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DIFFERENT APPROACHES TO CONTROL OF TISO THERMAL SYSTEM

RŮZNÉ PŘÍSTUPY K ŘÍZENÍ TISO TEPELNÉHO SYSTÉMU

Abstract

The contribution is aimed on problematic of multivariable control. Multivariable system can be controlled by multivariable controller or we can use decentralized control. Control of thermal system with two inputs and one output is shown in the paper. The goal of paper is to find what sort of results we can get by classical approaches and by more sophisticated strategies. Two discrete-time PID controllers are selected as a representative of classical approach and split-range with discrete-time PID controller is selected as a representative of more sophisticated strategy. Control strategies are compared in the view of control quality and costs, information and knowledge required by control design and application.

Abstrakt

Příspěvek se zabývá problematikou vícerozměrného řízení. Vícerozměrný systém může být řízen pomocí vícerozměrného regulátoru nebo lze použít tzv. decentralizované řízení. V článku je ukázáno řízení tepelné soustavy se dvěmi vstupy a jedním výstupem. Cílem článku je zjistit, jakých výsledků lze dosáhnout klasickými přístupy a jakých výsledků lze dosáhnout při použití sofistikovanějších strategií. Reprezentantem klasického přístupu jsou dva diskrétní PID regulátory, split-range s diskrétním PID regulátorem je vybrán jako zástupce sofistikovanější strategie. Strategie řízení jsou porovnány z pohledu nákladů a kvality řízení; informací a znalostí, které umí využít při návrhu a aplikaci řízení.

1 INTRODUCTION

We can define a multivariable Multi-Input Multi-Output system (MIMO) as system which has more inputs and outputs, whereas more output variables are influenced with one input [1].

If we have smaller number of inputs than outputs it is not possible to get zero steady state control error on all output variables. The solution is to specify request on degree of proximity to the set-point. This can be solved as an optimization problem dependent – solution and result depend on a criterion formulation.

If the system has more inputs than outputs the situation is more positive. This case is more interesting from practical point of view because we can get set-point with infinitely combinations of inputs. This admits to formulate additional control requirements (e.g. cost minimization). This case also leads to an optimization problem [2].

Two different control strategies for system with one controlled variable and two manipulated variables (TISO) are demonstrated on practical example of thermostatic bath control. Two discrete-time PID controllers and split-range with discrete-time PID controller are described, designed, applied and compared.

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2 CONTROLLED SYSTEM

Imperfect insulated basin filled with water C is placed in environment with temperature T_0. Electrical heating element A and coil B (pipe with flowing water) are dipped in the water. Measurement cell (element) D is also dipped in the water. Defined system has four input variables – environment temperature T_0, heating power E, temperature of cooling water T_{B0} and cooling water flow rate Q. Output variable is temperature of water T_C or temperature of measurement cell T_D.

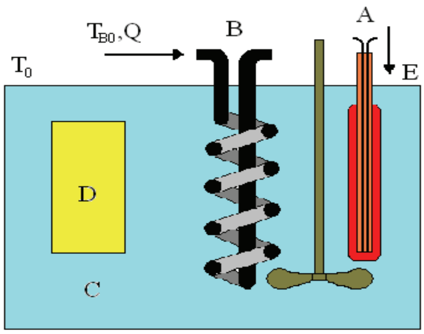

Fig. 1 Scheme of thermostatic bath

Mathematical model [3] can be derived under above stated assumptions, based on of thermal balance of heating element (1), coil (2), water in thermostatic bath (3) and dipped measurement cell (4).

$$E = \alpha_A S_A (T_A - T_C) + m_A c_A \frac{dT_A}{dt} \tag{1}$$

$$Q c_B T_{B0} + \alpha_B S_B (T_C - T_B) = Q c_B T_{B0} + m_B c_B \frac{dT_B}{dt} \tag{2}$$

$$\alpha_A S_A (T_A - T_C) + \alpha_D S_D (T_D - T_C) = \alpha_B S_B (T_C - T_B) + \alpha_C S_C (T_C - T_0) + m_C c_C \frac{dT_C}{dt} \tag{3}$$

$$0 = \alpha_D S_D (T_D - T_C) + m_D c_D \frac{dT_D}{dt} \tag{4}$$

where T_x are characteristic temperatures (state variable), m_x are masses, c_x are specific thermal capacities, S_x are areas for heating transfer, α_x are heat transfer coefficients between adjacent capacities, index x substitutes individual capacities A, B, C and D.

Integral part of the process properties is information about the constraints. Parameters of model are given in table 1, range of input variables and working point are given in table 2 and steady state in working point in table 3.

