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REGRESSION MODEL FOR DEPTH OF CUT OF WATER JET INTO ROCK MATERIALS

REGRESNÍ MODEL PRO HLOUBKU ZÁŘEZU VODNÍHO PAPRSKU DO HORNIN

Abstract

This article is aimed at derivation of a semiempirical and subsequently a regression model for cutting of rock materials by high-speed water jet. These models are similar to the ones derived for the same reason by other researchers. The searched variable is a depth of cut depending on pressure in the nozzle, diameter of the nozzle and traverse speed of the nozzle above rock material. The models are based on the dynamic pressure of water jet acting on rock material and comparison with experiment. The regression model is supported by simple theoretical model.

Abstrakt

V tomto článku je odvozen semiempirický a následně regresní model pro řezání horninových materiálů vysokorychlostním vodním paprskem, podobný jako pro tento účel odvodili jiní výzkumníci. Hledanou veličinou je hloubka zářezu paprsku do materiálu, která závisí na tlaku v trysce, průměru trysky a rychlosti posuvu trysky nad horninovým materiálem. Modely jsou založeny na působení dynamického tlaku kapaliny na horninu a porovnání s experimentem. Vytvořený regresní model má oporu v jednoduchém teoretickém modelu.

1 INTRODUCTION

Machining of materials by high-speed water jet is one of the topics in our laboratory at the Department of Liquid Jets of the Institute of Physics. A new theoretical model for cutting of rock materials by use of this tool was derived and it is presented in this article. The main searched quantity was the depth of cut *h*. The created model was motivated by model of Rehbinder who solved the same problem [1]. The model was compared with experiments being done on a rock sample presented by Hlaváč [2]. The researchers Summers & Blain assessed simple form of a regression model for cutting of materials [3]. Our approach to the problem is based on connection of the mean two accesses, the theoretical and the regression ones, and their combination into one regression model. All relationships are derived for simple conditions as a continual flow of jet, perpendicular impingement of jet to the rock, the best stand-off distance of the nozzle (1 cm). The nozzle moves above the rock material parallel to the surface. Model of Hlaváč presented in [4] solves this problem taking into account many parameters of water, nozzle and cut material as well as various conditions of cutting. Nevertheless, although Hlaváč's model yields very good results in comparison with experiments, it is

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rather complicated for quick use in practice. Therefore, this new approach can yield a certain simplification of calculations and quick application of the model in practice even by non-trained operators.

2 CUTTING OF ROCKS BY WATER JET

2.1 Semiempirical Model for Rocks

The high-speed water jet acts along a short length Δs on the rock surface during a time

$$t = \frac{kd}{v}, \tag{1}$$

where:

t - time of impingement of water jet into a small place of rock within movement of the nozzle [s],
k - coefficient of the jet spread [-],
d - diameter of the nozzle [m],
v - traverse speed the nozzle above material [m/s].

During the time t the part of a kinetic energy expressed using coefficient C creates a new surface, exactly

$$C\frac{1}{2}mu^2 = \sigma A = \Phi h, \tag{2}$$

where:

m - mass of the water flow [kg],
u - speed of water jet outside the nozzle $\left[\mathrm{m}\cdot\mathrm{s}^{-1}\right]$,
σ - the specific fracture energy of the rock material [$\mathrm{J}\cdot\mathrm{m}^{-2}$],
A - magnitude of the new area of a rock surface $\left[\mathrm{m}^2\right]$,
Φ - $\Phi = f(\sigma, a)$ constant phenomenological drag force of penetration of water jet into the rock [N],
a - rock (material) grain size [m],
h - depth of water jet penetration into the rock material $\left[\mathrm{m}\right]$.

The specific fracture energy of rock material σ contains a surface tension γ and some specific heat q (work for a plastic deformation of material), next $\sigma = \gamma + q$. The fracture energy for brittle materials is described in [5].

Rehbinder in [1] assumed that the dynamic pressure of jet p decreases with increasing distance from the nozzle according to the function for general attenuation $p = \mu p_0 e^{-\lambda h}$. The constant pressure will be assumed for simplicity in further derivation

$$p = \mu p_0 = \text{konst}, \tag{3}$$

μ - discharge coefficient of the nozzle $\mu = 0.6$ [-],
p_0 - pressure of water before nozzle [Pa],
λ - attenuation coefficient [m^{-1}].

