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EFFECT OF INPUT LINE ON EIGENFREQUENCY OF ROTARY HYDRAULIC MOTOR

VLIV VSTUPNÍHO VEDENÍ NA VLASTNÍ FREKVENCI ROTAČNÍHO HYDROMOTORU

Abstract

Hydraulic systems are applied in many branches of industry. Hydraulic elements belong to basic elements of the systems. They have a big influence on the correct working and the failure-free operation of the systems. Dynamic properties are one of important criterions at projection of hydraulic systems. Pressure or flow pulses have a negative influence on dynamic loading of hydraulic elements and system tightness. The aim of the paper is to evaluate an effect of input line on eigenfrequency of a given rotary hydraulic motor in concrete working conditions. The eigenfrequency of the investigated hydraulic elements was simulated using Mathcad software.

Abstrakt

Hydraulické systémy jsou aplikovány v mnoha oblastech průmyslu. Hydraulické prvky patří k základním prvkům těchto systémů. Mají velký vliv na správný pracovní a bezporuchový provoz systémů. Dynamické vlastnosti jsou jedním z nejdůležitějších kritérií při projektování hydraulických systémů. Tlakové a průtokové kmity mají negativní vliv na dynamické namáhání hydraulických prvků a těsnost systému. Předmětem tohoto příspěvku je vyhodnotit vliv vstupního vedení na vlastní frekvenci daného rotačního hydromotoru při konkrétních pracovních podmínkách. Vlastní frekvence zkoumaných hydraulických prvků byla simulována v programu Mathcad.

1 INTRODUCTION

Hydraulic elements are applied to performance of different functions in hydraulic systems (e. g. to pressure limitation, liquid line and flow adjustment). The correct function of the elements is a fundamental prerequisite of failure-free operation of hydraulic systems. Dynamic properties belong to these prerequisites. The eigenfrequency is used for the description of dynamic behaviour of hydraulic systems. If the eigenfrequency is equal to the excitation frequency, generated pressure or flow pulses can have a negative influence on service reliability. It can come to an accident too [1]. The eigenfrequency can be investigated by experimental measurements or mathematical simulations in suitable computer programs.

This paper deals with influence of input line on eigenfrequency of rotary hydraulic motor. The influence was mathematically simulated using Mathcad software for a given working condition.

2 MATHEMATICAL DESCRIPTIONS OF HYDRAULIC ELEMENTS

For mathematical simulations, it is necessary to describe single hydraulic elements by simplified mathematical equations. Subsequently, it is possible to simulate single hydraulic elements using a suitable simulation program.

2.1 Rotary Hydraulic Motor

Hydraulic motors belong to output appliances of hydraulic systems. They are used for the transformation of pressure head into mechanical energy. The simulation of dynamic properties was

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applied on the rotary hydraulic motor of MRAK 5-40 type [2]. The dynamic behaviour of the motor is expressed by the differential equations [3]:

$$\frac{dQ_M}{dt} = \frac{\Delta p_M}{L_M} - \frac{\Delta p_{MZ}}{L_M} - \frac{R_M}{L_M} \cdot Q_M, \tag{1}$$

$$\frac{d\Delta p_M}{dt} = \frac{Q}{C_M} - \frac{Q_M}{C_M} - \frac{Z_M}{C_M} \cdot \Delta p_M, \tag{2}$$

where:

Q_M – real flow through the hydraulic motor $\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$,

Δp_M – hydraulic gradient through the hydraulic motor $[\mathrm{Pa}]$,

Δp_{MZ}– hydraulic motor load $[\mathrm{Pa}]$,

L_M – resistance to acceleration of the hydraulic motor $\left[\frac{\mathrm{N}\cdot\mathrm{s}^2}{\mathrm{m}^5}\right]$,

R_M – resistance to motion of the hydraulic motor $\left[\frac{\mathrm{N}\cdot\mathrm{s}}{\mathrm{m}^5}\right]$,

C_M – capacity of the hydraulic motor $\left[\frac{\mathrm{m}^5}{\mathrm{N}}\right]$,

Z_M – linearized leakage permeability of the hydraulic motor $\left[\frac{\mathrm{m}^5}{\mathrm{N}\cdot\mathrm{s}}\right]$.

