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MATHEMATICAL MODELLING OF A SHORT MAGNETORHEOLOGICAL DAMPER

MATEMATICKÉ MODELOVÁNI KRÁTKÉHO MAGNETOREOLOGICKÉHO TLUMIČE

Abstract

Properties of magnetorheological liquids are changed by application of a magnetic field. This enables to control damping of mechanical vibration of rotating and stationary machines. The damping effect of dampers intended for attenuation of lateral vibration of rotors is produced by squeezing a thin film of a magnetorheological liquid. The presented article deals with development of their mathematical model. This requires to determine a pressure distribution in the lubricating film. The cavitation and dependance of magnetic flux on the variable film thickness around circumference of the damper had to be taken into account.

Abstrakt

Vlastnosti magnetoreologických kapalin se mění působením magnetického pole. To umožňuje řídit tlumení mechanického kmitání rotačních i stacionárních strojů. Tlumicí účinek tlumičů určených k potlačení příčného kmitání rotorů vzniká v důsledku stlačování tenkého filmu magnetoreologické kapaliny. Předložený článek se zabývá vypracováním matematického modelu takovýchto tlumičů. To vyžaduje stanovit rozložení tlaku v mazacím filmu. Do úvahy musí být vzata i kavitace a závislost magnetického toku na tloušťce filmu po obvodu tlumiče.

1 INTRODUCTION

Magnetorheological fluids are colloidal suspensions. If they are not effected by a magnetic field, they behave as a normal Newtonian one. But if the magnetic field is applied, the shear stress between the neighbouring layers increases. If a limit value (yield shear stress) is reached, the layers start to move together. This arrives at occurrence of a core in which the liquid moves as a solid body. In mathematical models the magnetorheological liquid is usually represented by a Bingham one.

The change of rheological properties of the liquid by application of a magnetic field can be utilized in semiactive devices for mitigation of mechanical vibration. In the field of rotor dynamics dampers working on the principle of squeezing a thin oil film are used. The change of magnitude of the damping force is controlled by the change of intensity of the magnetic flux.

The squeeze film magnetorheological dampers consist of two rings between which there is a thin film of a magnetorheological liquid (Fig.1). Both rings are coupled with the stationary part of the rotating machine, the outer one directly, the inner ring by a flexible spring. The shaft is supported by a rolling element bearing whose outer race is coupled with the inner ring of the damper. Vibration of the inner ring relative to the outer one squeezes the liquid and this produces the damping force. In the stationary part of the damper there are placed the coils. The damping effect can be controlled by the change of magnitude of the applied electric current in the coils.

At present time the magnetorheological dampers in the field of rotor dynamics are a subject of intensive research, both experimental and theoretical. In 2003 [1] Wang and Meng experimentally studied the vibration properties and control method of a flexible rotor supported by a magnetorheological fluid squeeze film damper. In 2004 [2] Forte et al. presented results of the theoretical and

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experimental investigations of a long magnetorheological damper. In 2006 Wang et al. [3] developed a mathematical model of a long squeeze film magnetorheological damper based on modification of a Reynolds' equation in which they included influence of the change of the width of the gap around the damper circumference on the yield shear stress of the magnetorheological liquid.