Tab. 1 Model parameters

Par.	Dimension	A heating	B cooling	C water	D element
m_x	kg	0.3	0.15669	4.0	8.93
c_x	$J.kg^{-1}.K^{-1}$	452	4180	4180	383
S_x	m^2	0.0095	0.065	0.24	0.06
α_x	$J.m^{-2}.s^{-1}.K^{-1}$	750	500	5	500

Tab. 2 Input variables – range and working point

Var.	E [W]	Q [kg.s^{-1}]	T_{B0} [°C]	T_0 [°C]
u_{max}	1000	0.5/60	20	25
u_0	250	0.5/60	15	25
u_{min}	0	0.5/60	5	25

Tab. 3 Steady state temperatures in working point

T_A [°C]	64.63
T_B [°C]	22.02
T_C [°C]	29.54
T_D [°C]=y_0	29.54

3 CONTROL DESIGN AND EXPERIMENTS

Following control demands and conditions are kept for all experiments. Temperature of the measurement cell T_D is selected as a controlled variable. Heating power E and temperature of cooling water T_{B0} are manipulated variables. Remaining variables are considered as disturbances. Under assumption that flow rate of the cooling water is constant, thermostatic bath is a linear system.

Control conditions are following:

- control starts from steady state - see tab. 3

- set-point is changed stepwise in time 20 minutes from value 29.54 to value 50 and in time 80 minutes set-point returns back to 29.54

- experiment lasts 140 minutes

- sample time is 20 seconds

3.1 Two discrete-time PID controllers

Equation of discrete-time PID controller is used in following form

$$u(k) = q_0 e(k) + q_1 e(k-1) + q_2 e(k-2) + u(k-1) \tag{5}$$

where q_0, q_1 a q_2 are constants, which are calculated from continue-time PID controller parameters according to following formulas [4]

$$\begin{aligned} q_0 &= r_0\left(1 + \frac{T_s}{2T_i} + \frac{T_d}{T_s}\right) \\ q_1 &= -r_0\left(1 - \frac{T_s}{2T_i} + \frac{2T_d}{T_s}\right) \\ q_2 &= \frac{r_0 \cdot T_d}{T_s} \end{aligned} \tag{6}$$

where r_0 is controller gain, T_i is integral time constant, T_d is derivate time constant and T_s is sample time. Parameters of continue-time PID controllers were tuned by trial-error method so control response is close to aperiodic.

Tab. 4 Parameters of PID controllers

Manipulated variable	r_0	T_i	T_d
Temperature of cooling water T_{B0}	4	4600	0
Heating power E	156	4800	0

Control response is depicted in Figure 2.

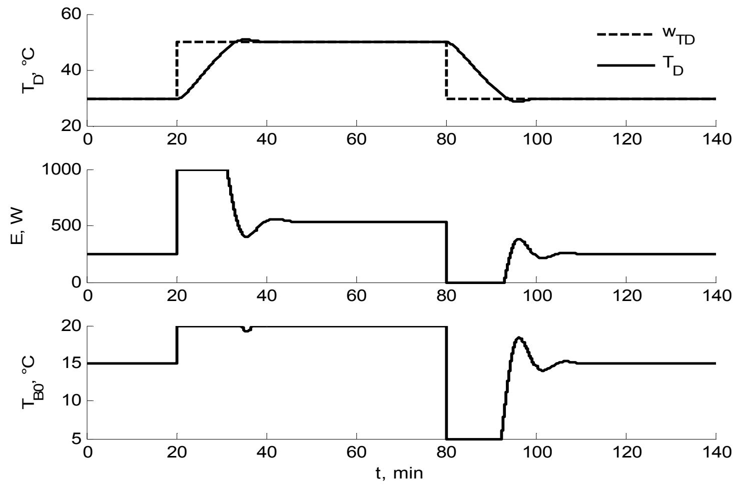

Fig. 2 Control with two PID controllers

3.2 Split-range control

Split-range strategy is often used in situations where one or more control variables should be used, depending on the operating scenario. There are several reasons for splitting the signals for example dividing output of one controller into two or more signals that are applied to different control actuators [5].

In our case Split-range is realized according to Figure 3.

