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CONTROL OF AIRFLOW SPEED IN HOT-AIR TUNNEL WITH UNCERTAIN PARAMETERS

ŘÍZENÍ RYCHLOSTI PROUDĚNÍ VZDUCHU V TEPLOVZDUŠNÉM TUNELU S NEURČITÝMI PARAMETRY

Abstract

This paper focuses on control of airflow speed in real laboratory model of hot-air tunnel using continuous-time robust controllers of standard PI and PID type designed via an algebraic approach. First, the controlled apparatus is identified and described by mathematical model with parametric uncertainty. Thereafter, various controllers are computed and robust stability of closed control loops is analyzed with the assistance of the value set concept in combination with the zero exclusion condition. The set of control experiments indicates practical applicability of considered synthesis method.

Abstrakt

Článek je zaměřen na řízení rychlosti proudění vzduchu v reálném laboratorním modelu teplovzdušného tunelu pomocí spojitých robustních regulátorů standardního typu PI či PID navržených prostřednictvím algebraického přístupu. Nejprve je řízená soustava identifikována a popsána matematickým modelem s parametrickou neurčitostí. Následně jsou počítány různé regulátory a s využitím principu množiny hodnot v kombinaci s větou o vyloučení nuly je analyzována robustní stabilita uzavřených řídicích smyček. Provedená sada regulačních experimentů naznačuje praktickou aplikovatelnost uvažované metody syntézy.

1 INTRODUCTION

The control of real technological processes is practically always bound up with some sort of uncertainty, which can be caused for example by imperfections and simplifications in modelling or changes in operating conditions. The common way how to include uncertainty in mathematical description of the controlled plant assumes no variations in the structure but only in individual parameters. This approach uses so-called parametric model of uncertainty. A prevalent task consists in design of cheap conventional PI or PID controller with fixed coefficients which guarantees closed-loop stability and conceivably also desired control behaviour for all expected values of the uncertain parameters.

A possible effective solution of this problem lies in the utilization of continuous-time controllers designed via general solutions of Diophantine equations in the ring of proper and stable rational functions (R_{PS}), Youla-Kučera parameterization and conditions of divisibility. The primary idea of this method is adopted from [11], [5] and control design is proposed and analysed for example in [8], [9]. The fractional approach brings a single scalar tuning parameter $m > 0$ which can be used to influence the dynamic, robustness or other properties of the control loop. Subsequently, robust stability can be tested through some standard tools [1], [2].

This paper aims to concisely outline the used synthesis method and primarily to apply it to control of airflow speed in laboratory model of hot-air apparatus. First, the controlled plant is identified and approximated by second order system with parametric uncertainty. In following control experiments, the controllers of PID and PI type for one degree-of-freedom (1DOF) and two degrees-of-freedom (2DOF) control configurations are designed, robust stability is graphically analysed and final control responses are measured and evaluated.

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2 MODEL OF HOT-AIR TUNNEL

The used laboratory model of hot-air system, made in VŠB-TU Ostrava [10], consists of the bulb, primary and secondary ventilator and an array of sensors covered by tunnel. The bulb functions as the voltage-controllable source of light and heat energy. The role of ventilators is to assure the air flow inside the tunnel. All parts are connected to the electronic circuits which adjust signals into the voltage levels suitable for CTRL 51 unit, constructed in the Academy of Sciences of the Czech Republic [4]. Finally, this control unit is connected with the PC via serial link RS232. The scheme of the plant and whole control system is shown in fig. 1.

