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Lukáš DVOŘÁK[*], Kamil FOJTÁŠEK[**]

COMPARISON OF TWO SIMPLIFIED MATHEMATIC MODELS OF PNEUMATIC SYSTEM

POROVNÁNÍ DVOU ZJEDNODUŠENÝCH MATEMATICKÝCH MODELŮ PNEUMATICKÉHO SYSTÉMU

Abstract

This paper deals with comparison of two methods of pneumatic systems modelling. The first one is mathematical modeling by means of *RHD* resistances, which are resistance to motion *R*, resistance to acceleration *H* and deformation resistance *D*. The second approach can be called as classical. This method is based on state equation of gas, continuity equation and motion equation. Output characteristics of models are compared with data obtained by measurement in the end.

Abstrakt

Tento článek se zabývá porovnáním dvou metod matematického modelování pneumatických mechanismů. První metodou je modelování pomocí *RHD* odporů, tj. pomocí odporu proti pohybu *R*, odporu proti zrychlení *H* a odporu proti deformaci *D*. Druhá metoda, kterou lze nazvat klasická, je založena na stavové rovnici plynu, rovnici kontinuity a pohybové rovnici. V závěru příspěvku jsou porovnány výstupní charakteristiky obou modelů.

1 MODELLING BY MEANS OF RHD RESISTANCES

This way of modelling is based on electro-pneumatic analogy. This method is used with the good results in the field of hydraulic systems [6] however in the area of pneumatic systems is not common. The reason can be in properties of the compressed air what makes difficult also much more simple calculations.

This method was more precisely described in Dvořák's doctoral thesis [3]. Program which computes pneumatic system characteristic by *RHD* resistance method in the software Matlab – Simulink was created. It consists of three parts i.e. model of compressed air source, model of valve and piping and model of pneumatic cylinder.

Into the model of air source it is necessary to enter the value of working pressure, which is input to the model of directional control valve and piping. Directional control valve can be described as a resistance to motion *R*. Resistance is caused by restriction when air flows through the valve. Quantity of resistance R can be calculated from the flow coefficient Kv and it also depends on the pressure ratio. Similarly the piping causes pressure losses by friction when air flows through it. Flow capacity of piping can be defined by flow coefficients for example Kv. Equivalent flow coefficient Kv of both elements can be then computed by methods described in literature [8], [10]. By the help of equivalent flow coefficient Kv it is possible to compute the total resistance of valve and piping, eq. (1)

[*] Ing., Ph.D. Department of Hydromechanics and Hydraulic Equipment, Faculty of Mechanical Engineering, VŠB-Technical University of Ostrava, 17. listopadu 15, Ostrava-Poruba, tel. (+420) 59 732 4384, e-mail lukas.dvorak@vsb.cz

[**] Ing. Department of Hydromechanics and Hydraulic Equipment, Faculty of Mechanical Engineering, VŠB-Technical University of Ostrava, 17. listopadu 15, Ostrava-Poruba, tel. (+420) 59 732 4314, e-mail kamil.fojtasek@seznam.cz

$$R_m = 10^9 \cdot \frac{\rho}{Kv^2 \cdot \sqrt{\frac{p_{2a}}{p_{1a}}}} \qquad (1)$$

In the equation (1) ρ is air density at working pressure, p_{1a} is absolute upstream pressure and p_{2a} is absolute downstream pressure. The formula was verified by experiment and it is possible to use it for subsonic and for choked (sonic) flow rate too.

Volume of air in pipes presents deformation resistance *D*. It is impossible to ignore it. Volume of pipeline can be added to dead volume of pneumatic cylinder. This modification does not influence results but makes the calculation simpler. Output value of valve and pipeline model is volumetric flow rate which is calculated from equation (2).

$$Q_1 = \sqrt{\frac{\Delta p}{R_m}} \qquad (2)$$

Pneumatic cylinder can be described as a combination of all resistance types. Resistance to motion *R* is caused by friction of piston and piston rod and its quantity can be calculated from efficiency of cylinder η, theoretical force F_{theor}, cylinder diameter D_p and piston velocity v_p.

