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# MEASUREMENT OF PHASE SHIFT BY USING A DSP

# MĚŘENÍ FÁZOVÉHO POSUNU UŽITÍM DSP

**Abstract**

The paper deals with design of the computer system for measurements of the phase shift between two harmonic signals using the Digital Signal Processor (DSP). The introducing part of the paper describes properties of the harmonic signals and the Hilbert transform. Concerning the Hilbert transform their two methods for computing, one is based on the Fourier transform while the second one benefits from the digital filters. The submitted paper deals with mentioned two methods for the phase evaluation as well. The phase shift between two harmonics signals is useful for rotors balancing. The algorithm of rotor balancing requires the amplitude of both the signals as well.

**Abstrakt**

Práce se zabývá návrhem počítačového systému pro měření fáze mezi dvěma harmonickými signály s použitím digitálního signálového procesoru (DSP). V úvodu příspěvku se popisují vlastnosti harmonického signálu a Hilbertovy transformace. Ohledně Hilbertovy transformace jsou zde dvě metody pro výpočet, první na základě Furierovy transformace, zatímco druhá používá výhod digitálních filtrů. Jsou popsány dvě metody pro výpočet fáze. Měření fáze mezi dvěma signály se v praxi používá při dynamickém vyvažování rotorů. Hlavní část práce je zaměřena na návrh programu pro DSP a PC.

## 1 INTRODUCTION

Balancing of rotors requires the measurement of the phase difference between two harmonic signals. There are many approaches to compute the phase difference, but this paper is focused to using of the Hilbert transform. The method is suited for using Digital Signal Processor (DSP) as it is shown in Fig. 1.

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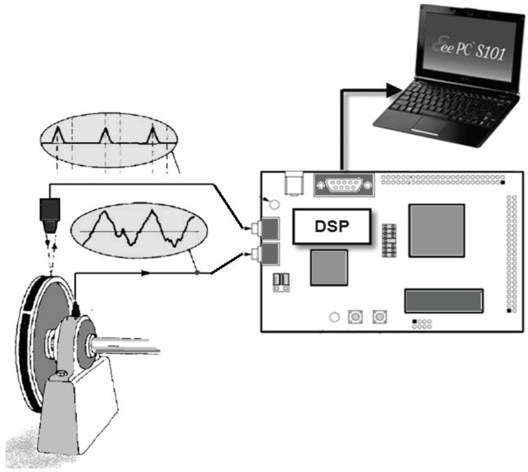


**Fig. 1** A balancing system based on DSP and PC

### 1.1 Analog Devices signal processors

The digital signal processor is a well known microprocessor, which is specialized for signal processing. The main advantage of DSP is that they are able to process data in the real time. A typical task for signal processing is digital filtration and spectral analysis.

DSP is enough fast and code executing is not interrupted by the operating system except for repetition of interrupting calls from the input sample measurement. DSP is more powerful than the PC microprocessors due to the fact that the operating systems fulfil only simple tasks, which does not require a lot of time. Thanks to their power, DSP are able to process a huge data stream.

DSP is an IC, which is replacing traditional analogue circuits, even the specialized digital computers. Now, there are DSP based on the fixed-point arithmetic and on the floating-point arithmetic.

DSP is differing from the PC microprocessor, which is containing ALU, in extending hardware by Shifter and MAC unit. MAC provides high-speed multiplication, multiplication with cumulative addition, multiplications with cumulative substractions, saturation and clear-to-zero functions. Any of the registers associated with the MAC can be both read and written in the same cycle. The result of multiplication is 32-bit word stored in 40-bit register to prevent overflow it.

### 1.2 DSP programming

Programming of the fixed-point DSP is based on a language, which can be described as simplified C code. This language is more powerful than an assembler, but identifiers are only the names of registers or addresses of memory cells. The advantage of the DSP statements in the simplified C code is that they are directly corresponding to the assembler statements. The statements can be conditional (IF). Jumps DO UNTIL loops, and subroutine calls are belonging to the DSP language. DSP benefits from interrupting system. The statements of the fixed-point DSP language are easily under control of programmers.

An extra computational power of DSP is resulting from computing a product and accumulation of two operands in one processor cycles simultaneously with register loading and index register modification in such a way that in the next cycle the next sample or the next filter coefficient is automatically read from the memory cells, which addresses are incremented by the given number. The samples and the filter coefficients are stored in circular buffers. These features and loops allow to code convolution of signals for computing the FIR and IIR filter output.

$$y_k = \sum_{i=0}^{m} b_i x_{k-i} \quad (1)$$

Register loading and one multiplying and addition is in one computer clock cycle, so the number of cycles needed for evaluation of a digital filter is equal to the filter order in the other word to the number of taps.

### 1.3 Communication between DSP and PCT

DSP works after program loading undependably as PC. The DSP kit is intended for processing a stereo-audio signal. This means simultaneous conversion two analogue signals into digital samples, filtration and conversion into analogue signals. DSP exchanges data with a host PC via RS 232 serial bus.

