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DISTRIBUTION OF GAS PRESSURE ALONG THE PIPE LENGTH IN THE CASE OF STATIONARY TURBULENT FLOW AT CONSTANT TEMPERATURE

PRŮBĚH TLAKU PLYNU PODÉL DÉLKY POTRUBÍ PŘI STACIONÁRNÍM TURBULENTNÍM PROUDĚNÍ ZA STÁLÉ TEPLOTY

Abstract

The paper deals with the mathematical modelling of dependence of pressure of a gas flowing in simple circular pipes at constant temperature on distance x from the beginning of the pipes and on changes in values of gas pressure p1 and velocity v1 at the beginning of the pipes, friction coefficient λ and angle of pipe inclination α. Considered dependences of gas pressure on changes in values of the above-presented quantities and parameters are expressed graphically.

Abstrakt

Práce se zabývá matematickým modelováním závislosti tlaku plynu proudícího jednoduchým kruhovým potrubím za stálé teploty t a to na vzdálenosti x od počátku potrubí a na změnách hodnot tlaku p1 a rychlosti v1 plynu na počátku potrubí, součinitele tření λ a úhlu α sklonu potrubí. Uvažované závislosti tlaku plynu na změnách hodnot uvedených veličin a parametrů jsou vyjádřeny graficky.

1 INTRODUCTION

When a gas flows through the pipes, the pressure of it changes as a result of conversions between individual forms of energy, i.e. kinetic, pressure potential and positional potential, and furthermore, as a result of pressure losses due to friction along the pipe length and local losses. Friction losses Z_t are produced along the whole pipe length as a consequence of friction between the layers of the gas and friction between the gas and the pipe wall. The local losses Z_m are caused by deformation of the velocity field, i.e. they thus occur at a change in cross-sectional area of flow and at a change in direction of flow, and by vortex generation and dissipation [1], [2], [3].

We shall determine the distribution of gas pressure along the pipe length on the following assumptions and simplifications:

- The gas flows isothermically through the pipes of constant cross section, without any withdrawal of gas along the pipe length. By isothermic flow, the real flow of gas can be replaced approximately if the temperature of the gas at the beginning of the pipes is equal to the ambient temperature and the pipes are sufficiently long [4] so that as a consequence of exchange of heat with the ambient air, the temperatures of gas and the ambient air equalize.
- We shall replace the three-dimensional stationary turbulent flow through the pipes by the one-dimensional stationary gas flow [1].
- The stationary infrasonic flow of gas belongs to the square area of friction losses, i.e. the friction coefficient λ is a function of relative density ε of the pipes

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$$\lambda = f(\varepsilon). \tag{1.1}$$

- The flow of real gas is replaced by an idealized flow. The idealized isothermic flow of gas is regarded as flow, in the case of which the gas energy dissipation due to internal gas friction and friction between the gas and the pipe walls is taken into account, and simultaneously this flow is described by the equation of state for ideal gas and the equation of isothermal process. The utilization of equations valid for ideal gas is based on knowledge stated in the specialized literature, according to which the results of such calculations, in the region of low values of Mach number similarity criterion

$$Ma \prec 1 \tag{1.2}$$

comply with technical practice [5].

- In the state equation, we shall not include the coefficient of compressibility Ks, because the value of it is often close to one, and in cases of gas mixtures the coefficient of compressibility is difficult to determine [6].
- We assume that the values of friction coefficient λ of the gas, angle of pipe inclination α, inner pipe diameter d, and pressure p1, density ρ1, velocity v1 of the gas at the beginning of pipes are known.