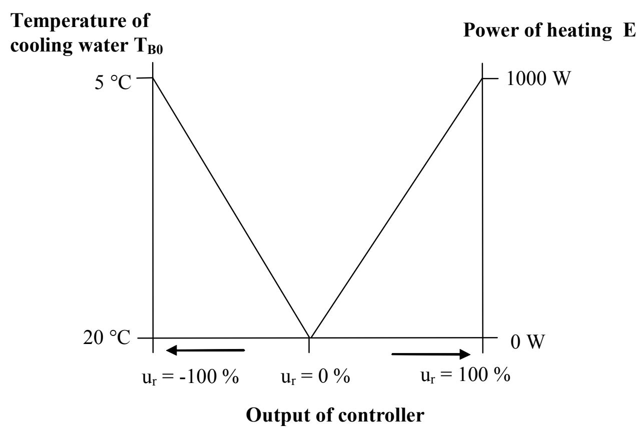

Fig. 3 Split-range scheme

We suppose that the output from controller is between -100 % and +100 %. If the controller output is negative the system is only cooled and the heating is off. Similarly if the controller output is positive cooling is on its minimal value and the heating is active. Manipulated variables are linearly interpolated according to Figure 3. If the controller output u_r = -100 % then the heating power E = 0 W and the temperature of cooling water is T_{B0} = 5 °C. If u_r = 0 % then the temperature of cooling water T_{B0} = 20 °C and the heating power E = 0 W. If u_r = 100 % then T_{B0} = 20 °C and E = 1000 W.

Discrete-time PID controller is used with Split-range. Parameters of the controller are calculated from continues-time PI controller parameters tuned with trial-error method to get aperiodic control responses. The controller gain is r_0 = 24 and integral time constant is T_i = 1250 s. Control response is depicted in Figure 4.

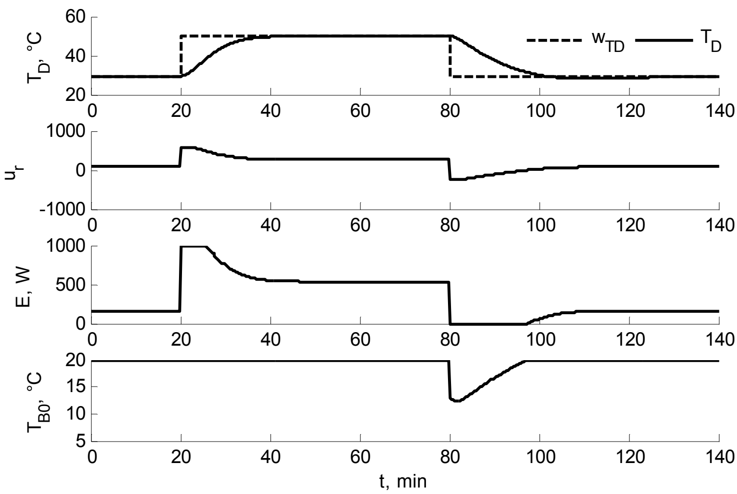

Fig. 4 Control with Split-range

4 CONTROL PERFORMANCE MEASURES

Three measures are computed to compare discussed control methods from the view of control quality and heating and cooling costs.

4.1 Control quality measure K

This measure is defined as a square root from mean quadratic control error

$$K = \sqrt{\frac{1}{N}\sum_{i=1}^{N} e^2(i)} \tag{7}$$

where N is a number of samples in the experiment.

4.2 Heating cost N_h

Heating cost is calculated directly from the price of electric energy

$$N_h = c_j \cdot T_s \cdot \sum_{i=1}^{N} E(i) \tag{8}$$

where T_s is a sample time.

4.3 Cooling cost N_c

Cooling cost is calculated in following way. To cool down the water the same amount of energy is necessary as to heat it up plus energy to respect lower efficiency of the cooling compared to the heating.

$$N_c = c_j \cdot \frac{Q \cdot c_{H_2O}}{\varepsilon} \cdot T_s \cdot \sum_{i=1}^{N} \left(T_{B0\max} - T_{B0}(i) \right) \quad (9)$$

where $T_{B0\max}$ is maximal cooling water temperature and $T_{B0}(i)$ is temperature of the cooling water.

4.4 Evaluated performance measures

Above stated measures are computed for all experiments. We can compare the control quality and costs of individual control methods according to Table 5.

Tab. 5 Control performance measures

Experiment	K	N_h (Kč)	N_c (Kč)	N_h+N_c(Kč)
Two discrete-time PID controllers	5,45	3,62	3,06	6,68
Split-range + discrete-time PID controller	5,78	3,10	0,42	3,52

5 CONCLUSION

The paper is aimed on temperature control in thermostatic bath as a system with two inputs and one output. The system is controlled by two discrete-time PID controllers and by split-range with discrete-time PID controller. If we compare these control responses in term of control quality and costs (heating cost and cooling cost), we can say that the control response with two discrete-time PID controllers is better in the case of control but this response is worse in the case of costs (two discrete-time PID controllers do not fulfil condition of optimal cost in the steady state - manipulated variables freeze after the control error is zero).

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