2.2 Input Line

There was used the pressure rubber hose with the length l = 1.05 m, the inside diameter d = 12 mm and the wall thickness t = 5.5 mm as the input line of the hydraulic motor for the investigation of dynamic behaviour. The dynamic properties were simulated by the lumped parameter model which is applicable on the following condition [2]:

$$\lambda = \frac{a}{f} > l, \tag{3}$$

where:

λ – wave length $[\mathrm{m}]$,

a – speed of sound $\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$,

f – frequency $[\mathrm{Hz}]$.

If the previous equation is not valid, it is necessary to divide the line into smaller segments (i. e. the model with continuously distributed parameters). In the case of the lumped parameter model, it is possible to simulate the line by a suitable connection of resistance to motion, resistance to acceleration and capacity of the line. There are different schematic connections of hydraulic lines (e. g. T-junction and π-junction). The T-junction physical model is shown in Fig. 1. The dynamics of the model is expressed by the following equation [3]:

$$\frac{C \cdot L}{2} \cdot \frac{d^2(\Delta p)}{dt^2} + \frac{C \cdot R}{2} \cdot \frac{d(\Delta p)}{dt} + \Delta p = \frac{C \cdot L^2}{4} \cdot \frac{d^3 Q_1}{dt^3} + \frac{R \cdot L \cdot C}{2} \cdot \frac{d^2 Q_1}{dt^2} + \left(\frac{C \cdot R^2}{4} + L \right) \cdot \frac{dQ_1}{dt} + R \cdot Q_1, \tag{4}$$

where:

L – resistance to acceleration of the input line $\left[\frac{\mathrm{N \cdot s^2}}{\mathrm{m^5}}\right]$,

R – resistance to motion of the input line $\left[\frac{\mathrm{N \cdot s}}{\mathrm{m^5}}\right]$,

C – capacity of the input line $\left[\frac{\mathrm{m^5}}{\mathrm{N}}\right]$.

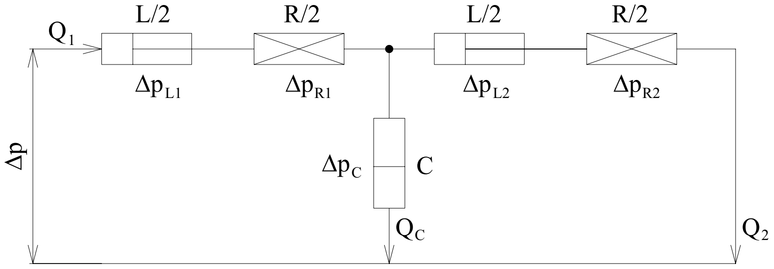

Fig. 1 T-junction model of line

2.3 Rotary Hydraulic Motor with Influence of Input Line

The mathematical model of the rotary hydraulic motor with the influence of the input line is described by the equations (1) and (2) as in the case of the single rotary hydraulic motor. The resistances to motion and acceleration of the hydraulic motor are constant. Only the liquid volume of the input line has to be included in the capacity of the hydraulic motor in this case. Then the capacity of the hydraulic motor with the input line is higher in comparison with the single hydraulic motor.

3 SIMULATION OF TIME–RESPONSE CHARACTERISTICS OF HYDRAULIC ELEMENTS

There are a lot of computer programs (e. g. Matlab, Simulink, Mathcad, Dynast and Flowmaster) for mathematical simulations of dynamic behaviour at different working conditions [2]. The dynamic behaviour of the single hydraulic elements was performed using Mathcad software in this case. The simulation was applied at the hydraulic gradient through the hydraulic motor $\Delta p_M = 8$ MPa. Mineral oil with the modulus of elasticity $K = 7.343 \cdot 10^8$ Pa and the speed of sound $a = 387.6\ \mathrm{m \cdot s^{-1}}$ was used as the working liquid [4] in the investigated hydraulic elements.

