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FATIGUE LIFE VISUALISATION

VIZUALIZACE ÚNAVOVÉ ŽIVOTNOSTI

Abstract

The visualization of cycle number (fatigue life) approach before crack initiation in seamless cylinders for natural gas provides a new, complex view of limit states in material fatigue. Method, described in the article, includes amplitudes, average values of stress and strain tensors, and load history. We are able to see cycle number distribution before the crack initiation and therefore it is possible to decide much more effectively on relevant modifications in the seamless cylinder design. In this paper we described the method of cycle number before the crack initiation calculation and offered an appropriate iterative operation. Material fatigue model is highly sensitive to input data and very often a divergent solution occurs, and that is a problem the proposed calculation method solves. Thanks to such visualization approach we are now able to assess limit states in material fatigue much more effectively.

Abstrakt

Přístup vizualizace počtu cyklů do iniciace trhliny tlakové láhve poskytuje nový komplexní pohled na mezní stav únavy materiálu. Způsob popsaný v tomto článku zahrnuje amplitudy a střední hodnoty tenzorů napětí a deformací a i historii zatěžování. Můžeme tedy objektivně vidět rozložení počtu cyklů do iniciace trhliny a tím mnohem lépe rozhodnout o případných konstrukčních změnách tlakové láhve. V tomto příspěvku je popsán vlastní způsob výpočtu počtu cyklů do iniciace trhliny a navrženo vhodné iterační řešení. Model únavy materiálu je totiž obecně velmi citlivý na vstupní hodnoty a velmi často dochází k divergenci řešení což navržený způsob výpočtu řeší. Díku tomuto přístupu vizualizace jsme nyní schopni mnohem lépe posoudit mezní stav únavy materiálu.

1 INTRODUCTION

This paper deals with possible approach to cycle number distribution visualization before crack initialization with a view to seamless cylinder LA 4 – 0216, produced by VÍTKOVICE Lahvárna a.s. Visualization of cycle number before crack initiation in the whole volume of seamless cylinder is a new approach to fatigue life of technical components evaluation. Critical locations identification was in the past based for example on maximum stress or strain values. However, the locations with maximal values don't have to correspond to locations with lowest cycle number before crack initialization and that's because the fatigue life calculation relationship is influenced by several factors. In other words, for example maximal value of reduced stress doesn't necessary be in the same location, where the critical location from the material fatigue point of view is located. The cycle number visualization approach before crack initialization, described in following chapters, offers very complex view to given topic and thus makes stated results rather more authentic.

The paper resumes a previous article "Fatigue life determination of seamless cylinder from material fatigue aspect", which deals with determination of fatigue life principles of seamless cylinder in given operating conditions.

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2 PRINCIPLE OF CYCLE NUMBER CALCULATION BEFORE CRACK INITIATION

The principle of cycle number calculation before crack initiation is described in (fig. 1). This approach takes into account non-symmetric cycle of stress curve (strain), flexible and plastic strains and multi-axial in phase stress. The relation between these factors and cycle number before the crack initiation N_f could be expressed as (1).

$$(\sigma_a + \sigma_m)\varepsilon_a = \sigma_f \left(2N_f\right)^b \left[\frac{\sigma_f \left(2N_f\right)^b}{E} + \varepsilon_f \left(2N_f\right)^c \right] \tag{1}$$