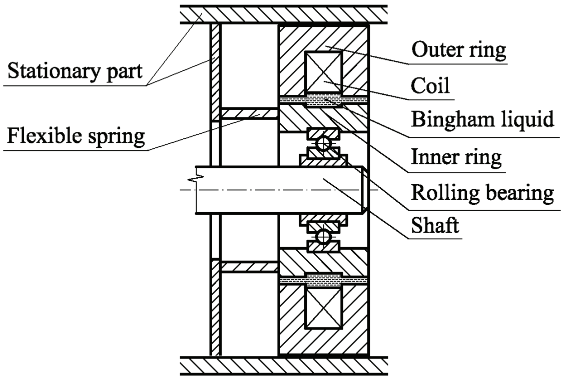

Fig. 1 Scheme of a MR damper

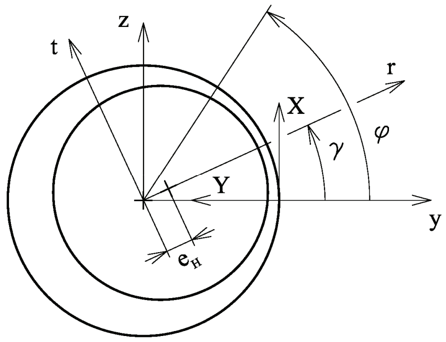

Fig. 2 The damper coordinate system

2 DETERMINATION OF THE DAMPING FORCES

In the mathematical model it is assumed that (i) the inner and outer rings of the damper are absolutely rigid and smooth, (ii) the width of the damper gap is very small relative to the radii of both rings, (iii) ratio of the length of the damper to the diametre of its outer and inner rings is small, (iv) the lubricant is a Bingham liquid, (v) the yield shear stress depends on magnitude of the magnetic flux, (vi) the flow in the oil film is laminar and isothermal, (vii) pressure of the lubricant in the radial direction is constant, (viii) the lubricant is considered to be massless, and (ix) influence of the curvature of the oil film is negligible.

The thickness of the lubricating film depends on the position of the inner damper ring relative to the outer one

$$h = c - e_H \cos(\varphi - \gamma) \tag{1}$$

h is the thickness of the film, c is the width of the gap between the inner and outer rings of the damper, e_H is the eccentricity, φ is the circumferential coordinate, and γ is the position angle of the line of centres (Fig.2).

As it is assumed that the length to diametre ratio of the damper is small (less that 0.5), the flow prevails in the axial direction and such damper can be considered as short. The flow of the lubricant is then governed by the equation of continuity and by the equation of equilibrium of a small element specified in the oil film

$$\frac{\partial v}{\partial Y} + \frac{\partial w}{\partial Z} = 0 \tag{2}$$

$$\frac{\partial p}{\partial Z} = \frac{\partial \tau}{\partial Y} \tag{3}$$

Y, Z are radial and axial coordinates, v, w are the velocity components in Y and Z directions (Fig.2), p is the pressure, and τ is the shear stress between the fluid layers in the axial direction.

The constitutive equations of a Bingham fluid can be expressed

$$\tau = \pm\tau_y + \mu \frac{\partial w}{\partial Y} \quad \text{for} \quad |\tau| > \tau_y \tag{4}$$

$$\frac{\partial w}{\partial Y} = 0 \quad \text{for} \quad |\tau| \le \tau_y \tag{5}$$

τ_y and μ are the yielding shear stress and Bingham viscosity respectively.

The boundary conditions for the velocity profile are given by the following relations

$$Y = 0 \qquad v = 0 \qquad w = 0 \tag{6}$$

$$Y = h_1 \qquad\qquad w = w_C \tag{7}$$

$$Y = h_2 \qquad\qquad w = w_C \tag{8}$$

$$Y = h \qquad v = \dot{h} \qquad w = 0 \tag{9}$$

where

$$\dot{h} = \frac{\partial h}{\partial t} \tag{10}$$

h_1 and h_2 are radial coordinates in the damper gap between which the core occurs ($0 < h_1 < h_2 < h$), w_C is the core velocity.

Substitution of (4) into (3), double integration, and application of the boundary conditions (6) - (9) give the relation for the flow velocity in the axial direction

$$w = \frac{1}{2\mu} p' \left(Y^2 - h_1 Y \right) + \frac{w_C}{h_1} Y \tag{11}$$

where

$$p' = \frac{\partial p}{\partial Z} \tag{12}$$

Integration of the equation of continuity (2) assuming that the velocity profile is symmetric arrives at relations

$$\int_0^{\dot{h}} dv = -\int_0^h \frac{\partial w}{\partial Z} dY \tag{13}$$

$$\dot{h} = -2 \frac{\partial}{\partial Z} \left(\int_0^{h_1} w \, dY + \int_{h_1}^{\frac{h}{2}} w \, dY \right) \tag{14}$$

Substitution of (11) into (14) and performing appropriate manipulations produce the modified Reynolds equation

$$\frac{\partial}{\partial Z} \left[\frac{1}{6\mu} p' h_1^3 - w_C \left(h - h_1 \right) \right] - \dot{h} = 0 \tag{15}$$

At the edge of the core ($Y = h_1$) the shear stress is equal to the yield one τ_y. Making use of (11) it holds for the core velocity

$$w_C = -\frac{p'}{2\mu} h_1^2 \tag{16}$$

The core velocity w_C is oriented in the opposite direction than the pressure gradient p'.