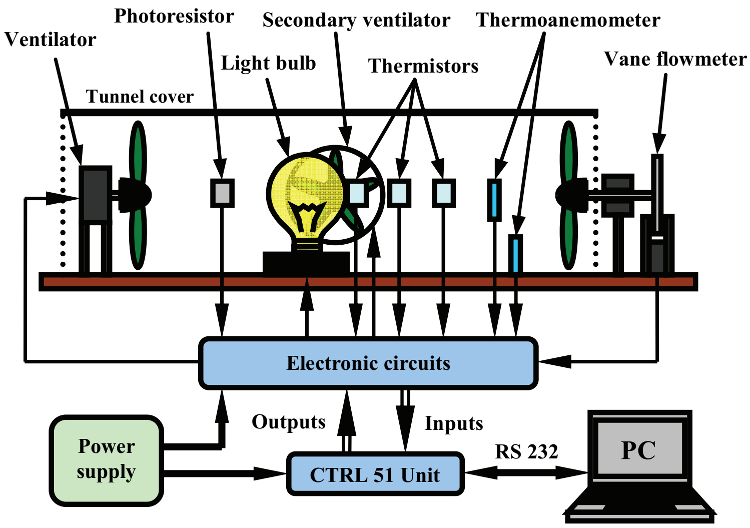

Fig. 1 Diagram of hot-air model and control system

The controlled object can be generally considered as multi-input multi-output (MIMO) system. Nevertheless, the published control experiments have been accomplished in one selected single-input single-output (SISO) loop consisting of primary ventilator voltage $u2$ (control signal) and air-flow speed measured by vane flowmeter $y7$ (controlled variable). The designation of variables corresponds to real connection of input and output signals of CTRL 51 unit [10]. The other actuating signals were set to constant values – bulb voltage $u1$ to 0 V and secondary ventilator voltage $u3$ to 0 V (in spite of it, the secondary fan revolved anyway).

All presented identification and control experiments were performed using the notebook HP Compaq nc6120 with Intel Pentium M processor 1.86 GHz, 512 MB DDR-333 SDRAM, Windows XP and MATLAB 6.5.1. The communication between MATLAB and CTRL 51 unit was arranged through four user function (for initialization, reading and writing of data and for closing) and the synchronization of the program with real time was done via „semaphore" principle (forthermore, the utilization of MATLAB functions „tic" and „toc" as an alternative was tested). To ensure the sufficient emulation of the continuous-time control algorithms, the sampling time 0.1 s was preset. The detailed information about utilization of serial link under MATLAB including mentioned user routines, program synchronization mechanism and several tests can be found in [3]. The discretization of integrative parts of control laws was carried out by left rectangle approximation method (the trapezoid method was also tried with the very similar results).

3 PLANT IDENTIFICATION

First, the attention was paid to identification of static and dynamical properties of controlled system. The fig. 2 depicts the trio of static characteristics measured during three different days. Note that behaviour of the system markedly depends on current conditions and operating point (the reference signal in control experiments will be 5 and 6 V). Thus, an array of step responses for various points has been measured and processed. The fig. 3 shows three examples of these responses for step-changes of $u2$ from 1V to 2V, from 5V to 6V and from 9V to 10V.

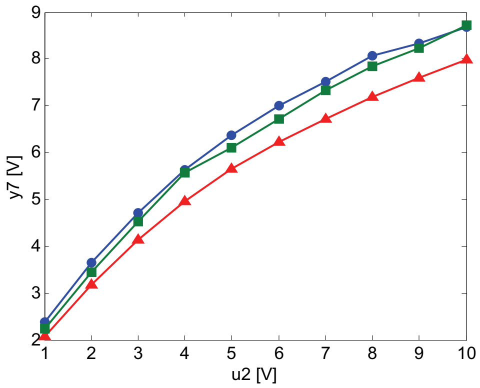

Fig. 2 Static characteristics of the system

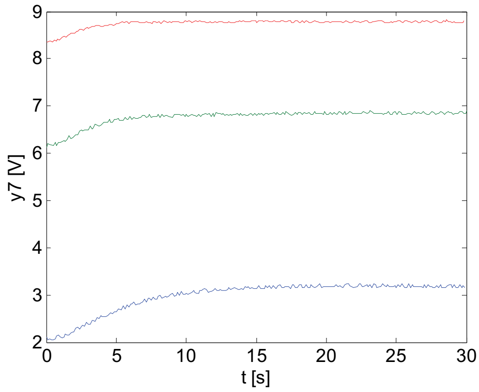

Fig. 3 Examples of step responses

The obtained data were normalized and approximated by step responses of second order system with double time constant:

$$G(s) = \frac{K}{(Ts+1)^2} \tag{1}$$

The least squares method was employed for identification. The example of approximation by (1) is given in fig. 4 where measured (jagged curve) and computed (smooth curve) values are compared. It belongs to $u2$ step-change from 1 to 2 V. In this particular case, the obtained function equals to:

$$G(s) = \frac{1.0714}{(2.8485s+1)^2} \tag{2}$$

The complete identification results for step responses depicted in fig. 3 are given in tab. 1.