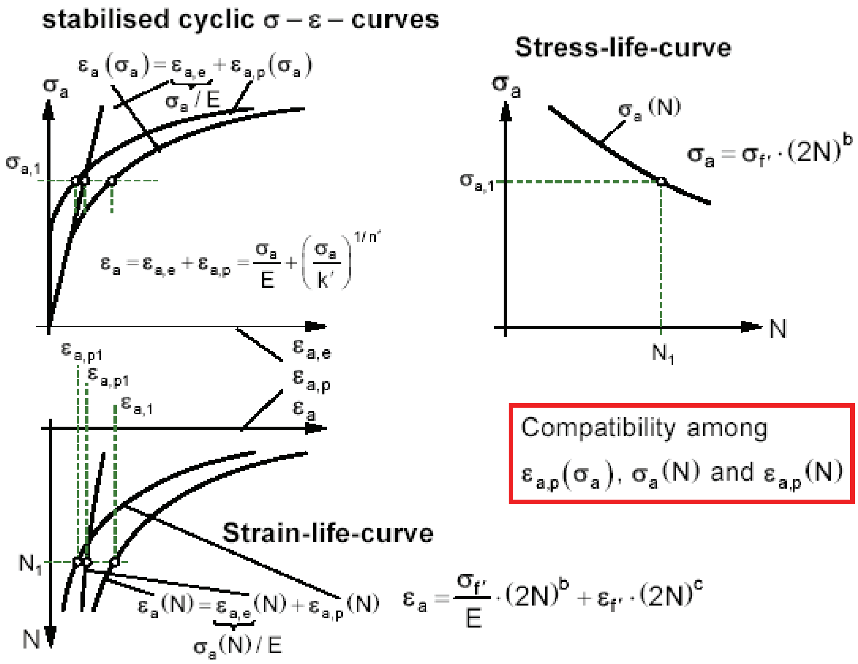

Fig. 1 Model of cycle number calculation before crack initiation

In case of multi-axial stress we can reduce stress amplitude and determine its middle value according to equations (2) and (3).

$$\sigma_a = \left\{ \frac{1}{2} \left[\left(\sigma_{x,a} - \sigma_{y,a}\right)^2 + \left(\sigma_{y,a} - \sigma_{z,a}\right)^2 + \left(\sigma_{z,a} - \sigma_{x,a}\right)^2 + 6\left(\sigma_{xy,a}^2 + \sigma_{yz,a}^2 + \sigma_{xz,a}^2\right) \right] \right\}^{\frac{1}{2}} \tag{2}$$

$$\sigma_m = \sigma_{x,m} + \sigma_{y,m} + \sigma_{z,m} \tag{3}$$

Total reduced strain amplitude e_a is determined by adding plastic and elastic strain. Poisson number in the elastic strain calculation corresponds to Poisson number of given material in elastic field, in plastic strain calculation will amount to 0.5.

$$\varepsilon_a = \varepsilon_a^{el} + \varepsilon_a^{pl} \tag{4}$$

$$\varepsilon_a^{el} = \frac{1}{1+\mu} \left\{ \frac{1}{2} \left[\left({\varepsilon^{el}}_{x,a} - {\varepsilon^{el}}_{y,a}\right)^2 + \left({\varepsilon^{el}}_{y,a} - {\varepsilon^{el}}_{z,a}\right)^2 + \left(\varepsilon_{z,a} - \varepsilon_{x,a}\right)^2 + 6\left({\varepsilon^{el2}}_{xy,a} + {\varepsilon^{el2}}_{yz,a} + {\varepsilon^{el2}}_{xz,a}\right) \right] \right\}^{\frac{1}{2}} \tag{5}$$

$$\varepsilon^{pl}{}_a = \frac{1}{1+0,5}\left\{\frac{1}{2}\left[\left(\varepsilon^{pl}{}_{x,a}-\varepsilon^{pl}{}_{y,a}\right)^2+\left(\varepsilon^{pl}{}_{y,a}-\varepsilon^{pl}{}_{z,a}\right)^2+\left(\varepsilon^{pl}{}_{z,a}-\varepsilon^{pl}{}_{x,a}\right)^2+6\left(\varepsilon^{pl2}{}_{xy,a}+\varepsilon^{pl2}{}_{yz,a}+\varepsilon^{pl2}{}_{xz,a}\right)\right]\right\}^{\frac{1}{2}} \quad (6)$$

Cycle number calculation N_f in equation (1) is rather not simple, therefore the cycle number will be calculated by using iterative approach according to bellow mentioned relations. The equation (1) will be rewritten into (7).