The simplification expressed by equation 3 is used, because in shallow depth of cut the pressure does not decrease markedly.

Fig. 1 shows a scheme of the water jet tool. The operator needing to adjust the tool can utilize three basic parameters. The first one is the pressure in the nozzle, the second one is the diameter of the nozzle and the third one is the traverse speed of the nozzle above rock. Therefore, the semiempirical model should be based on these three parameters.

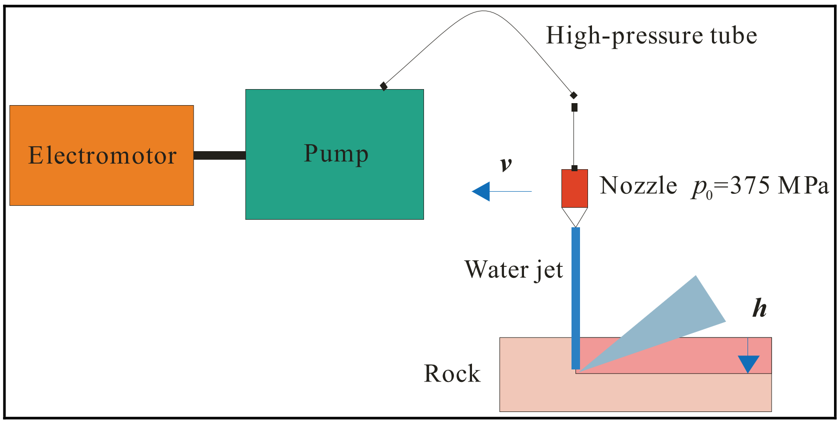

Fig. 1 High-speed water jet during penetration into the rock material

Based on the Bernoulli equation, when both the kinetic energy of water ahead the nozzle and the potential pressure energy of water passing the nozzle are neglected (sufficiently correct for pressures above 100 MPa), the *u* declared in (2) can be expressed as

$$u=\sqrt{\frac{2\mu p_0}{\rho}} \tag{4}$$

and the mass of water *m* in the same equation (2) is described as

$$m=Q_{\mathrm{m}}t=\rho S u\frac{kd}{v}, \tag{5}$$

for variables

ρ - density of water $\left[\mathrm{kg}\cdot\mathrm{m}^{-3}\right]$.

Q_m - mass flow rate of water $\left[\mathrm{kg}\cdot\mathrm{s}^{-1}\right]$,

S - sectional area of the nozzle opening [m^2].

It is considered that the water jet acts on a small area $\Delta S = k\Delta xd = kad$ and cuts a slot of depth *h*. The *kd* is the width of the slot and Δx represents a small growth of a discharged surface area in the straight direction (direction of traverse speed above material). Assuming that small cubes are braked from the rock an amount of $kadh/a^3$ cubes is created below the ΔS into the depth *h*. Subsequently, the equation (2) can be transformed into the form $\sigma\Delta A=\sigma 6khd$. Introducing equations (1), (3), (4) and (5) into the equation (2) the relationship marked the semiempirical model for the depth of cut made in rock material by a high speed water jet can be written in the form

$$h=K\frac{p_0^{1.5}d^2}{v}, \tag{6}$$

where:

K - is associated coefficient $K=C\xi\frac{1}{\sigma\cdot\rho^{0.5}}$ $\left[\mathrm{kg}^{-1.5}\cdot\mathrm{s}^2\cdot\mathrm{m}^{1.5}\right]$,

ξ - is general associated coefficient too [-], $\xi = 0.086$.

We declared *K* in (6) due to hard determination of *σ* and low estimation possibility and non-science of coefficient *C*.

2.2 Comparison of the Semiempirical Model with Experiment

Hlaváč performed experiments cutting the sandstone with the experimental number 1780 by water jet. The specific mass of the declared sandstone 1780 was 2.6 g·cm^{-3}, the average grain size was 0.52 mm, the compressive strength was 150 MPa and the tensile strength was 10 MPa. He observed relationships between depth of cut and cutting variables $h = f(p_0), h = f(d), h = f(v)$; trends can be observed in Fig. 2, 3, 4. Ten measurements of the cuts were accomplished and subsequently an average value was adjusted for each combination of parameters.