It holds for the shear stress on the boundary of the core

$$Y = h_1,\quad \tau = +\tau_y \quad \text{for} \quad p' < 0 \quad \text{and} \quad \tau = -\tau_y \quad \text{for} \quad p' > 0 \tag{17}$$

$$Y = h_2,\quad \tau = -\tau_y \quad \text{for} \quad p' < 0 \quad \text{and} \quad \tau = +\tau_y \quad \text{for} \quad p' > 0 \tag{18}$$

Making use of the symmetry of the velocity profile the relations for the radial coordinate of the core edge h_1 are obtained by integration of the equilibrium equation (3).

$$\int_{\tau_y}^{-\tau_y} d\tau = 2p' \int_{h_1}^{\frac{h}{2}} dY \quad \text{for} \quad p' < 0 \quad \text{and} \quad \int_{-\tau_y}^{\tau_y} d\tau = 2p' \int_{h_1}^{\frac{h}{2}} dY \quad \text{for} \quad p' > 0 \tag{19}$$

After performing the integrations one obtains the relation for the radial coordinate of the core edge

$$h_1 = \frac{h}{2} + \frac{\tau_y}{p'} \quad \text{for} \quad p' < 0 \quad \text{and} \quad h_1 = \frac{h}{2} - \frac{\tau_y}{p'} \quad \text{for} \quad p' > 0 \tag{20}$$

In the middle plane of the damper the liquid does not flow in the axial direction and therefore the core takes the all width of the damper gap

$$h_1 = 0 \quad \text{for} \quad Z = 0 \tag{21}$$

Integration of the modified Reynolds equation (15) with respect to the axial coordinate Z taking into account (16), (20) and the boundary conditions (21) arrives at the relations for the distribution of the pressure gradient in the axial direction

$$h^3 p'^3 + 3\left(h^2 \tau_y - 4\mu \dot{h} Z\right)p'^2 - 4\tau_y^3 = 0 \quad \text{for} \quad p' < 0 \tag{22}$$

$$h^3 p'^3 - 3\left(h^2 \tau_y + 4\mu \dot{h} Z\right)p'^2 + 4\tau_y^3 = 0 \quad \text{for} \quad p' > 0 \tag{23}$$

The yield shear stress τ_y depends on the magnetic flux that is given by the electric current in the coil, thickness of the lubricating film, and by the design parameters of the damper

$$\tau_y = k_\tau \left(\frac{I}{h}\right)^2 \tag{24}$$

I is the electric current and k_τ is the coefficient depending on the damper design.

Determination of the pressure gradient as a function of the axial coordinate Z leads to repeated solving a cubic algebraic equations (22) and (23). Solution of each of them gives three roots. The searched one must satisfy the following conditions

(i) the root must be real (not complex),

(ii) $0 < h_1(p') < \frac{h}{2}$,

(iii) $w_C(p') > 0$ for $p' < 0$ which is equivalent to $\dot{h} > 0$,

(iv) $w_C(p') < 0$ for $p' > 0$ which is equivalent to $\dot{h} < 0$.

The pressure is obtained by integration

$$p = \int p' \, dZ \tag{25}$$

with the boundary condition expressing that the pressure at the edge of the damper is equal to the atmospheric one

$$p = p_a \quad \text{for} \quad Z = \frac{L}{2} \tag{26}$$

p_a is the pressure in the surrounding space (atmospheric pressure) and L is the length of the damper.

If pressure at some location in the oil film drops to a critical level, a cavitation takes place. The observations showed that pressure of the medium in cavitated areas remained constant. Then it holds with enough accuracy

$$p_d = p \quad \text{for} \quad p \geq p_{CAV} \tag{27}$$

$$p_d = p_{CAV} \quad \text{for} \quad p < p_{CAV} \tag{28}$$

p_d is the pressure distribution in the layer of lubricant and p_{CAV} is the pressure in the cavitated area.

Components of the damping force are obtained by integration of the pressure distribution around the circumference and along the length of the damper

$$F_{dy} = -R \int_0^{2\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} p_d \cos\varphi \, dZ \, d\varphi \,, \quad F_{dz} = -R \int_0^{2\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} p_d \sin\varphi \, dZ \, d\varphi \tag{29}$$

F_{dy}, F_{dz} are the y, z components of the damping force, R is the inner ring radius.

