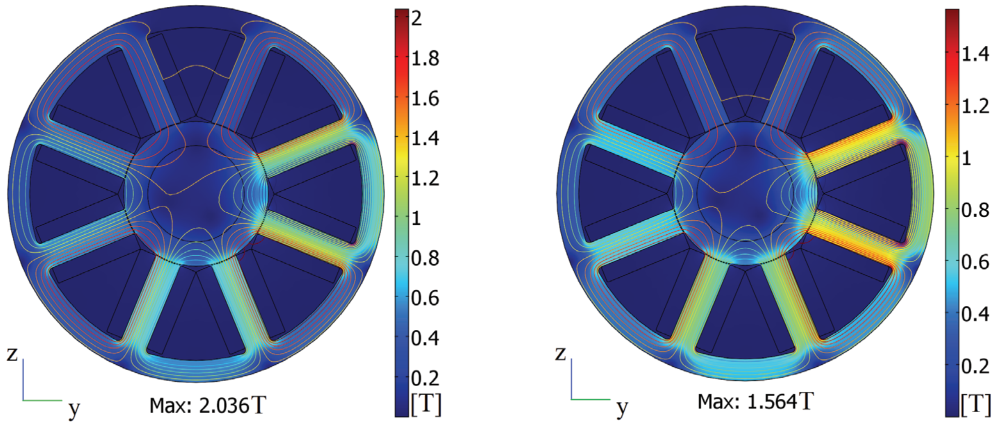

Fig. 4 The distribution of the magnetic flux density and the magnetic potential on the two-dimensional model of the radial AMB with the linear (left) and the nonlinear (right) material characteristic

The two-dimensional discretized model of the radial AMB has 102 629 and 404 631 DOF. There is the distribution of the magnetic flux density for model with 102 629 DOF in the Fig. 4. Values of the magnetic flux density increase on the bearing edges. This effect can be explained by the material saturation of the edges vicinities. There is the highest value of the magnetic flux density in the right electromagnet. This is because the right electromagnet is supplied by the highest current. If the nonlinear material B-H characteristic is taken into consideration, the highest value of the magnetic flux density is lower than that in the model using the linear characteristic. That is because the nonlinear material assumes the values of the magnetic flux density which cannot be higher then a certain limit (effect called the saturation).

The electromagnetic forces components computed by both methods (VWM, MSTM) are written in the Table 1. The sign of the component defines orientation of the component acting. The sign varies from the body (stator or rotor) where the force acts. Both methods give almost equal forces values. The difference of the values is up to 1 %. In the model using the nonlinear material model, the forces values are lower than those in the model considering the linear material. The reason is described above (effect called the saturation). The difference of the values is up to 5 % with the respect to the values of the linear model. The forces values computed in the models with 102 629 and 404 631 DOF are almost equal (difference is up to 1 %).

In the case of the nonlinear model, the stator material is saturated in the fillets on the sharp edges (Fig. 4). The distribution of the magnetic flux density is identical in the both material models. The distribution of the magnetic flux density is nearly identical along the width of the bearing pole in the both material models.

Table 1 The electromagnetic forces calculated in the two-dimensional model of the radial AMB with 102 629 DOF

Electromagnetic forces					
Force component		Linear material characteristic		Nonlinear material characteristic	
		Rotor	Stator	Rotor	Stator
VWM	$f_{m,y}$ [N]	1780.186	-1791.794	1719.775	-1731.002
	$f_{m,z}$ [N]	-1247.273	1256.774	-1208.430	1217.644
MSTM	$f_{m,y}$ [N]	1780.198	-1792.216	1719.788	-1730.824
	$f_{m,z}$ [N]	-1247.288	1263.402	-1208.445	1217.653

Table 2 The electromagnetic forces calculated in the three-dimensional model of the radial AMB

Electromagnetic forces						
Force component/ Number of DOF			Linear material characteristic		Nonlinear material characteristic	
			Rotor	Stator	Rotor	Stator
VWM	234 850	$f_{m,x}$ [N]	-0.105	0.226	-0.101	0.217
		$f_{m,y}$ [N]	1493.520	-1490.411	1441.423	-1438.517
		$f_{m,z}$ [N]	-1062.987	1058.252	-1029.623	1025.213
	411 099	$f_{m,x}$ [N]	0.0665	-0.00768	-	-
		$f_{m,y}$ [N]	1407.489	-1413.429	-	-
		$f_{m,z}$ [N]	-985.881	1015.915	-	-

The electromagnetic forces components computed by the VWM with two number of DOF are written in the Table 2. Both models with different number of DOF give almost equal values of the horizontal and vertical component of the electromagnetic force. In the model considering the nonlinear material properties, the forces nature is similar in both models.

There is the distribution of the magnetic flux density for the linear model with 411 099 DOF and for the nonlinear model with 234 850 DOF in the Fig. 5. Calculation of the nonlinear model with 411 099 DOF could not be carried out because of too high hardware demands (Table 2). In case of the linear material characteristic, the magnetic flux density is uniformly distributed along the length of the bearing (except for the vicinity of the bearing faces), see left Fig. 5. In the nonlinear model, the material is saturated just in a neighborhood of the symmetry plane, see Fig. 5. The saturation is the highest in the fillet between the stator ring and the bearing pole, see right Fig. 5.