The measurement represents 350 values obtained for 15 cuts. The standard conditions of experiment were $p_0 = 375\,\text{MPa}$, $d = 0.25\,\text{mm}$, $v = 25\,\text{mm}\cdot\text{s}^{-1}$. Only one variable was changed in each experimental step. Based on experimental rock the value *K*= 0.327 is set out for equation (6) - computed according an average $h = f(p_0)$. Fig. 2, 3 and 4 show comparison of the derived semiempirical model (6) and Hlaváč's experimental data for the rock. The black line is calculated from the regression model specified in the last chapter of this article.

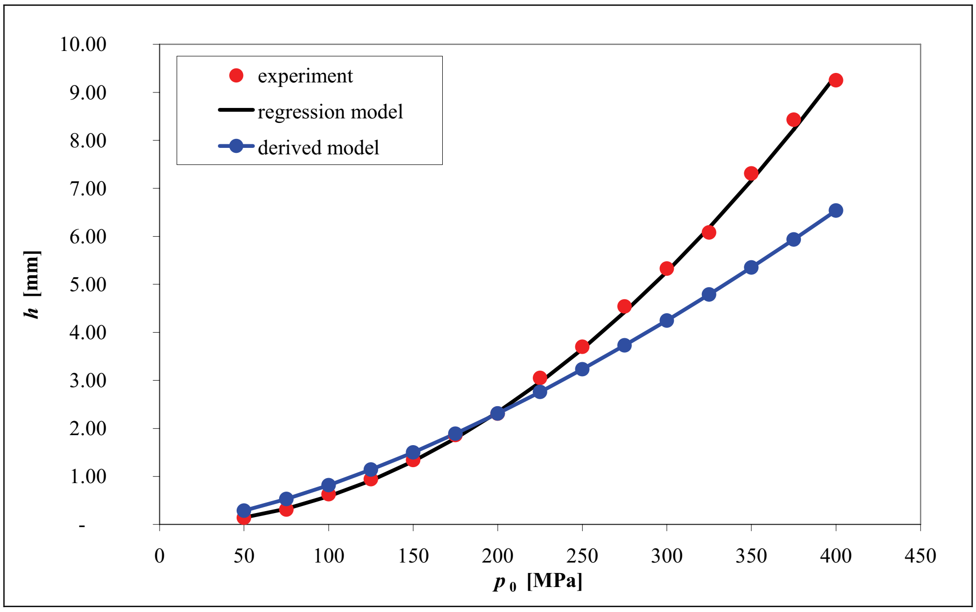

Fig. 2 Depend. of the depth of cut on pressure before the nozzle for sandstone samp (1780)

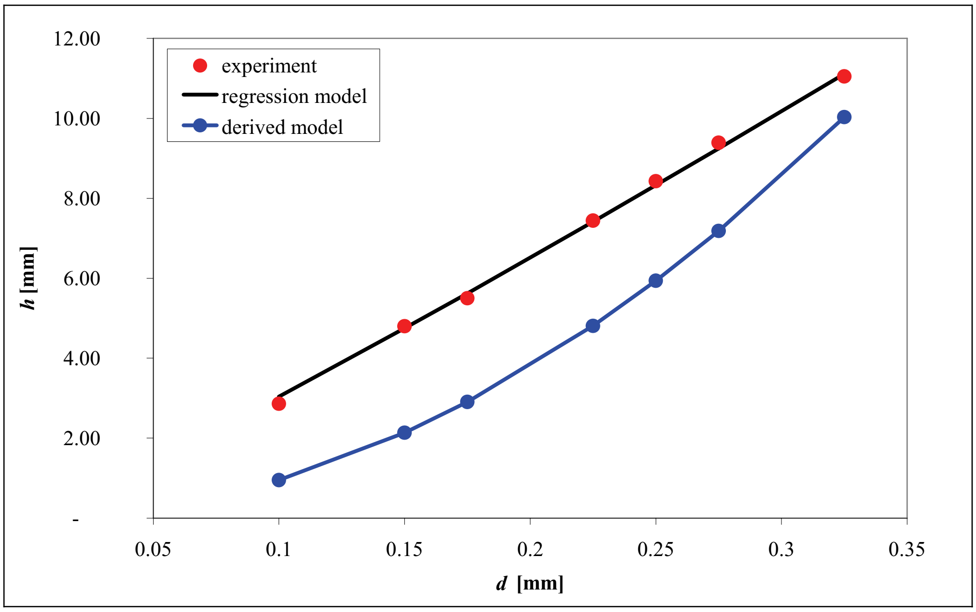

Fig. 3 Dependency of the depth of cut on the nozzle diameter for sandstone sample 1780