3.1 Simulation of Dynamic Behaviour of Rotary Hydraulic Motor

The mathematical simulation was performed for the load jumping $\Delta p_{MZ} = 5$ MPa on the hydraulic motor at the pre-determined parameters [2] of the hydraulic motor (i. e. $R_M = 10^{10}\ \mathrm{N \cdot s \cdot m^{-5}}$, $L_M = 1.529 \cdot 10^9\ \mathrm{N \cdot s^2 \cdot m^{-5}}$, $C_M = 2.68 \cdot 10^{-14}\ \mathrm{N^{-1} \cdot m^5}$, $Z_M = 2.604 \cdot 10^{-13}\ \mathrm{N^{-1} \cdot m^5 \cdot s^{-1}}$) on the condition that the outlet pressure behind the hydraulic motor $p = 0$ MPa. The hydraulic gradient through the hydraulic motor Δp_M was changed from 3 MPa to 8 MPa in this case. There were applied the equations (1) and (2) for the eigenfrequency determination. In Figure 2 is shown the time dependence of the hydraulic gradient change Δp_M for the above-mentioned load jumping. It is evident from the demonstrated dependence that the simulated quantity oscillates with the eigenfrequency $f_0 = 24.86$ Hz.

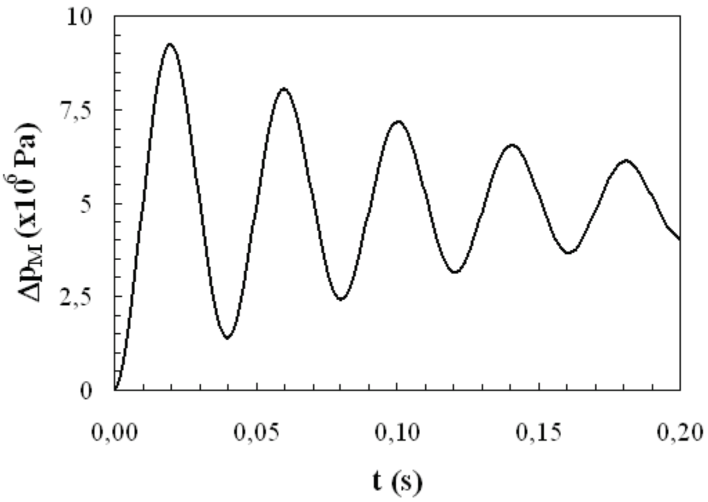

Fig. 2 Time dependence of hydraulic gradient change through rotary hydraulic motor

3.2 Simulation of Dynamic Behaviour of Input Line

There were firstly determined the basic parameters [2] of the input line (i. e. $R = 1.067 \cdot 10^8$ N·s·m^{-5}, $L = 7.687 \cdot 10^6$ N·s^2·m^{-5}, $C = 9.547 \cdot 10^{-13}$ N^{-1}·m^5) for the mathematical simulation which is described by the equation (4). The eigenfrequency of the input line was investigated for the hydraulic gradient jumping $\Delta p = 1$ MPa on the input line. The hydraulic gradient Δp was changed from 7 MPa to 8 MPa in this case. The time dependence of the flow change ΔQ_1 through the input line is shown in Fig. 3. It is evident that the flow Q_1 was increased from the steady state at the hydraulic gradient jumping. Its change converges to the constant value $\Delta Q_1 \cong 9.37 \cdot 10^{-3}$ m^3·s^{-1} (see Fig. 3). Besides the flow Q_1 is periodically changed with the eigenfrequency of the input line $f_0 = 117.49$ Hz which is higher in comparison with the eigenfrequency of the single rotary hydraulic motor. Further, it is necessary to control the wave length of the line for the given working conditions according to the equation (3):

$$\lambda = \frac{387.6}{117.49} = 3.3\ m > 1.05\ m. \tag{5}$$

It can be concluded from the above-mentioned equation, that the condition of the lumped parameter model was observed.