$$\frac{(\sigma_a+\sigma_m)\varepsilon_a}{\frac{\sigma_f{}^2}{E}+\varepsilon_f\sigma_f\left(2N_f\right)^{c-b}} = \left(2N_f\right)^{2b} \quad (7)$$

We calculate the left-hand side of equation, using the cycle number determined in previous steps, i.e. based on N_f^{i-1}. Consequently, cycle number in step i can be expressed in the right-hand side of equation according to relation (8).

$$N_f{}^i = \frac{1}{2}\exp\left(\frac{\log\left[\frac{(\sigma_a+\sigma_m)\varepsilon_a}{\frac{\sigma_f{}^2}{E}+\varepsilon_f\sigma_f\left(2N_f{}^{i-1}\right)^{c-b}}\right]}{2b}\right) \quad (8)$$

Afterwards the equation (1) will be expressed in figures (9).

$$\frac{(\sigma_a+\sigma_m)\varepsilon_a}{\frac{\sigma_f{}^2}{E}\left(2N_f\right)^{b-c}+\varepsilon_f\sigma_f} = \left(2N_f\right)^{c+b} \quad (9)$$

In the right-hand side of equation (9) we calculate the cycle number using the same method as in equation (8).

$$N_f{}^i = \frac{1}{2}\exp\left(\frac{\log\left[\frac{(\sigma_a+\sigma_m)\varepsilon_a}{\frac{\sigma_f{}^2}{E}\left(2N_f{}^{i-1}\right)^{b-c}+\varepsilon_f\sigma_f}\right]}{b+c}\right) \quad (10)$$

While the calculation according to equation (8) shows us the top estimation of cycle number before crack initiation, equation (10) gives us on the contrary the lowest estimation. By using both the (8) and (10) equation, very often a long iterary time occurs, or in worst cases a divergent solution occurs. For this reason it is advisable to use combination of both relations according to (11).

$$N_f{}^i = \frac{1}{4}\exp\left(\frac{\log\left[\frac{(\sigma_a+\sigma_m)\varepsilon_a}{\frac{\sigma_f{}^2}{E}+\varepsilon_f\sigma_f\left(2N_f{}^{i-1}\right)^{c-b}}\right]}{2b}\right)+\frac{1}{4}\exp\left(\frac{\log\left[\frac{(\sigma_a+\sigma_m)\varepsilon_a}{\frac{\sigma_f{}^2}{E}\left(2N_f{}^{i-1}\right)^{b-c}+\varepsilon_f\sigma_f}\right]}{b+c}\right) \quad (11)$$

$$(\sigma_a+\sigma_m)\varepsilon_a = \sigma_f\left(2N_f\right)^b\left[\frac{\sigma_f\left(2N_f\right)^b}{E}+\varepsilon_f\left(2N_f\right)^c\right] \quad (12)$$

In case of multi-axial stress we can reduce stress amplitude and determine its middle value according to equations (2) and (3).

$$\sigma_a = \left\{ \frac{1}{2} \left[\left(\sigma_{x,a} - \sigma_{y,a} \right)^2 + \left(\sigma_{y,a} - \sigma_{z,a} \right)^2 + \left(\sigma_{z,a} - \sigma_{x,a} \right)^2 + 6\left(\sigma_{xy,a}^2 + \sigma_{yz,a}^2 + \sigma_{xz,a}^2 \right) \right] \right\}^{\frac{1}{2}} \tag{13}$$

$$\sigma_m = \sigma_{x,m} + \sigma_{y,m} + \sigma_{z,m} \tag{14}$$

Total reduced strain amplitude e_a is determined by adding plastic and elastic strain. Poisson number in the elastic strain calculation corresponds to Poisson number of given material in elastic field, in plastic strain calculation will amount to 0.5.