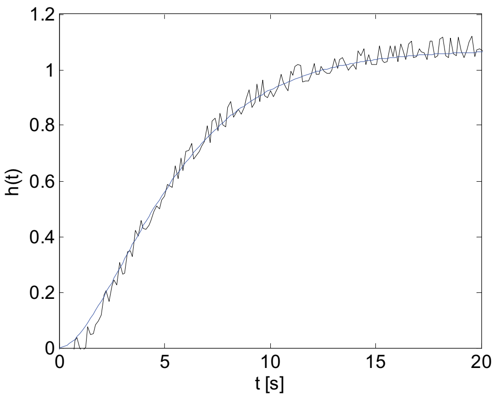

Fig. 4 Example of approximation

Tab. 1 Selected results of identification

$u2$ [V]	K [-]	T [s]
1 – 2	1.0714	2.8485
5 – 6	0.7205	1.7621
9 – 10	0.4377	1.2568

Further, the mathematical model affected by parametric uncertainty was constructed on the basis of data from the tab. 1 and also other, not depicted results. Although the intended working point corresponds to reference values of $y7$ at 5 and 6 V, the model is going to cover all mensurable area of revolutions. Therefore, the final model is:

$$G(s,K,T)=\frac{K}{(Ts+1)^2}=\frac{[0.3;1.2]}{([1;3]s+1)^2} \tag{3}$$

3 CONTROL DESIGN METHOD

The applied controller design method is based on fractional approach developed by Vidyasagar [11] and Kučera [5] and discussed e.g. in [8], [9]. It supposes transfer functions of continuous-time linear causal systems described in R_{PS}, i.e. expressed as:

$$G(s)=\frac{b(s)}{a(s)}=\frac{\frac{b(s)}{(s+m)^n}}{\frac{a(s)}{(s+m)^n}}=\frac{B(s)}{A(s)} \tag{4}$$

where $n=\max\{\deg(a),\deg(b)\}$ and $m>0$.

Consider 2DOF (FBFW) control system shown in fig. 5. Take notice that the traditional 1DOF (FB) system is obtained simply by $R=Q$.

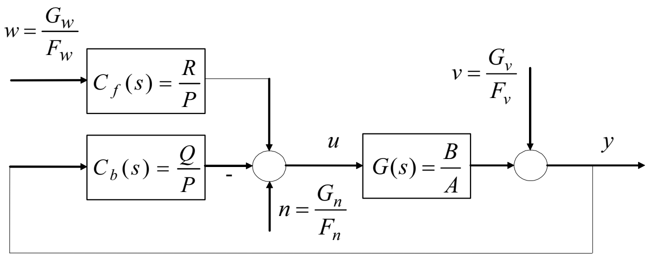

Fig. 5 2DOF control system configuration

External signals $w=\frac{G_w}{F_w}$, $n=\frac{G_n}{F_n}$ and $v=\frac{G_v}{F_v}$ represent the reference, load disturbance and disturbance signal, respectively. The most frequent case is a stepwise for reference and load disturbance signal and a harmonic signal for disturbance. Denominators of their transfer functions are then $F_w=F_n=\frac{s}{s+m}$ and $F_v=\frac{s^2+\omega^2}{(s+m)^2}$, respectively.

Under assumption of no disturbances ($n=v=0$), the general 2DOF control law is governed by:

$$Pu=Rw-Qy \tag{5}$$

Basic relations following from fig. 5 are:

$$y=\frac{B}{A}u;\quad u=\frac{R}{P}w-\frac{Q}{P}y \tag{6}$$

Further, the following equations hold:

$$y=\frac{BR}{AP+BQ}\frac{G_w}{F_w} \tag{7}$$

$$e=w-y=\left(1-\frac{BR}{AP+BQ}\right)\frac{G_w}{F_w} \tag{8}$$

For the structure 1DOF ($R = Q$), the last relation gives the form:

$$e = \frac{AP}{AP + BQ}\frac{G_w}{F_w} \tag{9}$$

The basic task is to ensure stability of the system in fig. 5. All stabilizing feedback controllers are given by all solutions of the linear Diophantine equation:

$$AP + BQ = 1 \tag{10}$$

with a general solution $P = P_0 + BT$, $Q = Q_0 - AT$, where T is free in R_{PS} and P_0, Q_0 is a pair of particular solutions (Youla – Kučera parameterization of all stabilizing controllers). Details and proofs can be found e.g. in [8], [9]. Then relations (8) and (9) take the form:

$$e = \left(1 - BR\right)\frac{G_w}{F_w} \tag{11}$$

$$e = AP\frac{G_w}{F_w} \tag{12}$$

For asymptotic tracking then either F_w divides AP for 1DOF or F_w divides $\left(1 - BR\right)$ for 2DOF. The last condition gives the second Diophantine equation for 2DOF structure:

$$F_w S + BR = 1 \tag{13}$$

Generally, the controller can be also designed for disturbance rejection and attenuation.