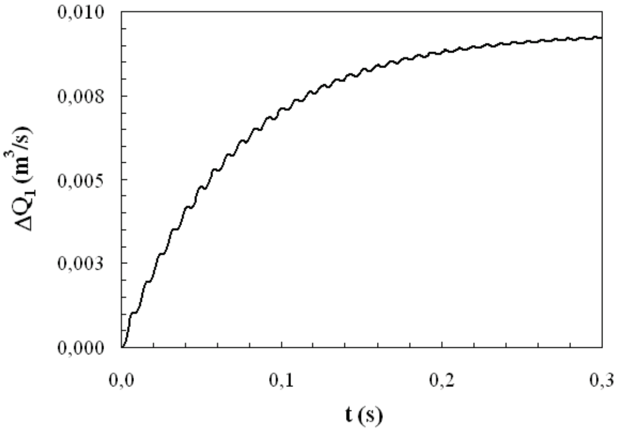

Fig. 3 Time dependence of flow change through input line

3.3 Simulation of Dynamic Behaviour of Rotary Hydraulic Motor with Influence of Input Line

The mathematical simulation of the hydraulic motor with the influence of input line was performed analogous to the simulation of the single hydraulic motor. Only the hydraulic motor capacity was increased (i. e. $C_M = 2.297 \cdot 10^{-13}\ N^{-1} \cdot m^5$) in this case [2]. It is given by a higher liquid volume of the hydraulic motor with the input line [5]. The simulation of dynamic behaviour was performed for the same load jumping $\Delta p_{MZ} = 5$ MPa on the hydraulic motor.

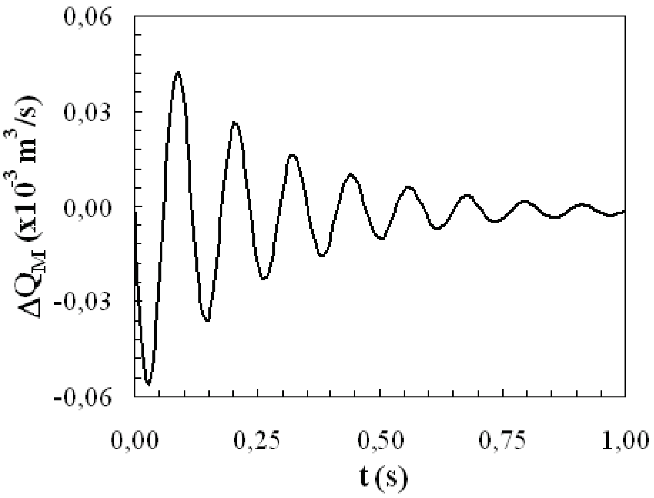

Fig. 4 Time dependence of flow change through rotary hydraulic motor with influence of input line

The time dependence of the flow change Q_M through the hydraulic motor is shown in Fig. 4. It is evident that the flow Q_M is periodically changed around the steady state value with the eigenfrequency $f_0 = 8.48$ Hz. Consequently, the eigenfrequency of the system of the hydraulic motor and the input line is lower in comparison with the investigated single hydraulic elements.

The eigenfrequency was also experimentally determined at a similar hydraulic system [2] with the same parameters of the hydraulic motor and the input line. There was obtained the measured value of the eigenfrequency $f_0 = 8.20$ Hz at the same working conditions. It is evident that the differences between the simulated and measured values are slight.