$$\varepsilon_a = \varepsilon_a^{el} + \varepsilon_a^{pl} \tag{15}$$

$$\varepsilon_a^{el} = \frac{1}{1+\mu} \left\{ \frac{1}{2} \left[\left(\varepsilon^{el}{}_{x,a} - \varepsilon^{el}{}_{y,a} \right)^2 + \left(\varepsilon^{el}{}_{y,a} - \varepsilon^{el}{}_{z,a} \right)^2 + \left(\varepsilon_{z,a} - \varepsilon_{x,a} \right)^2 + 6\left(\varepsilon^{el2}_{xy,a} + \varepsilon^{el2}_{yz,a} + \varepsilon^{el2}_{xz,a} \right) \right] \right\}^{\frac{1}{2}} \tag{16}$$

$$\varepsilon_a^{el} = \frac{1}{1+\mu} \left\{ \frac{1}{2} \left[\left(\varepsilon^{el}{}_{x,a} - \varepsilon^{el}{}_{y,a} \right)^2 + \left(\varepsilon^{el}{}_{y,a} - \varepsilon^{el}{}_{z,a} \right)^2 + \left(\varepsilon_{z,a} - \varepsilon_{x,a} \right)^2 + 6\left(\varepsilon^{el2}_{xy,a} + \varepsilon^{el2}_{yz,a} + \varepsilon^{el2}_{xz,a} \right) \right] \right\}^{\frac{1}{2}} \tag{17}$$

$$\varepsilon^{pl}{}_a = \frac{1}{1+0{,}5} \left\{ \frac{1}{2} \left[\left(\varepsilon^{pl}{}_{x,a} - \varepsilon^{pl}{}_{y,a} \right)^2 + \left(\varepsilon^{pl}{}_{y,a} - \varepsilon^{pl}{}_{z,a} \right)^2 + \left(\varepsilon^{pl}{}_{z,a} - \varepsilon^{pl}{}_{x,a} \right)^2 + 6\left(\varepsilon^{pl2}_{xy,a} + \varepsilon^{pl2}_{yz,a} + \varepsilon^{pl2}_{xz,a} \right) \right] \right\}^{\frac{1}{2}} \tag{18}$$

Cycle number calculation N_f in equation (1) is rather not simple, therefore the cycle number will be calculated by using iterative approach according to bellow mentioned relations. The equation (1) will be rewritten into (7).

$$\frac{(\sigma_a + \sigma_m)\varepsilon_a}{\frac{\sigma_f{}^2}{E} + \varepsilon_f \sigma_f \left(2N_f\right)^{c-b}} = \left(2N_f\right)^{2b} \tag{19}$$

We calculate the left-hand side of equation, using the cycle number determined in previous steps, i.e. based on N_f^{i-1}. Consequently, cycle number in step i can be expressed in the right-hand side of equation according to relation (8).

$$N_f{}^i = \frac{1}{2}\exp\left(\frac{\log\left[\frac{(\sigma_a + \sigma_m)\varepsilon_a}{\frac{\sigma_f{}^2}{E} + \varepsilon_f \sigma_f \left(2N_f{}^{i-1}\right)^{c-b}} \right]}{2b} \right) \tag{20}$$

Afterwards the equation (1) will be expressed in figures (9).

$$\frac{(\sigma_a + \sigma_m)\varepsilon_a}{\frac{\sigma_f{}^2}{E}\left(2N_f\right)^{b-c} + \varepsilon_f \sigma_f} = \left(2N_f\right)^{c+b} \tag{21}$$

In the right-hand side of equation (9) we calculate the cycle number using the same method as in equation (8).

$$N_f{}^i = \frac{1}{2}\exp\left(\frac{\log\left[\frac{(\sigma_a + \sigma_m)\varepsilon_a}{\frac{\sigma_f{}^2}{E}\left(2N_f{}^{i-1}\right)^{b-c} + \varepsilon_f \sigma_f} \right]}{b+c} \right) \tag{22}$$

While the calculation according to equation (8) shows us the top estimation of cycle number before crack initiation, equation (10) gives us on the contrary the lowest estimation. By using both the

(8) and (10) equation, very often a long iterary time occurs, or in worst cases a divergent solution occurs. For this reason it is advisable to use combination of both relations according to (11).