4 ROBUST CONTROL EXPERIMENTS

First, the parametrically uncertain model (3) and nominal system (for controller design):

$$G_N(s) = \frac{0.7}{\left(1.9s + 1\right)^2} = \frac{0.1939}{s^2 + 1.0526s + 0.277} \tag{14}$$

have been assumed. The experimental choice of tuning parameter $m = 0.6$ results in the regulator:

$$C_b(s) = \frac{\tilde{q}_2 s^2 + \tilde{q}_1 s + \tilde{q}_0}{s^2 + \tilde{p}_1 s} = \frac{2.3967s^2 + 2.5311s + 0.6684}{s^2 + 1.3474s} \tag{15}$$

The closed-loop characteristic polynomial for plant (3) and controller (15) can be easily formulated as:

$$\begin{aligned} p\left(s, K, T\right) &= \left(Ts + 1\right)^2 \left(s^2 + \tilde{p}_1 s\right) + K\left(\tilde{q}_2 s^2 + \tilde{q}_1 s + \tilde{q}_0\right) = \\ &= T^2\left(s^4 + \tilde{p}_1 s^3\right) + T\left(2s^3 + 2\tilde{p}_1 s^2\right) + K\left(\tilde{q}_2 s^2 + \tilde{q}_1 s + \tilde{q}_0\right) + \left(s^2 + \tilde{p}_1 s\right) \end{aligned} \tag{16}$$

The moderately zoomed value sets of this family with polynomial (quadratic) uncertainty structure, plotted via the Polynomial Toolbox for Matlab [7] for non-negative frequencies, are depicted in fig. 6. The zero exclusion condition indicates that polynomial (16) and thus also the whole control system is robustly stable, i.e. the controller (15) stabilizes the plant model (3) for all possible values of its parameters, because the origin of the complex plane is not included in the value sets and the polynomial (16) has a stable member. For details about this very universal and effective technique for graphical testing of robust stability and related topics see e.g. [1], [2]. The real control behaviour of the laboratory model and the controller (15) is shown in fig. 7.

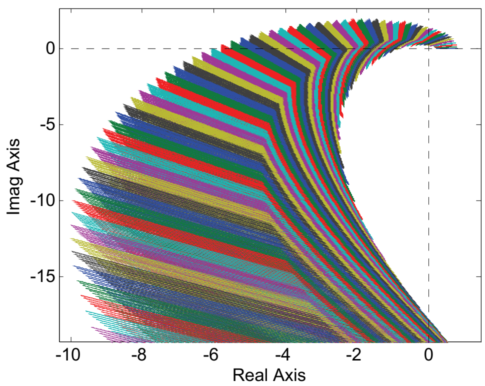

Fig. 6 Value sets for (16)

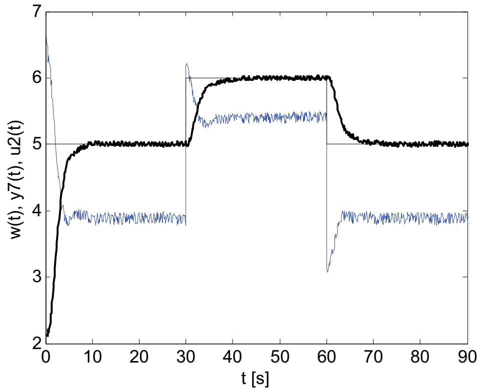

Fig. 7 Control of airflow speed by regulator (15)

Possibly, the controller with both feedback and feedforward part (for 2DOF configuration and the same parameter $m = 0.6$) is given by:

$$C_b(s) = \frac{2.3967s^2 + 2.5311s + 0.6684}{s^2 + 1.3474s}; \quad C_f(s) = \frac{1.8566s^2 + 2.228s + 0.6684}{s^2 + 1.3474s} \tag{17}$$

The feedforward part $C_f(s)$ does not influence neither closed-loop characteristic polynomial and so nor its robust stability. Now, the final control results can be seen in fig. 8.

