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SHAPE OPTIMIZATION – INFLUENCE OF APPLIED MODEL SYMMETRY
TVAROVÁ OPTIMALIZACE – VLIV UŽITÉ SYMETRIE VÝPOČTOVÉHO MODELU

Abstract

The contribution presents the cooling rib optimization on the two numerical models with the aim to show the influence of used model geometry on the optimization problem properties. The parameters of used computational models (the models differ in used symmetry conditions) and the application of boundary conditions are presented. The obtained results show the necessity to take into account the authentic rib space arrangement in model simulations of optimization problem.

Abstrakt

Článek prezentuje optimalizaci chladicího žebrování na dvou výpočtových modelech, s cílem poukázat na vliv rozdílnosti modelů na vlastnosti optimalizační úlohy. Jsou popsány parametry použitých počítačových modelů (modely se liší v užitých podmínkách symetrie) a aplikace okrajových podmínek. Získané výsledky dokumentují nutnost zohlednění věrného prostorového uspořádání žeber v modelových simulacích úlohy optimalizace.

1 INTRODUCTION

The present state of knowledge and mathematical description of physical reality together with available computational equipment make possible to perform more and more complicated and realistic numerical simulations of observed processes. Those continuously developing possibilities of numerical analysis are followed by the endeavour of the engineers to reach the best possible behaviour of modified and new developed technical solutions. This challenge leads usually to the formulation of optimization problem whose solution imposes even higher requirements on whole IT structure. In order to shorten the time of numerical simulations and to accelerate the development process of the technical solutions it is still attractive and effective to use the appropriate simplifying assumptions (adequate BCs, symmetry etc.) especially for those numerical optimization problems whose computational and time demand exceeds the demand of direct numerical analysis problem. Both the severity of optimization problem and the risk of incorrect application of simplifications and/or boundary conditions predestinate the more complicated technical problems to be solved within the research projects of universities. The aim of the work is to show the significant influence of axial and plane symmetry application in the numerical model. This influence will be presented on the optimization problem of apparently simple heat task.

The numerical modelling of physical heat phenomenon is closely related to the mathematical description of heat source, heat convection and heat transfer. The mathematical model is based on the equations describing appropriate phenomena. Those phenomena are in general way three-dimensional and time dependent and can be described by the system of differential equations which is solvable by numerical methods. Due to the fact that the solution of heat problem can not be explicit expressed for given boundary conditions, the following optimization problem is performed in the numerical way as well.

2 MATHEMATICAL OPTIMIZATION AND MODEL EQUATIONS

2.1 Thermal Analysis of Solid Body

The object of the work whose temperature field is the objective of this paper is a solid body. Let us consider the heat convection equation [2] for description of temperature field in the form

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$$\frac{\partial}{\partial t}\left(\rho\, c_p T\right)=\frac{\partial}{\partial x_j}\left(\lambda\,\frac{\partial T}{\partial x_j}\right) \tag{1}$$

where the symbols are: c_p specific heat capacity $[\mathrm{J\cdot kg^{-1}\cdot K^{-1}}]$, t time [s], ρ density $[\mathrm{kg\cdot m^{-3}}]$, T temperature [K], x_j coordinate in j direction [m] and λ thermal conductivity $[\mathrm{W\cdot m^{-1}\cdot K^{-1}}]$.

The equation systems for solid and fluid phase are mutually linked by boundary conditions at the boundary shared by both phases, i.e. at the surface of the body flowed by liquid. After completing of equation system by appropriate outer boundary conditions (for the flow problem and for problem of heat convection and transfer) it is possible to approach the numerical simulation of boundary or mixed problem. In this paper the optimization based on steady-state thermal analysis is performed so in the eq. (1) the time dependent member on the left side becomes equal to zero.

2.2 Numerical Optimization – Goal Driven Optimization

According to [5] the goal driven optimization (GDO) is a constrained, multi-objective optimization technique in which the "best" possible designs are obtained from a sample set regarding to the goals which were set for parameters. The sample set can be generated either by the Screening or Advanced way. The GDO allows to determine the effect on input parameters applied for the output parameters. The screening approach is a non-iterative direct sampling method by a quasi-random number generator based on Hammersley algorithm. The Advance approach is an iterative Multi-objective Genetic Algorithm. Usually the Screening approach is used for preliminary design which may lead to applying of the Advanced approach for more refined optimization results.

