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NEW 2DOF PI AND PID CONTROLLERS TUNING METHOD FOR INTEGRATING PLANTS

NOVÁ METODA SEŘÍZENÍ 2DOF PI A PID REGULÁTORŮ PRO INTEGRAČNÍ REGULOVANÉ SOUSTAVY

Abstract

The paper deals with a new 2DOF PI and PID controllers tuning method for integrating plants. The described approach is derived from the multiple dominant pole method and it enables the achievement of an aperiodic servo and regulatory step responses.

Abstrakt

V příspěvku je popsána nova metoda seřízení 2DOF PI a PID regulátorů pro integrační regulované soustavy. Uvedený přístup je odvozen z metody násobného dominantního pólu a umožňuje dosažení aperiodického průběhu pro skokovou změnu polohy žádané veličiny i poruchové veličiny působící na vstupu regulované soustavy.

1 INTRODUCTION

The use of PI and PID controllers for integrating plants is not often discussed in the control system literature. At the same time their tuning does not belong among simple problems [Åström, Hägglund 2006; O´Dwyer 2006; Vítečková, Víteček 2008; Hudzovič, Kozáková 2001; Rosinová, Markech 2008]. It is given by the degree of the astatism $q \geq 2$, which induces a predisposition to oscillations and big overshoots [Vítečková, Víteček 2008; Vítečková, Víteček 2009].

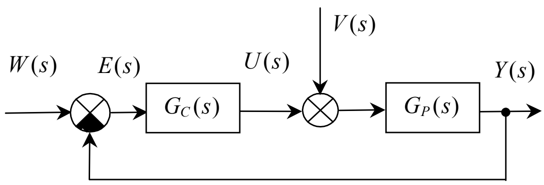

Fig. 1 Control system with standard controller

It is considered the control system in Fig. 1, where: $E(s)$, $W(s)$, $V(s)$ and $Y(s)$ are the transforms of the error, the desired variable, the disturbance variable and the output variable; $G_C(s)$ – the controller transfer function; $G_P(s)$ – the plant transfer function.

It is known that it is impossible to obtain for the standard PI or PID controllers the aperiodic servo step response without the overshoot [Vítečková, Víteček 2008; Vítečková, Víteček 2009]. If the

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overshoot is inadmissible, then it is necessary to use a limitation of the desired variable velocity or the input filtration of the desired variable. In this case it is very favourable to use the 2DOF (two degrees of freedom) controllers, where by the suitable choice of the weights of the desired variable in the proportional and derivative terms, it is possible to obtain corresponding input filtration [Vítečková, Víteček 2008; Vítečková, Víteček 2009].

From the above mentioned it follows that the servo and regulatory step responses cannot be simultaneously aperiodic without overshoots for the integrating plants and the standard PI or PID controllers.

It is obvious that the same conclusion holds for the integrating plants of arbitrary orders with a time delay too and the corresponding digital controllers as well [Krokavec, Filasová 2006].

Below the first order plants with integration will be considered, i.e. the degree of astatism $q = 2$ is supposed.

2 MULTIPLE DOMINANT POLE METHOD

In the paper the first order plant with integration without time delay is supposed

$$G_P(s) = \frac{k_1}{s(T_1 s + 1)} \tag{1}$$

and the standard PID controller

$$G_C(s) = r_0 + \frac{r_{-1}}{s} + r_1 s = r_0\left(1 + \frac{1}{T_I s} + T_D s\right) \tag{2}$$

$$T_I = \frac{r_0}{r_{-1}}, \quad T_D = \frac{r_1}{r_0} \tag{3}$$

where k_1 is the plant gain, T_1 – the time constant, r_0 – the proportional term weight (the controller gain), r_{-1} – the integral term weight, r_1 – the derivative term weight, T_I – the integral time, T_D – the derivative time.

It is obvious that for $r_1 = 0$ or $T_D = 0$ from (2) the transfer function of the standard PI controller

$$G_C(s) = r_0 + \frac{r_{-1}}{s} = r_0\left(1 + \frac{1}{T_I s}\right) \tag{4}$$

is obtained.

The multiple dominant pole method supposes the existence of a stable real dominant pole with the multiplicity increased by 1 over the number of the adjustable parameters of the chosen controller [Górecki 1971; Vítečková, Víteček 2008; Vítečková, Víteček 2009].