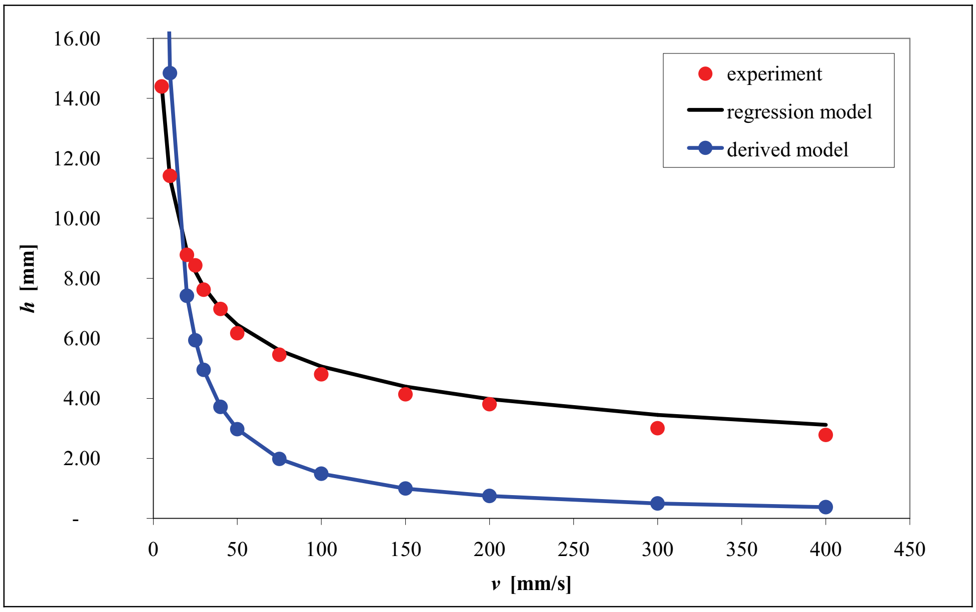

Fig. 4 Dependency of the depth of cut on traverse speed of the nozzle above material for sandstone sample 1780: regression curve lies above the measured points for high traverse speeds because the average rounded values of coefficients are used for the presented more complex regression based model

The figures show that the semiempirical model (6) is only the first approximation of the real dependency. On the other side the model is considered to be a good base for the estimation of the regression model. Nevertheless, the semiempirical model does not solve the boundary conditions as the value of the depth of cut for $v = 0\,\mathrm{m\cdot s^{-1}}$ or cutting with a critical pressure (it should be assumed that for some non-zero pressure the rock still is not cut). Via these notices the model contents information about manner of rock machining by water jet.

2.3 Influence of Cavitation

The graph in Fig. 2 shows that presented model is not fully reliable in dependency $h = f(p_0)$. The interaction process is not sufficiently described. Rehbinder assumed a great influence of cavitation during the cutting process. However, he did not incorporated description of this phenomenon to his relationships. Nevertheless, instead specific fracture energy used in presented model he assumed other influences, e.g. permeability of the rock. Because there is a substantial difference between the theory and experiment for $h = f(p_0)$ another phenomena acting on rock should be assumed than a simple dynamic pressure of liquid jet. One of the main phenomena could be the cavitation [6]. The material removal per time induced by activity of cavitation depends on speed of water *v*: $\Delta m/\Delta t = \delta v^6$. Using the pressure instead the jet velocity the equation can be rewritten to the form $\Delta m/\Delta t = \kappa p_0^3$. The quantities δ and κ are coefficients. Such a consideration could explain the difference between results obtained from the derived theory and experiment regarding the pressure before nozzle.

The difference between results of a semiempirical model and the experimental data for relationship $h = f(v)$ maybe explained by acting of the disintegrated rock - the water jet is edged out from the rock. Therefore, the jet acting area Δs is no more so small as it was assumed in the beginning. The area is larger and so the interaction time should be longer. Nevertheless, the difference between theoretical and experimental relationships $h = f(d)$ has no sound explanation.