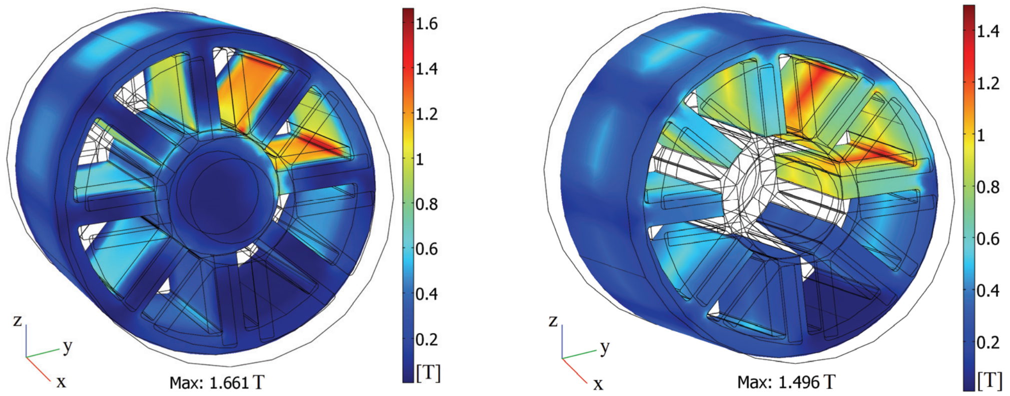

Fig. 5 Distribution of the magnetic flux density in the three-dimensional model of the radial AMB with the linear (left) and the nonlinear (right) material characteristic

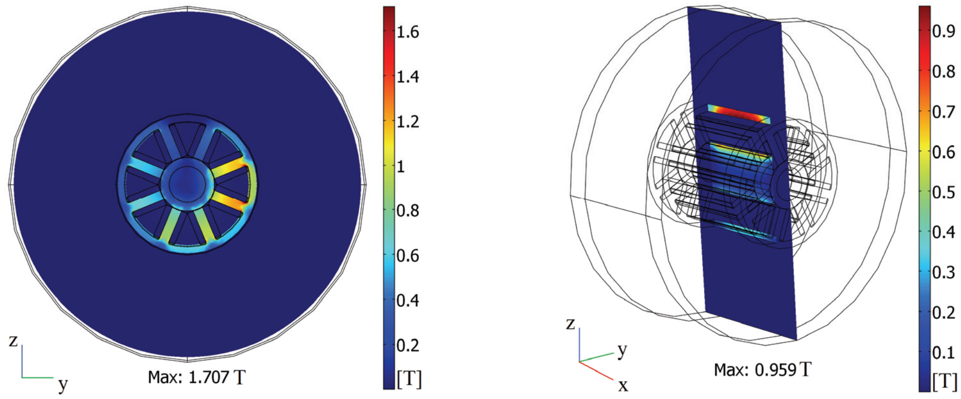

Fig. 6 Distribution of the magnetic flux density in the axial middle (left) and the longitudinal section (right)

There are the distributions of the magnetic flux density in the axial middle section and longitudinal section of the linear model with 411 099 DOF in the Fig. 6. The distribution in axial section (left Fig. 6) is similar to the two-dimensional model, see Fig. 4. Absolute values of the magnetic flux density are lower in case of the three-dimensional model (there is higher leakage of the magnetic field in the three-dimensional model). The distribution of the magnetic flux density in the stator ring (see right Fig. 6) is uniform in the neighborhood of the electromagnet with the highest current supply (except for the vicinity of the bearing faces). Situation is the opposite in the rotor (see right Fig. 6), where the distribution is considerably non-uniform. The highest magnetic flux density value is in the neighborhood of the bearing faces.

There are the electromagnetic forces values in the Table 3. These values are determined by formulas (6). The magnitudes are approximately equal in comparing with the two-dimensional model but comparing with the three-dimensional model the difference is about 20 %.

Table 3 The electromagnetic forces calculated in the one-dimensional model

Electromagnetic forces		
Force component	$f_{m,y}$=1775.342 N	$f_{m,z}$ =-1183.561 N

6 CONCLUSIONS

In order to investigate a dynamical response of rotor systems supported by radial AMBs the electromagnetic forces components must be determined. The electromagnetic force computation is carried out by the MSTM and using the VWM in two-dimensional and the three-dimensional magneto-static field. Magnetic material properties of the AMB stator and outer part of rotor are described by the linear or nonlinear B-H characteristic.

Differences between results obtained by both methods (MSTM, VWM) are negligible. Material properties description makes differences in results up to 10 %. The most crucial aspect is the number of model dimensions, for instance one-dimensional, two-dimensional or three-dimensional. This brings differences in results up to 30 %. Computational demands are much higher in case of usage the finite element method then in case of usage the one-dimensional approximation of the magnetic circuit. These demands rise even more if the nonlinear material properties are considered.

To conclude, in dynamical problems, computation of the electromagnetic forces by the finite element method is too computing time demanding so it can be hardly put into practice.

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