## 2. METHODS FOR MEASURING THE PHASE BETWEEN TWO HARMONIC SIGNALS

The Hilbert transform has been chosen for calculation the phase difference between two harmonic signals

Let two harmonic signals be defined as follows (2)

$$x_1(t) = A_1 \cos(\omega t + \varphi),\ x_2(t) = A_2 \cos(\omega t) \quad (2)$$

The phase shift $\varphi$ is a quantity to be computed. To design an algorithm we turn our attention to the trigonometric formulas

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) \quad (3)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) \quad (4)$$

Substitution and extension of the formulas (2), (3), (4) we get expressions (5) and (6).

$$A_1 A_2 \sin(\varphi) = A_1 A_2 \sin(\omega t + \varphi)\cos(\omega t) - A_1 A_2 \cos(\omega t + \varphi)\sin(\omega t) \quad (5)$$

$$A_1 A_2 \cos(\varphi) = A_1 A_2 \cos(\omega t + \varphi)\cos(\omega t) + A_1 A_2 \sin(\omega t + \varphi)\sin(\omega t) \quad (6)$$

Since both signals represent cosine function (2) and formula (5) and (6) also contains the sine function, we need shift the phase between the input signals using the Hilbert transform. This transformation will be implemented FIR filter type Hilbert transformer (7), (8).

$$A_1 A_2 \sin(\varphi) = HT\{x_1(t)\}x_2(t) - x_1(t)HT\{x_2(t)\} \quad (7)$$

$$A_1 A_2 \cos(\varphi) = x_1(t)x_2(t) + HT\{x_1(t)x_2(t)\} \quad (8)$$

In order to calculate the sine and cosine function of the phase (7) and (8) we need to determine the amplitude of each harmonic component. The amplitude of both the harmonic signals could be calculate using the formula (9)(10).

$$A_1^2 = (A_1 \cos(\omega t + \varphi))^2 + (A_1 \sin(\omega t + \varphi)) \quad (9)$$

$$A_2^2 = (A_2 \cos(\omega t))^2 + (A_2 \sin(\omega t))^2 \quad (10)$$

The previous formulas (9) and (10) may be rewritten using the Hilbert transform $HT\{...\}$ notation in the following form (11) and (12).

$$A_1^2 = (x_1(t))^2 + (HT\{x_1(t)\})^2 \quad (11)$$

$$A_2^2 = (x_2(t))^2 + (HT\{x_2(t)\})^2 \quad (12)$$

By using the square root of the expression (11) and (12) we get the amplitudes of both the harmonic signals. The amplitude $A_1$ and $A_2$ and the phase terms compose the expression $A_1 A_2 \sin(\varphi)$ and $A_1 A_2 \cos(\varphi)$. DSP calculates the phase shift between signals (2) and (3) then.

If the phase is evaluated from the expression $A_1 A_2 \sin(\varphi)$ then the sign of the expression $A_1 A_2 \cos(\varphi)$ specifies the angle quadrant.

This method makes it possible to measure the phase between two harmonic signals in the range from–180 ° (-π) to +180 ° (π).

## 2.1 Input signal

Connectors for analogue input and output are the type of stereo jack, they have left and right channel. The converters are of the type ΣΔ (sigma – delta). The sampling frequency can be set from the value 5.5125 kHz to 48 kHz. It is true that the higher sampling rate, the smaller the number of processor cycles between the measured samples. Codec characteristics are set in control registers DSP.

## 2.2 Digital filters

Generally, filters are used to adjust the frequency spectrum processed signal. The digital filter is a algorithm designed to process signals in real time with minimum number of computational operations, DSP cycles. They are a great advantage of Digital Signal Processors.

The main reasons for their use are as follows:

- high quality accuracy and the performance
- easy simulation and design, flexibility
- linear phase and constant delays (FIR), possible adaptive filtering

## 2.3 Design digital filters of the FIR type

The digital filter coefficients are designed by using Signal Analyzer, the indoor software. First, we will design the Hilbert transformer. The proposal method is the type of FIR filter. This filter is designed for filtering input harmonic signals to shift their phase by 90 degree. The filter impulse response and the filter coefficients are shown in Fig. 3. The frequency response is shown in Fig. 2. The order of the FIR filter is equal to 50. The frequency response shows that the Hilbert transformer does not distorts harmonic signals with the normalized frequency values from 0.05 to 0.95 of the Nyquist frequency.