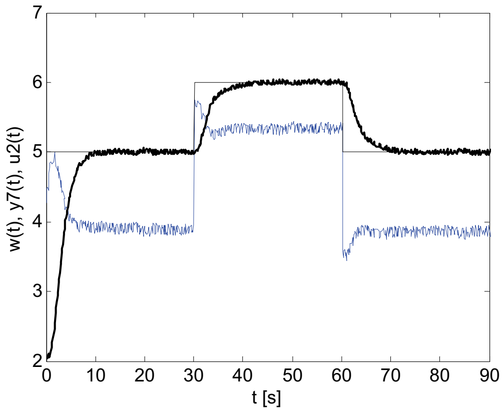

Fig. 8 Control of airflow speed by regulator (17)

The last experiment relies on simplification based on model order reduction. Hence, the trivial approximation results in new nominal system:

$$\frac{0.7}{(1.9s+1)^2} \approx \frac{0.7}{3.8s+1} = \frac{0.1842}{s+0.2632} = G_N(s) \tag{18}$$

The tuning parameter $m = 0.2632$, selected now according to recommendations from [6], represents the PI controller:

$$C_b(s) = \frac{\tilde{q}_1 s + \tilde{q}_0}{s} = \frac{1.4289s + 0.3761}{s} \tag{19}$$

and together with the uncertain model (3) it produces closed-loop characteristic polynomial:

$$p(s,K,T)=(Ts+1)^2 s+K(\tilde{q}_1 s+\tilde{q}_0)=T^2 s^3+T2s^2+K(\tilde{q}_1 s+\tilde{q}_0)+s \qquad (20)$$

Its value sets are shown in fig. 9. Similarly to the previous cases, the point zero is excluded from the value sets and thus the whole control system is robustly stable. And finally, fig. 10 depicts real control behaviour of the loop with the plant and regulator (19).

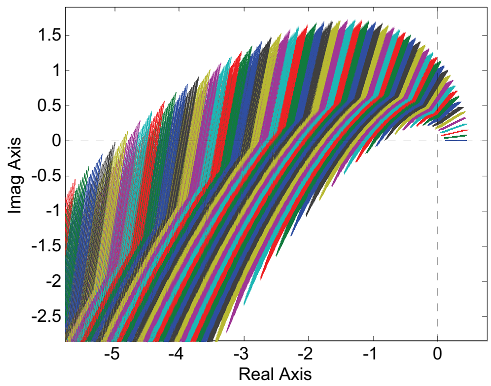

Fig. 9 Value sets for (20)

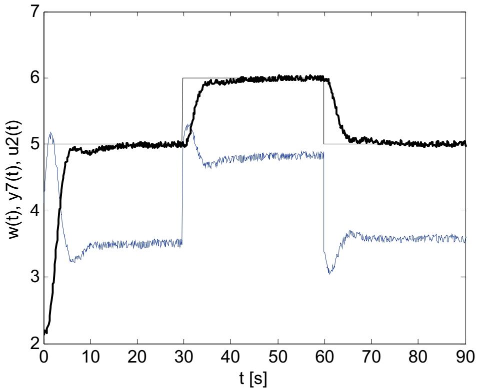

Fig. 10 Control of airflow speed by regulator (19)

The objective evaluation of quality has been performed by meaning of integrated squared error (ISE) criterion. The values of ISE can be found in tab. 2.

Tab. 2 Outcomes of ISE calculations

Controller	ISE
(15)	16.0646
(17)	21.1741
(19)	18.5004

As can be seen, there are no substantial differences among evaluated control responses. In comparison with 1DOF (15), 2DOF control structure (17) have brought modest reduction of control signals after step changes. However, not a (15) has generated any noticeable overshoot and respective closed-loop behaviour is, from the ISE viewpoint, better. PI controller (19) represents a little bit longer settling time owing to tiny undershoot during process of control.

5 CONCLUSIONS

The paper has been focused on utilization of an algebraic PI and PID regulator design method to control of airflow speed in real laboratory model of hot air tunnel. The controlled plant has been described by mathematical model with parametric uncertainty. Three controllers have been designed while closed-loop robust stability has been graphically tested with the help of the value set concept and zero exclusion condition. The obtained and evaluated control results affirm practical applicability of used approach.

Acknowledgment

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