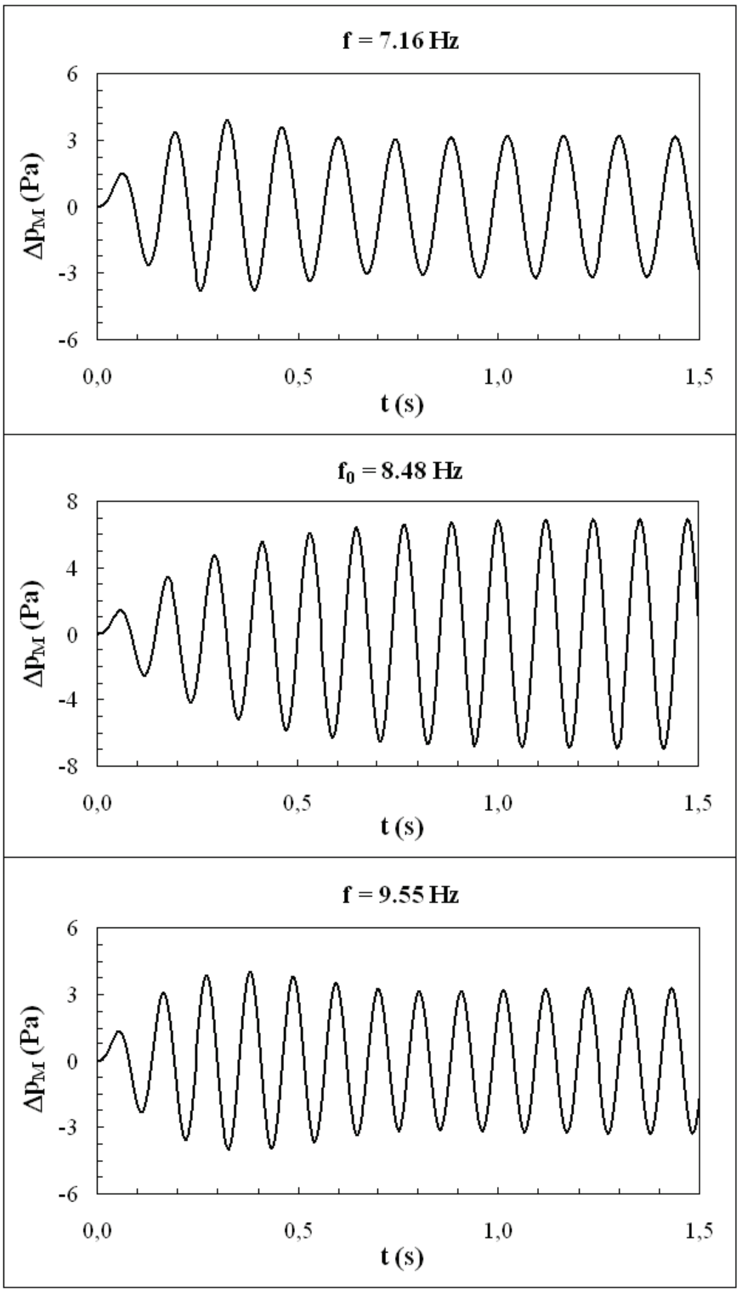

Fig. 5 Time dependencies of hydraulic gradient on rotary hydraulic motor with influence of input line at different frequencies

The mathematical simulation of the hydraulic motor with influence of the input line was similarly performed for the following harmonic load change [2] of the hydraulic motor:

$$\Delta p_{MZ} = \Delta p_{M0} \cdot \sin(2\pi \cdot f \cdot t) = 1 \cdot \sin(2\pi \cdot f \cdot t), \tag{6}$$

where:

Δp_{M0} – steady value of hydraulic motor load $[\mathrm{Pa}]$.

Fig. 5 demonstrates changes of the hydraulic gradient through the hydraulic motor at different frequencies. It is evident that the frequency has a big influence on amplitudes of pressure pulses. The maximum amplitude of pressure pulses ($\Delta p_M \cong 7$ Pa) is obtained at the eigenfrequency $f_0 = 8.48$ Hz. Lower amplitudes of pressure pulses are in general obtained at lower and higher frequencies in view of the eigenfrequency (see Fig. 5). Therefore the eigenfrequency is a very important quantity for the investigation of dynamic properties in hydraulic systems.

4 COMPARISON OF DYNAMIC PROPERTIES OF HYDRAULIC ELEMENTS

The mathematically simulated values of the eigenfrequeicies for the investigated hydraulic elements are compared in Tab. 1. The maximum value of the eigenfrequency was obtained for the input line. On the contrary, the minimum value of the eigenfrequency was acquired for the rotary hydraulic motor with the influence of the input line. This fact is effected by a higher liquid volume of the system in comparison with liquid volumes in the single hydraulic elements. There are recommended to use hydraulic lines with short lengths and smaller volumes in hydraulic systems [6] on this account.

Tab. 1 Eigenfrequencies of hydraulic elements

Hydraulic element	Eigenfrequency [Hz]
Hydraulic motor + input line	8.48
Hydraulic motor	24.86
Input line	117.49

5 CONCLUSIONS

The aim of this study was to investigate the effect of the input line on the eigenfrequency of the rotary hydraulic motor. This influence was investigated in terms of pre-determined parameters of the hydraulic elements and the mathematical simulation of differential equations using Mathcad software. It was found that the input line has a big influence on the eigenfrequency decrease at the rotary hydraulic motor by reason of a higher liquid volume. The mathematically simulated value of the eigenfrequency was practically consistent with the experimentally measured value of the eigenfrequency on a hydraulic system with the same hydraulic elements.

A big accent is put on utilization of mathematical simulations at the present time. Low operating costs, labour protection, simulations at different working conditions and relatively fast results belong to the advantageous of the mathematical simulations in comparison with experimental measurements.

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