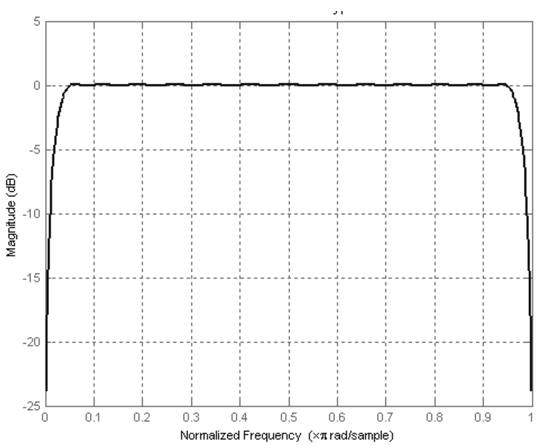


**Fig. 2** Frequency Response of the Hilbert transformer

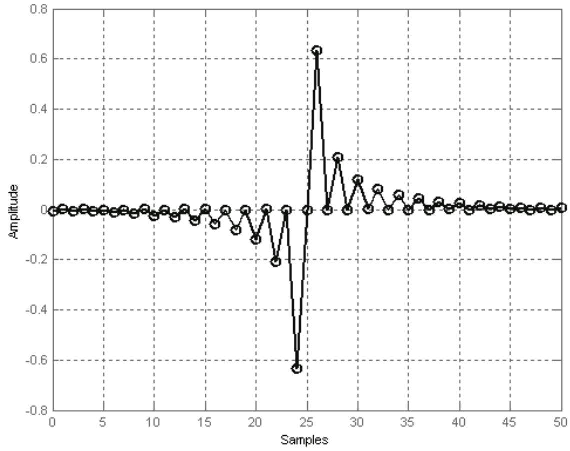


**Fig. 3** Impulse Response of the Hilbert transformer

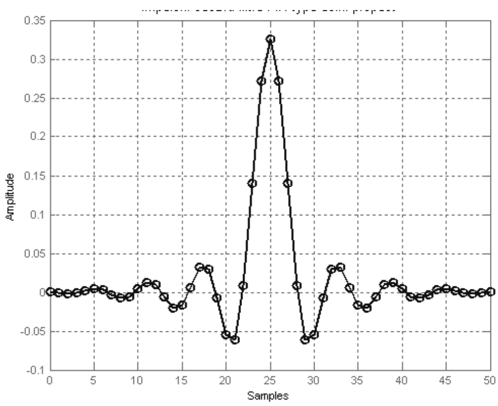


**Fig. 4** Impulse response of the low-pass filter

As the Hilbert transformer delays the input signal by half the filter order. To compensate this delay, the real part of the analytic signal has to be delayed by 25 samples as well.

The last digital filter is the low pass filter at the output of the block diagram in Fig 5 and 6. The impulse response of this filter is shown in Fig. 4. The particular results are transmitted ti the host computer after conversion the decimal format into the HEX format.

## 3. IMPLEMENTATION OF THE CALCULATION PHASE FOR DSP

Block diagram of operations for obtaining the value of $A_1 A_2 \sin(\varphi)$ and $A_1 A_2 \cos(\varphi)$ is shown in Fig.5 and 6. As the input signals for the phase calculation serves two harmonic signals $x_1(t)$ and $x_2(t)$, which are inputed to the left and right channel.

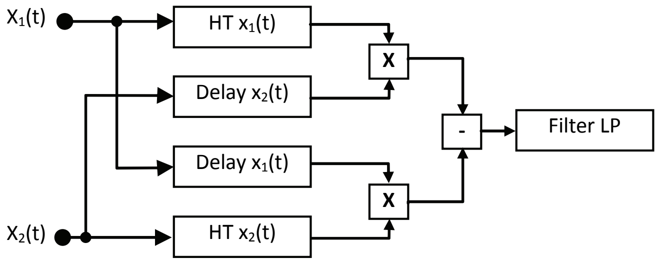


**Fig. 5** Block diagram of operations for calculation of the value $A_1A_2\sin(\varphi)$

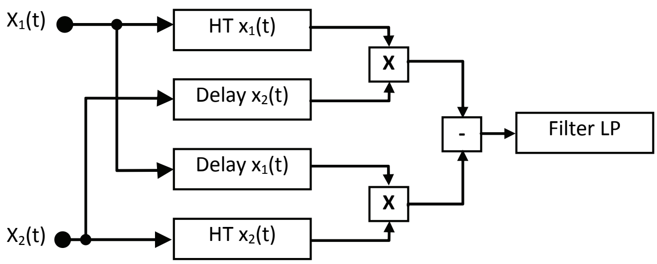


**Fig. 6** Block diagram of operations for calculation of the value $A_1A_2cos(\varphi)$

## 3 CONCLUSIONS

The paper is focused at creation an application, which is intended to do measurement of the phase shift between two harmonic signals using the signal processor (DSP).

As the appropriate method is selected the trigonometric formulas and the Hilbert transform. This method is coded into the DSP, which is sending particular results to a PC for processing and display the resulting phase.

In practice, this phase measurement between two signals can be found in the dynamic balancing of rotors.

The proposed system, using DSP, allows the measurement of phase between two harmonic signals in the range from +180 ° (π radians) to -180° (-π radians).

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