The multiple dominant pole and the adjustable parameter values can be obtained by solving the equation system

$$\frac{\mathrm{d}^i N(s)}{\mathrm{d}s^i} = 0 \tag{5}$$

where $N(s)$ is the characteristic polynomial of the control system with the plant (1) and chosen controller (2) or (4). For the plant (1) and the standard PID controller (2) the equation system

$$\begin{aligned} N(s) &= T_1 s^3 + (1 + k_1 r_1)s^2 + k_1 r_0 s + k_1 r_{-1} \\ \frac{\mathrm{d}N(s)}{\mathrm{d}s} &= 3T_1 s^2 + 2(1 + k_1 r_1)s + k_1 r_0 \\ \frac{\mathrm{d}^2 N(s)}{\mathrm{d}s^2} &= 6T_1 s + 2(1 + k_1 r_1) \end{aligned} \tag{6}$$

is obtained.

By solving that equation system (6) for the ratio

$$\alpha = \frac{r_1 r_{-1}}{r_0^2} = \frac{T_D}{T_I} \geq 0 \tag{7}$$

the triple dominant pole and weights can be expressed on the dependency of the ratio (7)

$$s_3^* = -\frac{1}{3T_1(1-3\alpha)}, \quad 0 \leq \alpha < \frac{1}{3} \tag{8}$$

$$r_0^* = \frac{1}{3k_1 T_1(1-3\alpha)^2}, \quad r_{-1}^* = \frac{1}{27k_1 T_1^2 (1-3\alpha)^3}, \quad r_1^* = \frac{3\alpha}{k_1(1-3\alpha)} \tag{9}$$

or after consideration of the relations (3)

$$T_I^* = 9T_1(1-3\alpha), \quad T_D^* = 9\alpha T_1(1-3\alpha) \tag{10}$$

may be obtained.

It is obvious, that for $\alpha = 0$ it is obtained tuning formulas for the standard PI controller (4).

The control system transfer function for the plant (1), the standard PID controller (2) and the computed controller adjustable parameters (7) – (10) have the form

$$G_{wy}(s) = \frac{Y(s)}{W(s)} = \frac{G_C(s)G_P(s)}{1 + G_C(s)G_P(s)} = \\ = \frac{\left[\frac{9}{2}T_1(1-3\alpha)\left(1+\sqrt{1-4\alpha}\right)s + 1\right] \cdot \left[\frac{9}{2}T_1(1-3\alpha)\left(1-\sqrt{1-4\alpha}\right)s + 1\right]}{[3T_1(1-3\alpha)s + 1]^3} \tag{11}$$

and the control system transfer function for the disturbance has the form

$$G_{vy}(s) = \frac{Y(s)}{V(s)} = \frac{G_P(s)}{1 + G_C(s)G_P(s)} = \frac{27k_1 T_1^2 (1-3\alpha)^3 s}{[3T_1(1-3\alpha)s + 1]^3} \tag{12}$$

The both relations (11) and (12), likely as relations (8) – (10), for $\alpha = 0$ hold for the standard PI controller ($r_1^* = 0,\ T_D^* = 0$)

From the relation (11) it follows that the nominator of the control system transfer function for $\alpha = 0.25$ (the value used by Ziegler and Nichols) has a stable double real zero and the corresponding standard PID controller adjustable parameters are given by the formulas

$$r_0^* = \frac{16}{3k_1 T_1}, \quad T_I^* = \frac{9}{4}T_1, \quad T_D^* = \frac{9}{16}T_1 \tag{13}$$

The servo and regulatory step responses for α = 0, 0.05, 0.1, 0.15, 0.2 a 0.25 are shown in Fig. 2.

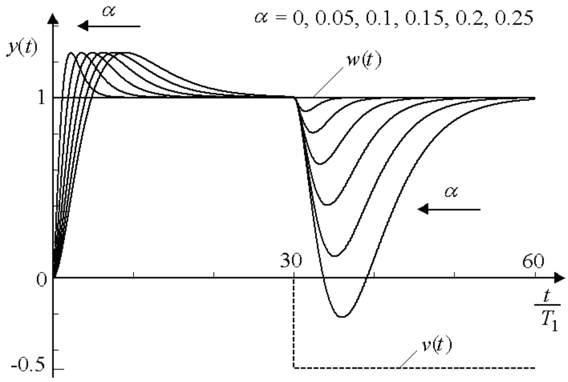

Fig. 2 Servo and regulatory step responses

From Fig. 2 it follows that the servo step responses for $0 \le \alpha \le 0.25$ show a practically constant overshoots about 25 %. For the value α = 0 the standard PI controller is obtained and the tuning is rather conservative. For positive values $0 < \alpha \le 0.25$ standard PID controller is obtained. For the value α = 0.25 the high-quality regulatory response is obtained. By the corresponding choice of value α, the controller tuning can conform to the limitation of the manipulated variable.