$$N_f^{\,i} = \frac{1}{4}\exp\left(\frac{\log\left[\dfrac{(\sigma_a+\sigma_m)\varepsilon_a}{\dfrac{\sigma_f^{\,2}}{E}+\varepsilon_f\sigma_f\left(2N_f^{\,i-1}\right)^{c-b}}\right]}{2b}\right)+\frac{1}{4}\exp\left(\frac{\log\left[\dfrac{(\sigma_a+\sigma_m)\varepsilon_a}{\dfrac{\sigma_f^{\,2}}{E}\left(2N_f^{\,i-1}\right)^{b-c}+\varepsilon_f\sigma_f}\right]}{b+c}\right) \qquad (23)$$

Initial cycle number N_f^0 was determined as 10^7. Convergency criteria (error of calculation due to iterative solution) was stated 10 cycles. Application of this relation usually leads to a solution within a few iterative steps, as shown in Fig. 2.

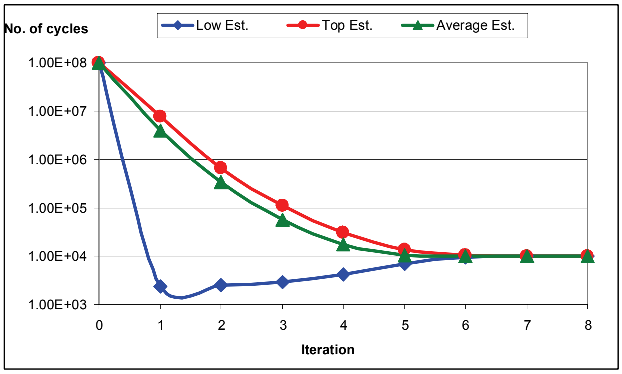

Fig. 2 Convergency of cycle number calculation before crack initiation

3 VIZALIZATION OF CYCLE NUMBER BEFORE CRACK INITIATION

For visualization of cycle number before crack initiation in the whole volume of analyzed seamless cylinder it is necessary to calculate cycle number N_f in all nodes of the finite element model. Given model contains a high number of nodes and calculation N_f proceeds iterative, therefore the time necessary for results evaluation (at all times and all nodes of the model) is much longer than the solution of stress strain analysis using the finite elements method. Figures bellow show cycle number distribution before crack initiation. They also show very critical locations from material fatigue aspect and minimal values of stated cycle number.

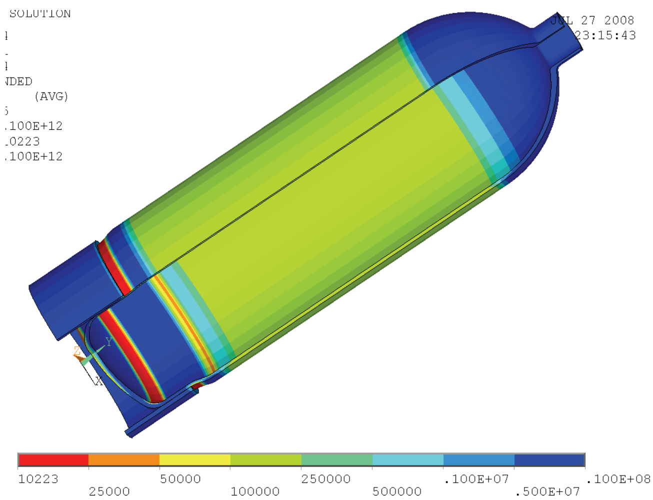

Fig. 3 Distribution of cycle number before crack initiation in seamless cylinder – overview