Principles

Regarding to the calculation procedure described bellow it is obvious that the Screening approach is in this case the most convenient tool in the searching for a new design. Based on this fact the basic mathematical background of Screening process will be presented in the following.

The solution of global minimization relies on the sequential generation of the search directions. For the first iteration the method of steepest descent was used. For following iterations the conjugate directions were formed. The principles of common optimization procedures (used in this contribution) are generally well known and can be found for example in [1] or [4].

Shifted Hammersley Sampling Method

The shifted Hammersley sampling method is the sampling strategy used for the Screening process. The conventional Hammersley sampling algorithm is a quasi-random number generator which has very low discrepancy and is used for quasi-Monte Carlo simulations. The low discrepancy is defined as a sequence of points that approximate the equidistribution in a multi-dimensional cube in an optimal way. In other worlds, the design space is populated almost uniformly by those sequences and, due to the inherent properties of Monte-Carlo sampling, dimensionality is not a problem (i.e. the number of points does not increase exponentially with an increase in the number of input parameters). The conventional Hammersley algorithm is constructed by using the radical inverse function. Any integer n can be represented as a sequence of digits $n_0, n_1, n_2, \dots, n_m$ by the following equation:

$$n = n_0 n_1 n_2 \dots n_m\,. \tag{2}$$

For example, lets consider the integer 687459, which can be represented this way as $n_0 = 6$, $n_1 = 8$, and so on. Because this integer is represented with radix 10, it is possible to write it as $687459 = 9 + 5\cdot 10 + 4\cdot 100$ and so on. In general, for a radix representation, the equation is

$$n = n_m + n_{m-1}\cdot R + \dots + n_0\cdot R^{(m-1)} \tag{3}$$

The inverse radical function is defined as the function which generates a fraction in (0, 1) by reversing the order of the digits in Eq. 2. about the decimal point, as shown bellow:

$$\Phi_R(n) = 0.n_m n_{m-1} n_{m-2} \dots n_0 = n_m \cdot R^{-1} + n_{m-1} \cdot R^{-2} + \dots + n_0 \cdot R^{-(m-1)} . \quad (4)$$

Thus, for a k-dimensional search space, the Hammersley points are given by the following expression:

$$H_k(i) = \left[\frac{i}{N}, \Phi_{R1}(i), \Phi_{R2}(i), .., \Phi_{Rk-1}(i) \right], \quad (5)$$

where $i = 0, \dots, N$ indicates the sample points. Now, from the plot of this points, it is seen that the first row (corresponding to the first sample point) of the Hammersley matrix is zero and the last row is not 1. This implies that, for the k-dimensional hypercube, the Hammersley sampler generates a block of points that are skewed more toward the origin of the cube and away from the far edges and faces. To compensate for this bias, a point shifting process is proposed that shifts all Hammersley points by the amount bellow:

$$\Delta = \frac{1}{2} N \quad (6)$$

This moves the point set more toward the center of the search space and avoids unnecessary bias. Thus, the initial population always provides unbiased, low-discrepancy coverage of the search space.

3 THERMAL PROBLEM – DESCRIPTIONS OF MODELLING APPROACHES

The subject of the practical application is the problem of electromotor cooling with aim to reach the possibly lowest service temperature of the rotor and stator winding. The significant limiting factors of possible technical solutions are in this case the cost factors, i.e. the mass, production costs etc. and the limitations resulting from appropriate norms and customers expectations. Assuming those limitations the formulation of optimization problem is highly complicated because it is necessary to find a compromise between the severity of formulation, preparation and solution of complex optimization and the utility value of the simplified partial optimizations results.

Regarding to the up to now experiences with severity of heat problems solution by CFD analysis and also because of the complex character of the limitations it was decided to realize the overall electromotor modification in the form of partial tasks. One of most important problem was the optimization of cooling ribs shape with the aim to increase the efficiency of heat removal from the motor to cooling air and simultaneously not to increase the mass of the motor. For the numerical solution of this optimization the following three variants can be used: full 3D model of the motor with ribbing, one-rib model, model of 1/8 of the motor frame with ribbing.

The first two variants represent the extreme limits of cooling ribs geometry modelling when the full 3D model offers the possibility to include all space details. Using this variant it is theoretically possible to design an optimal shape for every single rib. However the practical realization of such result is due to the high production- and technological costs almost impossible. Because of this fact the mentioned variant will be not realized.