$$R = \frac{p_{loss}}{Q} = \frac{F_{loss}}{S_p^2 \cdot v_p} = 1{,}621 \cdot \frac{F_{theor} \cdot (1-\eta)}{D_p^4 \cdot v_p} \qquad (3)$$

Resistance to acceleration *H* is caused by inertia of piston and piston rod mass and moving mass connected with piston rod.

$$H = \frac{m_{red}}{S_p^2} = \frac{m_{red} \cdot h^2}{V_1^2} = 1{,}621 \cdot \frac{m_{red}}{D_p^4} \qquad (4)$$

Deformation resistance *D* is caused by air compressibility. From experiment appears that it is possible to consider air compression in working chamber (chamber which is supplied with compressed air) as an isothermal process. Then the simply equation for resistance calculation can be used.

$$D_{\sup} = \frac{p_{2a}}{V_{\sup}} \qquad (5)$$

In the equation (5) p_{2a} is absolute pressure in the end of compression and V_{sup} is variable working chamber volume. It can be calculated with relation (6) where h_{act} is actual piston position and S_p is its area, constant 0,003 presents dead volume of cylinder and V_h is pipeline (hose) volume.

$$V_1 = S_p \cdot (h_{act} + 0{,}003) + V_h \qquad (6)$$

Actual quantity of exhaust chamber deformation resistance *D* can be calculated by the eq. (7)

$$D_{ex} = \frac{p_n}{V_{ex}} \cdot \left(\frac{p_{act}}{p_n} + 1 \right) \qquad (7)$$

where p_{act} is actual pressure in exhaust chamber, p_n is normal pressure and V_{ex} is half of exhaust chamber volume.

Working chamber can be described as a net of *RHD* resistances, see Fig. 1.

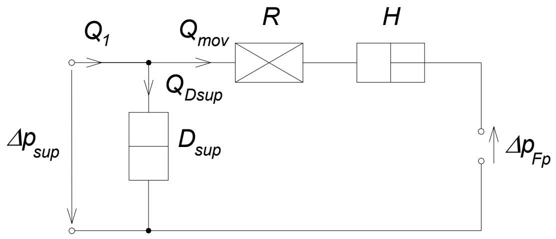

Fig. 1 Model of working chamber

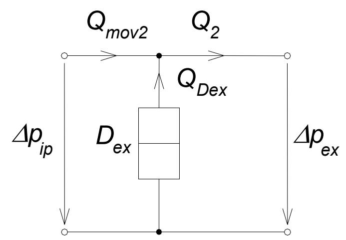

Fig. 2 Model of exhaust chamber

Operation of pneumatic cylinder can be distributed to three time parts. The first part concerns fulfilment of dead volume. Dead volume is added to volume of working chamber (equation (6)) and by the help of this volume the deformation resistance $D_{\sup}$ is calculated, equation (5). During the first time part the pressure increases (equation (8)) but the piston does not move yet.

$$\Delta p_{\sup} = D_{\sup} \cdot \int Q_1 \cdot dt \;\leq\; \Delta p_{Fp} \tag{8}$$

$$\Delta p_{Fp} = \frac{F + p_{ex} \cdot S_{p2}}{S_p} \tag{9}$$

When the pressure reaches Δp_{Fp} what is determined by force on piston and pressure in exhaust chamber the first time part is finished and starts the second part, i.e. movement of the piston. During the second time part inlet flow rate is shared into two branches. The flow rate called "flow rate to motion" Q_{mov} which flows through *R* and *H* resistances causes movement of piston. Piston velocity and position can be calculated from this flow rate, equation (11), (12).

$$h \in \langle 0; h_{\max} \rangle$$

$$Q_{mov} = \frac{D_{\sup}}{H} \int_0^t \left[\int_0^t Q_1 \cdot dt \right] \cdot dt - \frac{D_{\sup}}{H} \int_0^t \left[\int_0^t Q_{mov} \cdot dt \right] \cdot dt - \frac{R}{H} \int_0^t Q_{mov} \cdot dt \tag{10}$$

$$v_p = \frac{Q_{mov}}{S_p} \tag{11}$$

$$h_{act} = \int_o^t v_p \cdot dt \tag{12}$$

Flow rate $Q_{D\sup}$ influence pressure in working chamber which can be solved by relation (14).