2 GAS PRESSURE DEPENDING UPON DISTANCE FROM THE BEGINNING OF THE PIPES

The pressure loss Δp_t due to friction can be expressed with the adequate head loss Z_t [7]

$$\Delta p_t = \rho g Z_t , \tag{2.1}$$

for which the Darcy-Weisbach relationship is valid [3]

$$Z_t = \lambda \frac{l}{d} \frac{v^2}{2g} . \tag{2.2}$$

In the square flow region, the friction coefficient λ is a function of relative pipe roughness ε, which means that at the set pipe inner diameter d and the known relative roughness of the pipes ε, the head loss Z_t is a function of pipe length l and gas flow velocity v, i.e.

$$Z_t = Z_t(l, v). \tag{2.3}$$

We shall determine the distribution of gas pressure along the pipe length as a function of friction coefficient λ, angle of pipe inclination α, pipe inner diameter d, and further gas pressure p_1, density ρ_1 and velocity at the pipe beginning; it means that the gas pressure will be expressed by a functional relation as follows

$$p = p(\lambda, \alpha, d, p_1, \rho_1, v_1, x), \tag{2.4}$$

where x is the distance between the observed point and the beginning of the pipes. For the specifically set values of quantities λ, α, d, p_1, ρ_1, v_1 we shall obtain the dependence of gas pressure p on distance x from the beginning of the pipes, i.e.

$$p = p(x). \tag{2.5}$$

At determining the mentioned function, the energy balance of the gas in an elementary pipe segment dl is used as a basis. We shall use the continuity equation, Bernoulli equation and equation for losses (2.2), [2], [8], [9], [10]. After modifications we shall obtain a differential equation of the following form

$$vdv + \frac{dp}{\rho} + gdh + gdZ_t = 0\,. \tag{2.6}$$

The expression

$$dh = \sin\alpha \cdot dl \tag{2.7}$$

expresses a difference between the heights of segment *dl* of the pipes that make with the horizontal plane an angle α. By differentiating the head loss due to friction dZ_t, the state equation and the continuity equation, and by utilizing the equation for isothermal process, we shall get the following differential equation after modifications

$$\left(v - rT\frac{1}{v} + \frac{\lambda}{d}lv\right)dv + \left(g\sin\alpha + \frac{\lambda}{2d}v^2\right)dl = 0\,. \tag{2.8}$$

By solving this equation we shall receive a relation for the dependence of pressure on distance *x* from the beginning of the pipes

$$p = \left(\frac{K_1\left(1 + K_2 x\right)}{K_3 + K_4 \ln\dfrac{p_1}{p} - K_5 x}\right)^{\frac{1}{2}}, \tag{2.9}$$

where

$$K_1 = \frac{p_1^2 v_1^2}{2}, \qquad K_2 = \frac{\lambda}{d}, \qquad K_3 = \frac{v_1^2}{2}, \qquad K_4 = \frac{p_1}{\rho_1}, \qquad K_5 = g\sin\alpha\,. \tag{2.10}$$

The obtained equation is valid for horizontal pipes, downcomers and also for vertical pipes. According to the assumptions, it is a case of long pipes that are used e.g. in the mining [11] or gas industry [12].

The equation (2.9) applies to mixtures of gases of *n* components, if we determine the density ρ of gas mixture by means of the equation

$$\rho = \sum_{i=1}^{n} \frac{V_i}{V}\rho_i\,, \tag{2.11}$$

where ρ_i is the density, V_i is the volume of i-th component of gas mixture ($i = 1,2, \ldots ,n$) and V is the total volume of gas mixture.

3 GRAPHICAL REPRESENTATION OF GAS PRESSURE DISTRIBUTION ALONG THE PIPE LENGTH

The equation (2.9) is an implicit function $f(x,p) = 0$. For its graphical processing, the computer program MATLAB was used.

From many possible dependences offered by the equation (2.9) for processing, we shall select a drop in flowing gas pressure depending upon a change in one of the above-mentioned flow parameters or on a change in one of the above-mentioned quantities for graphical observation.

We shall observe the presented functional dependences in the course of methane flowing through simple circular pipes (henceforth referred to as pipes) of the length l, inner diameter d and angle of inclination α. The flow occurs at constant ambient temperature and constant temperature of pipe walls, and according to the assumptions, also at constant gas temperature. Values of quantities and parameters characterising the gas flow will be stated in the observed cases of flow.