On the other hand the second variant, see Fig. 1, offers a detail study of particular geometric parameters of the rib. This way is focused on the finding of ideal rib shape and its application on the whole circumference of the frame. Regarding to the small size of such mathematical model represents this variant the fastest and cheapest way of the solution. However it can be expected that the simple copy of one rib profile around the frame circumference will not offer the best possible results.

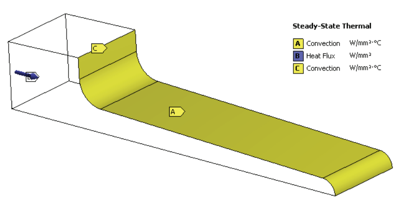


Fig. 1 Second modelling variant geometry boundary conditions

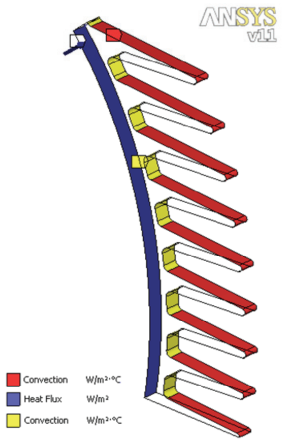


Fig. 2 Third modelling variant geometry; boundary conditions

The third variant, see Fig. 2, is an expansion of the second one into the minimal feasible segment where the plane symmetry conditions can be applied. From the modelling point of view this variant represents a compromise between the first two variants and simultaneously describes realistically the real rib orientation for present way of frame production so the results obtained in that way could be easily applied in production process.

The results of former CFD motor analyses [3] show that the numerical simulations of heat convection and heat transfer through the cooling ribs can be realized using the plane model. The mentioned CFD analyses offer among others also values for heat transfer boundary conditions on the circumfluenced rib surfaces for which it is within the optimization process supposed that those values will not be changed significantly due to the rib shape change. The optimization process is based on the results of plane thermal analysis for following boundary conditions: inner cylindrical surface – input heat flux, rib surfaces and surfaces between the ribs – heat convection, all other surfaces are considered as adiabatic.

4 OPTIMIZATION – DEFINITIONS AND RESULTS

The heat convection and heat transfer problem does not have the explicit solution for given geometry and boundary conditions. Due to this fact is the heat problem solution and following rib shape optimization performed in numerical way. For the numerical simulation the software system ANSYS Workbench 11 was used where the heat transfer problem was solved by FEM simulation of steady-state problem and I/O parameters are directly connected to the DesignXplorer modules which adds the optimization tools to the solution process.

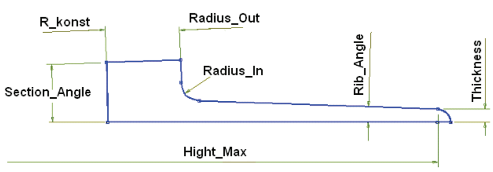
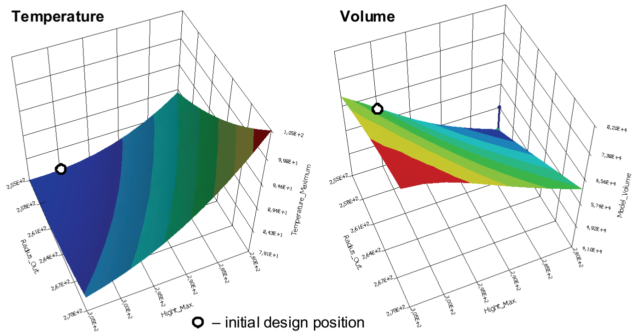


Fig. 3 First variant – design parameters

The first of solved optimizations is the profile optimization of one cooling rib in the sense of second rib modelling variant – see Chapter 3. In case of that variant it is supposed that the optimization allows to find the best suitable rib profile. Based on this fact the detailed rib shape is used for the optimization. The geometry is described by six dimensional parameters, i.a. radiuses at the foot and head of the rib, see Fig. 3. The remaining parameters are: rib volume and surface temperatures in certain chosen positions along the rib height.

Within the optimization itself the design space exploring was performed as the first. Its aim was to find the general responses of this optimization problem. The obtained sensitivities of design variables and also temperature and volume responses charts depending on most sensitive design variables can be considered as the most important results. The analysis of those responses charts show that already initial, i.e. current rib shape profile embodies good properties which are located close to the borders of design space, see Fig. 4. The significant reduction of rib surface temperature is according to those results possible only at the expense of inconsiderable rib volume growth. The found out facts show that for rib space placement according to Fig. 5 the rib profile can be considered as optimal.