$$Q_{D\sup} = Q_1 - Q_{mov} \tag{13}$$

$$\Delta p_{\sup} = \Delta p_{Fp} + D_{\sup} \int_0^t Q_{D\sup} \cdot dt \tag{14}$$

When the piston reaches end position the second time part is finished. All inlet flow rate causes growing of pressure in chamber (16).

$$h_{act} = h_{\max}$$
$$Q_1 = Q_{D\sup} \tag{15}$$

$$\Delta p_{\sup} = \Delta p_{Fp} + D_{\sup} \int_o^t Q_1 \cdot dt \tag{16}$$

Exhaust chamber can be described as one deformation resistance, see Fig. 2. Equation (17) allows to compute pressure during all time parts of cylinder operation.

$$\Delta p_{ex} = p_{ip} + D_{ex} \cdot S_{p2} \int_0^t v_p \cdot dt - D_{ex} \int_o^t Q_2 \cdot dt \tag{17}$$

In relation p_{ip} is initial pressure in chamber, S_{p2} is annulus area, v_p piston velocity and Q_2 is output flow which is calculated by the help of valve and pipe model.

2 CLASSICAL METHOD OF PNEUMATIC SYSTEMS MODELLING

As a classical method can be called method which is based on state equation, continuity equation and motion equation. This way of modelling is used since sixtieth years of the last century. This method is commonly known and described in many publications for example [2], [5], [9], [12]. The model of whole pneumatic system consists of directional valve model, pipelines model and model of pneumatic cylinder.

The flow rate through a pneumatic valve is represented in the following two formulas.

The case of sonic (choked) flow

$$Q_m = C \cdot p_1 \cdot \rho_0 \cdot \sqrt{\frac{T_0}{T_1}} \qquad p_2 / p_1 \le b \tag{18}$$

The case of subsonic flow

$$Q_m = C \cdot p_1 \cdot \rho_0 \cdot \sqrt{\frac{T_0}{T_1}} \cdot \sqrt{1 - \left(\frac{p_2/p_1 - b}{1-b} \right)^2} \qquad p_2 / p_1 > b \tag{19}$$

Where C is the sonic conductance which presents the flow passes ability and b is the critical pressure ratio.

Model of pipelines consists of three following equations [12]. The first of them is continuity equation,

$$\frac{\partial \rho}{\partial t} + \rho \cdot \frac{\partial w}{\partial z} + w \cdot \frac{\partial \rho}{\partial z} = 0 \tag{20}$$

The second is equation of state of an ideal gas

$$V \cdot \frac{dp}{dt} = R \cdot \frac{dT}{dt} - \frac{R \cdot T}{w} \cdot \frac{dw}{dt} \tag{21}$$

And the last one is equation of motion

$$\frac{\partial w}{\partial t} + w \cdot \frac{\partial w}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\lambda}{2 \cdot d_p} \cdot w \cdot |w| = 0 \tag{22}$$

In the equations ρ is density, z is position, p is absolute pressure, V is volume, R is gas constant, T is temperature w flow velocity and λ is friction coefficient.

Model of pneumatic cylinder consists of three following equations. The first of them is state equation of air in working chamber.

$$\frac{dp_s}{dt} = \frac{1}{V_s}\left(\frac{p_s \cdot V_s}{T_s}\frac{dT_s}{dt} + R \cdot T_s \cdot Q_s - p_s \frac{dV_s}{dt}\right) \quad (23)$$

Similarly the state equation of air in exhaust chamber is following

$$\frac{dp_e}{dt} = \frac{1}{V_e}\left(\frac{p_e \cdot V_e}{T_e}\frac{dT_e}{dt} - R \cdot T_e \cdot Q_e - p_e \frac{dV_e}{dt}\right) \quad (24)$$

The third of the pneumatic cylinder model equations is movement equation of piston.

$$m\frac{dv_p}{dt} = p_s \cdot S_s - p_e \cdot S_e - p_{atm} \cdot (S_s - S_e) - m \cdot g \cdot \sin\alpha - F_{fr} - F \quad (25)$$

In the equations (23, 24, 25) p is absolute pressure, V is volume of chambers, T is thermodynamic temperature, Q is mass flow rate, S is area of working and exhaust sides of piston, m is mass connected with piston rod, α is inclination angle of cylinder, F_{fr} is friction force and F is load force.