2.4 Regression Model

The regression model of Summers & Blain [3] was used for an insight to the semiempirical derivation of dependencies of the depth of cut created by water jet in this form

$$h = \frac{p_0^x d^y}{v^z}, \tag{7}$$

where x, y, z are regression exponents [-].

It is evident that the regression model has a formal similarity to presented semiempirical model. Only the coefficient is missing. The coefficient should be added as it is presented in the semiempirical model (6) because the coefficient *K* (below *Z*) characterizes the mechanical quality of rock whereas the fraction in the equations (6-8) belongs to the quality of the tool. The black lines in Fig. 2, 3 and 4 are calculated according to the presented regression model with exponents being as those in the equation (8)

$$h = Z\frac{p_0^2 d^{1.1}}{v^{0.35}}, \tag{8}$$

where *Z* is the regression coefficient [reg. dimension].

This regression model has the same form as the equation (6) so the model has held to the theory. The model does not solve the boundary conditions. For example, explanation of the $h = f(v)$ dependency should be more rigorous using the theoretical form of attenuation for both the pressure and the jet velocity e^{-bv} where *b* is the coefficient and the tool cannot cut for pressures below some $p \neq 0\,\mathrm{MPa}$. The value depends on the mechanical quality of rock. Therefore, the coefficient *Z* was added comparing to the regression model of Summers & Blain. The exponents found for regression model are assumed to be valid for each rock with a similar grain size, or better said for rocks with grain sizes comparable with water jet diameters [7] (in observed interval of variables). The regression model expressed by equation (8) yields proper results if the formats of variables are megapascals for the pressure p_0, millimetres for the nozzle diameter *d* and millimetres per second for the traverse speed *v*. Subsequently, the depth of cut *h* is obtained in millimetres.

2.5 Regression

The coefficients of the regression model (8) were calculated from data obtained from experiments performed on two sandstones – the first one designated 1780 (the specific mass $2.6\,\mathrm{g\cdot cm^{-3}}$) and the second one designated 1328 (the specific mass $2.62\,\mathrm{g\cdot cm^{-3}}$). Experimental cuts were performed so that only one of the parameters was changed from cut to cut and other parameters were adjusted to selected standard values. The regression coefficients and constants for the regression model (8) were determined using the software MatLab. The coefficient named pseudo *R*-squared (declared in the Tab. 1) characterizes concordance rate as a measure of goodness of fitting for non-linear regression. The values are in Tab. 1.

Tab. 1 Regression exponents and pseudo *R*-squared coefficients

Sandstone	***Z***	x	y	z	R^2
1780	0.00116	1.96	1.12	0.36	0.998
1328	0.00072	1.98	1.11	0.34	0.997

The assumption is that the exponents x, y, z are invariant for each rock. The small differences between x, y, and z are assumed to be caused by experimental uncertainties. The differences between coefficients *Z* determined for presented samples are wider confirming the assumption that the *Z* characterizes mechanical quality of respective rock. The operator driving the tool should know only the *Z* value for the rock machining. Because the exponents were rounded to values presented in the equation (8) the coefficient *Z* was recalculated for both experimental samples. The values of *Z* are $8.29\cdot10^{-4}$ for sandstone 1780 (*R*-squared coefficient R^2= 0.997) and $6.36\cdot10^{-4}$ for sandstone 1328 (*R*-squared coefficient R^2= 0.997).

3 CONCLUSIONS

The main aim of this contribution is the derivation of the regression model. The model has an initial formulation in a theoretical level. The regression model includes only one changing parameter - the coefficient related to the material properties *Z*. Therefore, the operator needs to change only the coefficient *Z* during machining of rocks.

It is clear that created semiempirical model does not describe the process of cutting of rock materials exactly. It was improved through the preparation of the regression model based on the up-to-date theory. Nevertheless, the regression method can yield only mathematical results within the accuracy of the input experimental data and cannot fully describe of the physics of the interaction processes during cutting. The agreement or disagreement of either theoretical or semiempirical models was testified by experiments and the validity of presumptions is proved. The presumption about influence of pressure onto the cutting process seems to be insufficient and description of the disintegration caused by cavitation phenomenon should be added into the presented model in future.

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