3 EXAMPLE

Applicability of the described approach was tested by means of computer simulations. Design parameters of the investigated damper are : width of the gap 0.2 mm, diametre of the inner ring 130 mm, length of the damper 60 mm. Bingham viscosity of the magnetorheological liquid and the design parametr of the damper are 0.06 Pas and 0.001 NA^{-2} respectively. The centre of the journal performs a circular orbit of constant eccentricity 0.08 mm and moves at a constant angular speed 150 rad/s. The task was to analyze the attenuation effect of the damper.

Fig.3 shows the time histories of the velocity component of the rotor journal and of the component of the damping force in y direction. It is evident that the force always acts in the direction opposite to the movement of the journal and it implies the damping effect is produced. The time history of y component of the damping force in dependence on magnitude of the electric current in the coils is drawn in Fig.4. If no current flows, the lubricant behaves as a newtonian liquid and the produced damping effect is low. The increase of the current leads to rising the damping. The pressure distribution in the oil film at the moment of time when the position angle of the line of centres is 176° can be seen in Fig.5. The dependence of the yield shear stress around the circumference of the damper for the same position of the shaft is drawn in Fig.6. The magnetic flux depends on the width of the damper gap and on magnitude of the electric current. If the inner ring is in the eccentric position, the gap width is variable around the damper circumference and this produces the change of the yield shear stress of the magnetorheological liquid.

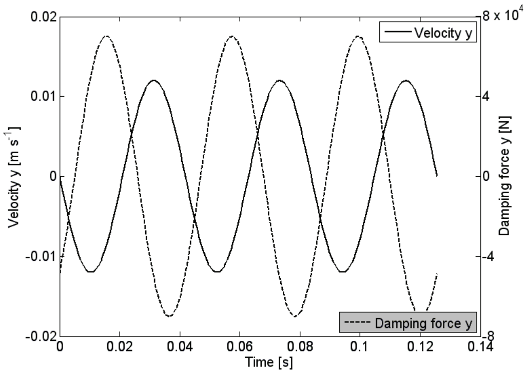

Fig. 3 Damping force and velocity

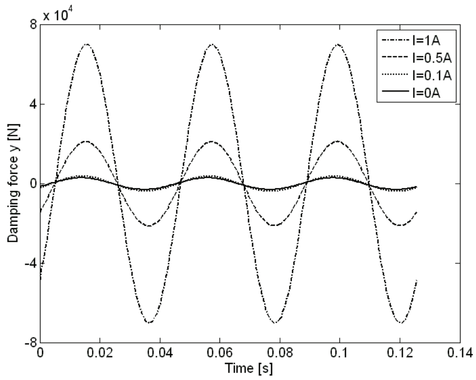

Fig. 4 Damping force - current relationship

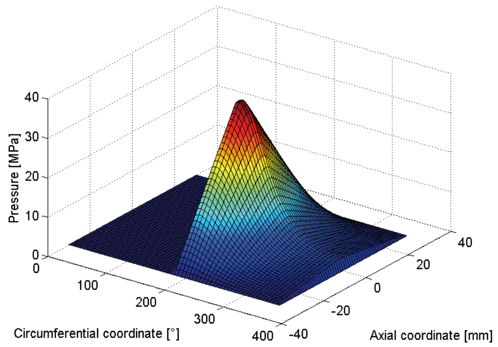

Fig. 5 Pressure distribution

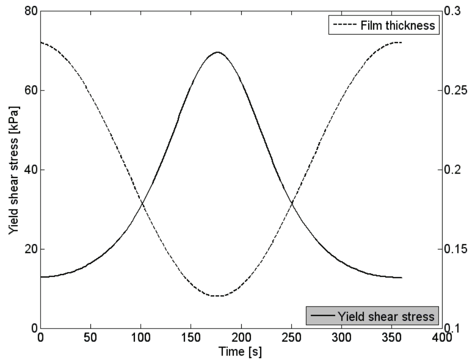

Fig. 6 Yield shear stress and the film thickness

4 CONCLUSIONS

The change of properties of magnetorheological liquids by application of a magnetic field enables the increase or decrease magnitude of the damping force and this makes possible to control damping of mechanical vibration. The magnetorheological dampers intended for attenuation of vibration of rotors work on a principle of squeezing a thin layer of magnetorheological liquid. The computer simulations showed that in the developed mathematical model the force produced by the damper acted always in the direction opposite to the shaft movement and it implies the damper attenuates the shaft vibration. The dependence of magnitude of the damping force on magnitude of the electric current that produces the magnetic field has been also proved.

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