A lot of simulation programs were compiled on the base of mentioned relations [1], [7], [11]. Programs can differ for example in description of the friction force however the principle of modelling is the same. The program based on the classical method was compiled within the scope of Fojtášek's [4] graduation thesis however model of pneumatic system was simplified. Simplifications were following: air temperature was considered a constant, piston friction force was considered a constant where $F_{fr} = 0{,}95 \cdot F_{theor}$ and model of pipes was not considered because pipes were short. It corresponds with simplifications of *RHD* resistance model. In the thesis the accuracy of classical and *RHD* resistance models was tested in comparison with results of measurement on the real mechanism. Some results are mentioned below.

3 COMPARISON OF MODELS RESULTS

Real mechanisms consisted of pneumatic cylinders (C92SDB-40-500, C95QDB 63-250CB both by SMC), directional valves (SMC - SYA3220-M5, Festo - SV-5-M5-B) and plastic hoses with inside diameter $4\,mm$ and length $0{,}5\,m$. Within experiment pneumatic cylinders were affixed on frame which allowed to load the piston rod. Scheme of mechanism is in Fig. 3 and photo of mechanism is in Fig.4. Load mass on the piston rod was $11{,}5\,kg$ and inclination angle of cylinder was $90\,deg$. Pneumatic cylinders and directional valves were combined and every combination of elements was measured with and without load mass on piston rod.

Input parameters of both models are described in table 1 for one combination. Into the model compiled by means of *RHD* resistances it is necessary to enter parameters as dimensions of cylinder, efficiency and valve flow coefficient which can be found out in catalogues of pneumatic components.

Classical model requires knowledge in the same parameters. The only difference is parameter of directional valve because it is necessary to know sonic conductance C and crit. pressure ratio b. These parameters of some directional valves can be found out in catalogues too however in the majority of cases the C and b have to be determined by experiment.

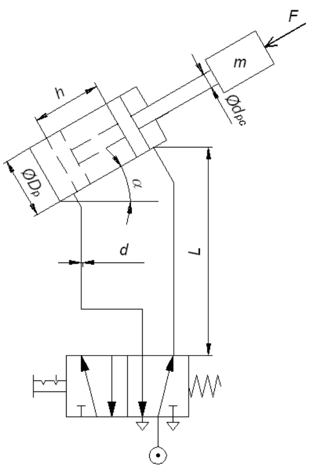

Fig. 3 Simple pneumatic system

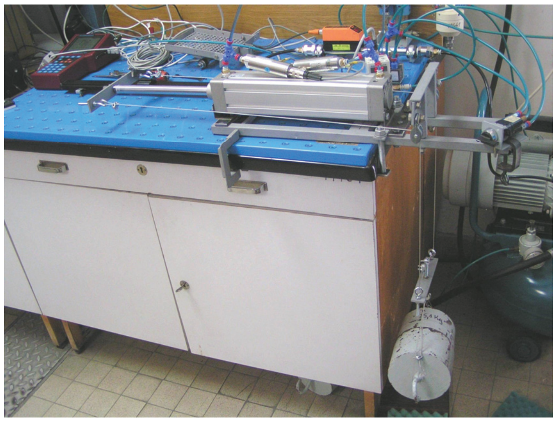

Fig. 4 Load of piston rod

Tab. 1 Input parameters

directional valve SYA 3220 SMC)	RHD method	*Kv*	0,1333	$m^3.h^{-1}$
	classical method	*C*	0,61	$dm^3.s^{-1}.bar^{-1}$
		b	O,44	-
polyurethane hoses		*L*	0,5	*m*
		d	0,004	*m*
cylinder C92SDB-40-500 (SMC)		D_p	0,04	*m*
		d_{pc}	0,016	*m*
		h	0,5	*m*
		F	0	*N*
		m	0	*kg*
		η	0,93	-
		α	90	*deg*
working pressure		*p*	$5 \cdot 10^5$	*Pa*