3.1 DROP IN GAS PRESSURE ALONG THE PIPE LENGTH DEPENDING UPON THE INITIAL VALUE OF PRESSURE

Through the pipes of length l = 400 m, inner diameter d = 0.3 m and friction coefficient λ = 0.048, methane of constant temperature t = 15°C and density ρ = 0.68 kg.m^{-3} flows. The velocity of the gas at the pipe inlet is v_1 = 30 m.s^{-1}. The graphical processing of a drop in methane pressure along the pipes will be performed for the following values of initial pressure: p_1 = (5.5; 5.0; 4.5; 4.0; 3.5; 3.0)x10^{5}Pa.

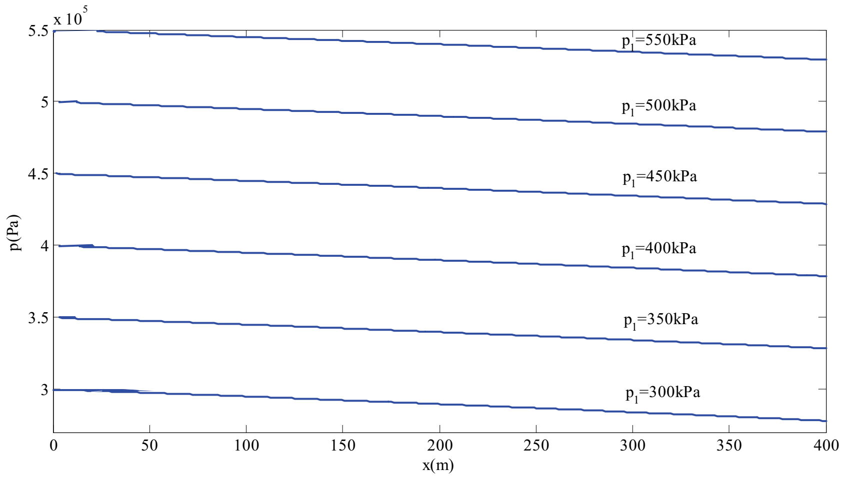

Fig. 1 Methane pressure p along the pipe length x for set input values of pressures p_1

Table 1 Difference Δp between pressures at the pipe beginning and the pipe end

i	p_1	p_2	Δp
	kPa		
1	550.0	529.2	20.8
2	500.0	479.2	20.8
3	450.0	428.9	21.1
4	400.0	378.7	21.3
5	350.0	328.5	21.5
6	300.0	278.0	22.0

The greatest drop in methane pressure is that for the input value of pressure p_1 = 300 kPa, the smallest then for the input pressure p_1 = 550 kPa. In Table 1 the summarization of differences between pressures $\Delta p = p_1 - p_2$ at the pipe beginning and the pipe end is given. Hence it is evident that curves in Fig. 1 are not parallel.

3.2 DROP IN GAS PRESSURE ALONG THE PIPE LENGTH DEPENDING UPON THE VALUE OF INITIAL VELOCITY

Methane of constant temperature t = 15°C and density ρ = 0.68 kg.m^{-3} flows through the pipes of length l = 400 m, inner diameter d = 0.3 m and friction coefficient λ = 0.048. Methane pressure at the beginning of the pipes is p_1 = 300 kPa. We shall represent graphically the pressure p of methane along the pipe length x for the values of initial velocity v_1 = (10; 15; 20; 25; 30) m.s^{-1}.

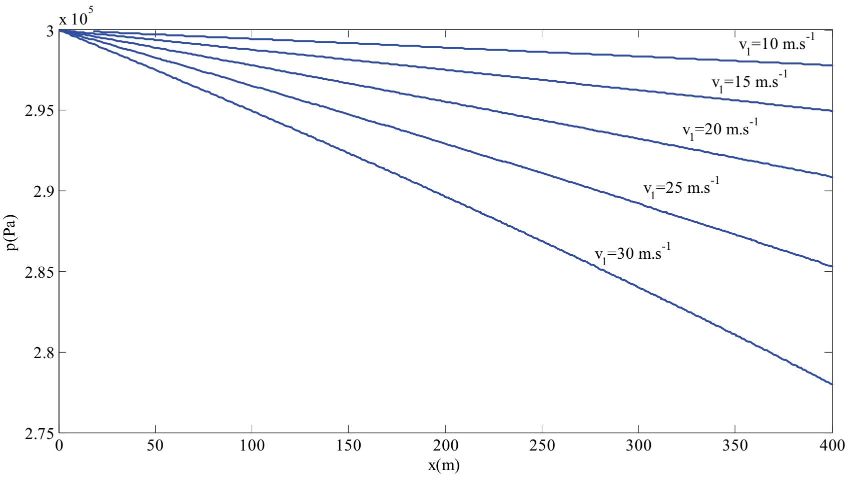

Fig.2 Methane pressure *p* along the pipe length *x* for set values of initial velocity v_1