4 CONTROLLERS WITH TWO DEGREE OF FREEDOM

If the overshoot for the servo response is inadmissibly big, then it is possible to use the 2DOF PID controller, which is described by relation [Åström, Hägglund 2006; Vítečková, Víteček 2008; Vítečková, Víteček 2009].

$$U(s) = r_0 \left\{ bW(s) - Y(s) + \frac{1}{T_I s} E(s) + T_D s [cW(s) - Y(s)] \right\} \quad (14)$$

where b is the weight of the desired variable for proportional term, c – the weight of the desired variable for derivative term.

For $b = c = 1$ the standard PID controller (2) is obtained and for $b = 1$ a $T_D = 0$ the standard PI controller (4) is obtained.

The control system with the 2DOF PID controller (14) can be transformed in the scheme in Fig. 3 with the input filter with the transfer function

$$G_F(s) = \frac{W'(s)}{W(s)} = \frac{cT_I T_D s^2 + bT_I s + 1}{T_I T_D s^2 + T_I s + 1} \quad (15)$$

and the PID standard controller with the transfer function (2).

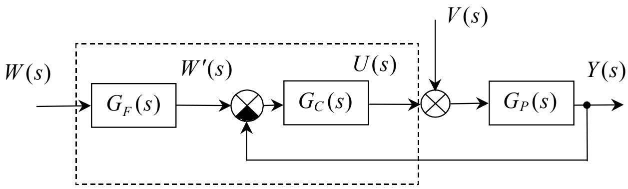

Fig. 3 Control system with 2DOF controller

For $T_D = 0$ from relation (14) the 2DOF PI controller can be obtained, which corresponds to the input filter in Fig. 3 with the transfer function

$$G_F(s) = \frac{W'(s)}{W(s)} = \frac{bT_I s + 1}{T_I s + 1} \tag{16}$$

and the standard PI controller with the transfer function (4).

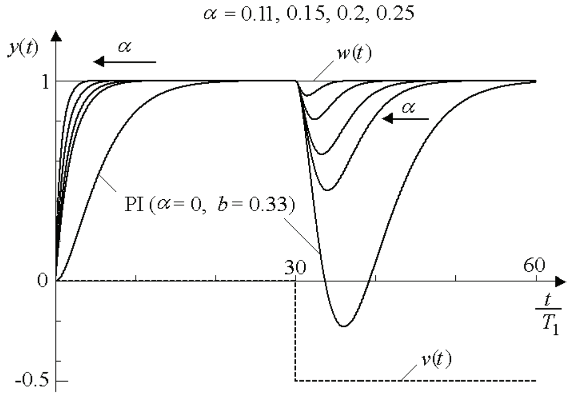

Fig. 4 Servo and regulatory responses for controllers with two degree of freedom

For the 2DOF PID controller the two poles of (11) can be compensated with two input filter zeros (15) by suitable choice of the set-point weights b and c. These weights can be obtained via comparison of the coefficients for the same power of the complex variable s, i.e.

$$\left(\frac{1}{|s_4^*|}s+1\right)^2 = \frac{2+\sqrt{3}}{6}T_d^2 s^2 + \frac{3+\sqrt{3}}{3}T_d s + 1 = cT_I^* T_D^* s^2 + bT_I^* s + 1 \tag{17}$$

After substitution (10) in the relation (17) the set-point weights

$$b = \frac{2}{3}, \qquad c = \frac{1}{9\alpha} \tag{18}$$

can be obtained.

Because $0 \le c \le 1$, the practical values of α for the 2DOF PID controller are

$$\frac{1}{9} \le \alpha \le \frac{1}{4} \tag{19}$$

Similarly for the 2DOF PI controller the set-point weight can be obtained from the relation

$$\frac{1}{|s_3^*|}s+1=3T_1s+1=bT_I^*s+1 \qquad (20)$$

It is obvious that is

$$b=\frac{1}{3} \qquad (21)$$

The servo and regulatory step responses for α = 0.11, 0.15, 0.2, 0.25 and k_1 = 1 for 2DOF PID and 2DOF PI ($\alpha = 0$ and $b = 0.33$) controllers are shown in Fig. 4. The response for the 2DOF PI controller is very conservative.

5 CONCLUSIONS

In this paper the new 2DOF controller tuning method for the first order plants with integration is derived, which comes from the multiple dominant pole method. The described method enables 2DOF PI and PID controllers tuning, which enables the achievement of aperiodic servo and regulatory step responses and by suitable choice of the derivative time to the integral time ratio to conform to the limitation of the manipulated variables.

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