In the following figures (Fig. 5 – Fig. 7) there are results of simulation by both methods and results of measurement. From the comparison of simulation and experimental results appears good correspondence of curves of piston position and velocity. Experimental stroke time was 1,32 s and values obtained by simulation was following: *RHD* method – 1,01 s, classical method 0,99 s. Stroke time difference between measurement and simulations was about 25% (0,3 s). In other cases differences were max. to 30%. This was caused by dumping of piston movement in the end of the stroke. Damping is not included into models. However difference between classical and *RHD* method was only 4% (0,04 s) and in other cases max. to 20%.

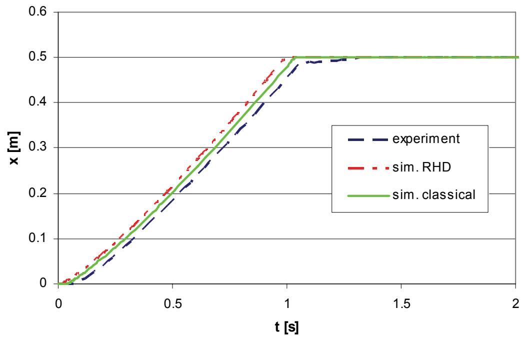

Fig. 5 Curves of piston position

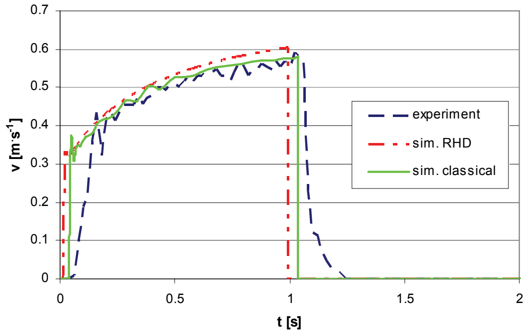

Fig. 6 Curves of piston velocity

The simulated curves of pressures in chambers differ from experiment results mainly in the cases of *RHD* simulation method however it does not influence dynamics of mechanism.

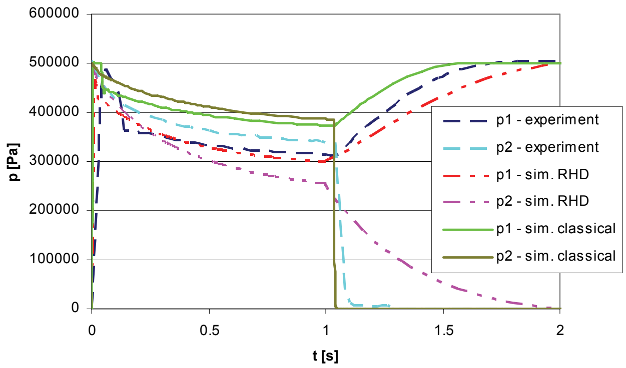

Fig. 7 Curves of pressure in chambers

4 CONCLUSIONS

In this paper were presented two methods of pneumatic systems modelling. The classical method which is based on general state equation, continuity equation and motion equation can be considered as accurate. However this method requires definition of certain element information that must be defined experimentally, as for example in case of directional control valves the sonic conductance C and critical pressure ratio b.

Method based on RHD models provides good results, too. Input parameters can be found out in catalogues of pneumatic components. Experiment isn't necessary. However up till now this method has been verified only for the double-acting cylinders with the piston diameter up to $D_p = 63\,mm$ and with stroke up to $h = 500\,mm$. Another limit of the model is the size of directional control valve. The good results were obtained with the models where the directional control valve with flow coefficient up to $Kv = 0{,}14\,m^3 \cdot h^{-1}$ was used.

On the base of models comparison it can be said that classical model is more accurate. However to obtain input parameters is more difficult in comparison with *RHD* simulation method. Both methods of modeling bring good information about pneumatic system behaviour. This information is needed for correct system design. Better results of both models can be obtained mainly by more accurate description of friction force.

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