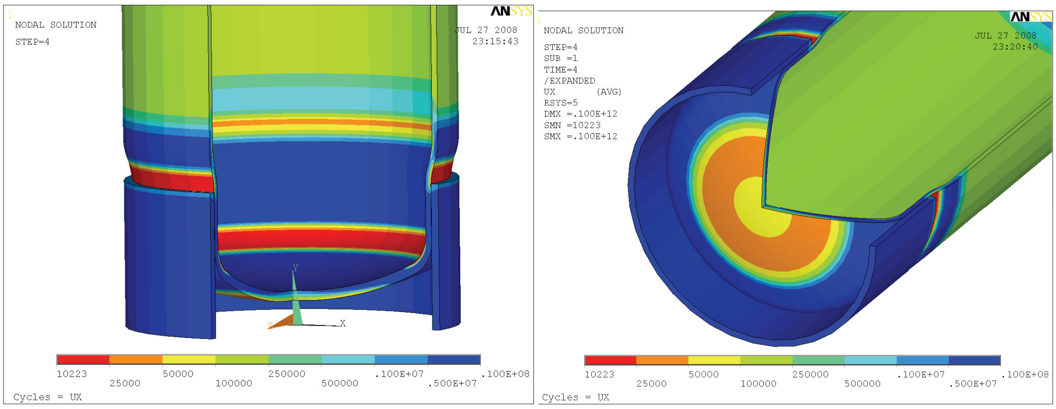

Fig. 4 Distribution of cycle number before crack initiation in seamless cylinder – bottom

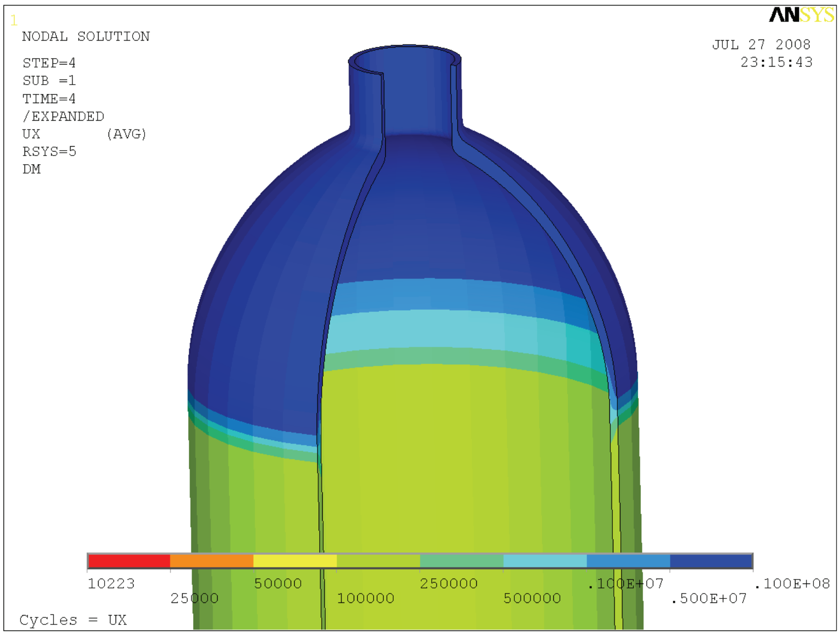

Fig. 5 Distribution of cycle number before crack initiation in seamless cylinder – neck

4 CONCLUSION

The visualization of cycle number (fatigue life) before crack initiation in seamless cylinders for natural gas provides a new, complex view of limit states in material fatigue. Method, described in the article, includes amplitudes, average values of stress and strain tensors and load history. We are able to see cycle number distribution before the crack initiation and therefore it is possible to decide much more effectively on relevant modifications in the seamless cylinder design, and we are able to decide whether evaluated seamless cylinder fulfils given criteria or not.

In this paper we described the method of cycle number before crack initialization calculation and offered appropriate iterative operation. Without the appropriate iterative solution the calculation wouldn't be possible. Material fatigue model is highly sensitive to input data and very often a divergent solution occurs, and that is a problem the proposed calculation method solves. Thanks to such visualization approach we are now able to assess limit states in material fatigue more effectively, before the production starts and before it is possible to carry out laboratory tests. Thanks to this approach we are also able to decide on further design modifications of seamless cylinder much more effectively.

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