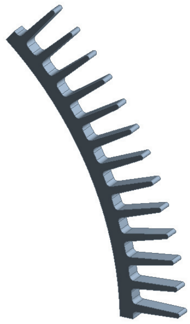


Fig. 4 1st Optimization: temperature and volume response charts **Fig. 5** Virtual 3D geometry

The second solved optimization problem is based on the third variant of cooling rib modelling. This geometric model describes the real product markedly better because the parallel orientation of the ribs along every frame quarter is taken in the account. According to the sensitivity analysis results of the first optimization in this case the geometry is described only by four design variables. The next defined parameters are: temperatures along the rib height, the ribbing volume and heat fluxes through the chosen heat transfer surfaces.

The optimization process was initiated again by the design space exploring to get the general problem behaviour. The response charts, see Fig. 6, in this case show that this optimization problem is not monotonous in given design space, both chart are antagonistic again. The shape optimization itself was consequently solved by certain methods available in ANSYS Workbench 11 whereas the technically most useful results were obtained by the GDO-screening method, see Fig. 7. The detail analysis of the results made possible to propose the optimal ribbing shape where the surface temperature was reduced about 5 per cent. Simultaneously the ribbing volume was reduced as well. This cutting down on the volume corresponds to the reduction of motor frame mass of approx. 4 per cent.

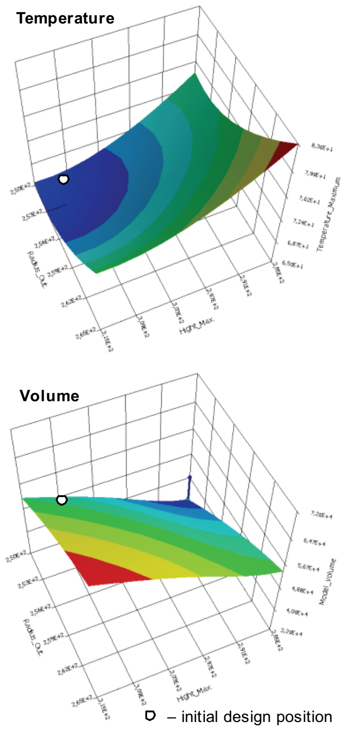


Fig. 6 2nd Optization: temperature and volume response charts

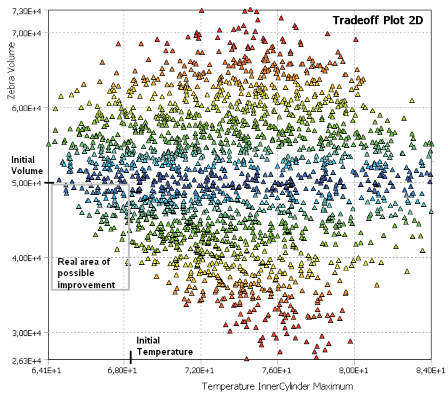


Fig. 7 GDO Screening: Tradeoff graph

4 CONCLUSION

The contribution presents the cooling rib optimization on the two numerical models with the aim to show the influence of used model geometry on the optimization problem properties. The parameters of used computational models (the models differ in used symmetry conditions), the application of boundary condition and obtained results are presented. The initial ribbing shape and boundary conditions values are based on current geometry and on corresponding CFD results.

It can be stated that:

1. The one-rib modelling simplification, i.e. use of axial symmetry, shows that the initial rib profile seems to be optimal, see Fig. 4. It can be taken into account that this profile is the result of similar optimization for the ribbing geometry used in the past, see Fig. 5.
2. Rather different properties of optimization problem were obtained using two plane symmetries and by optimization of 1/8 of frame circumference. Although the initial optimization point is located close to the temperature minimal value, see Fig. 6, this state can be even improved by optimization what is shown at the Fig. 7. The range of improvement compared to the former state in case of final geometry makes approx. 5 per cent temperature reduction and approx. 4 per cent mass reduction.
3. The obtained results show that although the profile of one rib is the building stone of cooling ribbing it is not possible to optimize the overall cooling properties only on the base of this single profile. It seems obvious that the proper space orientation of the ribs increases the cooling ability and this fact has to be taken into account for its optimization.

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