Table 2 Methane pressure loss Δp depending upon input values of velocity v_1

i	v_1	p_1	p_2	Δp
	m.s^{-1}	kPa		
1	10	300.0	297.8	2.2
2	15	300.0	295.0	5.0
3	20	300.0	290.9	9.1
4	25	300.0	285.3	14.7
5	30	300.0	278.0	22.0

It follows from Fig. 2 that with the increasing initial velocity of methane, a drop in methane pressure accelerates. The decrease in pressure $\Delta p = p_1 - p_2$ is shows in Table 2.

3.3 DROP IN GAS PRESSURE ALONG THE PIPE LENGTH DEPENDING UPON THE VALUE OF FRICTION COEFFICIENT

Through the pipes of length l = 400 m and inner diameter d = 0.3 m, methane of constant temperature t = 15°C and density ρ = 0.68 kg.m^{-3} flows. Input values of methane pressure and velocity at the pipe beginning are p_1 = 300 kPa and v_1 = 30 m.s^{-1}, respectively. We shall represent graphically the pressure of methane for the following values of friction coefficient λ = (0.04; 0.05; 0.06; 0.07; 0.08).

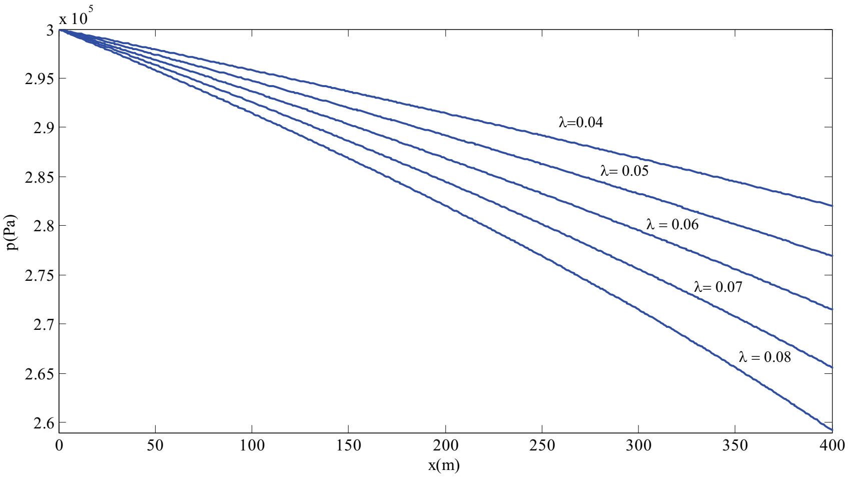

Fig. 3 Methane pressure p along the pipe length x depending upon the value of friction coefficient λ

Table 3 Methane pressure p and a decrease in it for set values of friction coefficient λ

i	λ	$p_{(x=200m)}$	$p_{(x=400m)}$	$\Delta p_{(x=200m)}$	$\Delta p_{(x=400m)}$
	-	kPa			
1	0.04	291.4	282.1	8.6	17.9
2	0.05	289.2	276.9	10.8	23.1
3	0.06	286.8	271.5	13.2	28.5
4	0.07	284.5	265.6	15.5	34.4
5	0.08	282.0	259.3	18.0	40.7

As can be seen in the graph, a drop in methane pressure with the growing pipe length is much more conspicuous in the case of higher values of friction coefficient. This fact can also be observed in Table 3 for two various distances from the pipe beginning.

3.4 DROP IN GAS PRESSURE ALONG THE PIPE LENGTH DEPENDING UPON PIPE INCLINATION

Methane of density $\rho = 0.68$ kg.m^{-3} flows at temperature $t = 15$ °C through the pipes of length $l = 400$ m, inner diameter $d = 0.3$ m and friction coefficient $\lambda = 0.048$. Gas pressure and velocity at the pipe inlet are $p_1 = 300$ kPa and $v_1 = 20$ m.s^{-1}, respectively. We observe the methane pressure graphically depending upon the angle of pipe inclination $\alpha = (0, \pi/12, \pi/6, \pi/4, \pi/3, \pi/2)$.

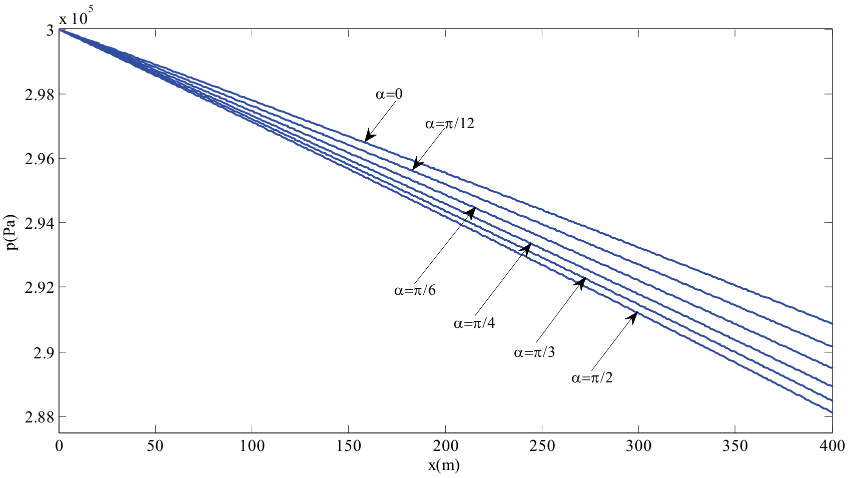

Fig. 4 Methane pressure p along the pipe length x depending upon the angle of pipe inclination α

Table 4 Pressure loss Δp of methane for set values of angle of pipe inclination α

i	α	p_1	p_2	Δp
	rad	kPa		
1	0	300.0	290.9	9.1
2	π/12	300.0	290.2	9.8
3	π/6	300.0	289.5	10.5
4	π/4	300.0	288.9	11.1
5	π/3	300.0	288.5	11.5
6	π/2	300.0	288.2	11.8

As can be seen in the graph, the velocity of decrease in gas pressure decelerates with the growing angle of inclination. This fact can be observed in Table 4 as well, where $\Delta p = p_1 - p_2$ is a difference between methane pressures at the pipe beginning and the pipe end.

4 CONCLUSION

The article deals with the graphical processing of functional dependence of pressure of gas flowing turbulently through the long pipes at constant ambient temperature upon distance from the pipe beginning. The functional dependence was derived from the law of conservation of energy when abstracting some properties of real gas. The adoption of simplifying conditions follows from the conclusions of experiments stated in the specialized literature.

The presented procedure can be used for the fast and preliminary observation of pressure distribution along the pipe length for the specifically set flow of gas. By simultaneous graphical representation of more functional dependences, the influence of change in value of one of the parameters or one of the quantities characterising the flow of gas on distribution of gas pressure in the pipes can be observed.

Theoretical conclusions will be further verified in technical practice, namely by the measurement of gas pressures in the above-mentioned conditions in mine and gas distribution networks.

USED SYMBOLS

d pipe inner diameter (m)

g gravitational acceleration ($m.s^{-2}$)

h height (m)

l pipe length (m)

Ma Mach number similarity criterion (1)

p pressure (Pa)

r specific gas constant ($J.kg^{-1}.K^{-1}$)

t temperature (°C)

T thermodynamic temperature (K)

v mean velocity of fluid flow ($m.s^{-1}$)

V volume (m^3)

Z_m local head loss (m)

Z_t head loss due to friction (m)

α angle (rad)

ε pipe relative roughness (1)

κ Poisson constant (1)

λ friction coefficient (1)

ρ density ($kg.m^{-3}$)

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