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**THE FATIGUE OF MATERIAL UNDER MULTIAXIAL SPECIMEN LOADING**  
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**Abstract**

This contribution describes experimental results under combined loading of specimens manufactured from common construction steel 11523 melting T31052. Specimens were loaded by amplitude of the torque, then by combination of torque and tension prestress. The last set of specimens was loaded in combination of torque and inner overpressure in the conditions of closed vessel. To obtain the required input values the stress-strain analysis of specimens was performed by finite element method in software Ansys. For evaluation of the results the Fuxa's criterion [3] was applied. The performed experiments and their results embody a good agreement with bellow mentioned conjugated strength criterion. The experiments were performed on modified testing machine equipped by overpressure chamber.

**Abstrakt**

Tento článek popisuje výsledky experimentů při kombinovaném zatěžování zkušebních vzorků vyrobených z běžné konstrukční oceli 11523 tavba T31052. Zkušební vzorky byly zatěžovány amplitudou krouticího momentu, dále pak kombinací amplitudy krouticího momentu s tahovým předpětím. Poslední sada zkušebních vzorků byla zatěžována amplitudou krouticího momentu v kombinaci s vnitřním přetlakem v podmínkách uzavřené nádoby. Pro získání potřebných vstupních hodnot do konjugovaného kriteria pevnosti [3] byla počítána napěťové deformační analýza zkušebních vzorků metodou konečných prvků v programu ANSYS. Na výsledky experimentů bylo aplikováno konjugované kritérium pevnosti. Zkoušky byly provedeny na upraveném zkušebním zařízení, které bylo doplněno o přetlakovou komoru.

**1 INTRODUCTION**

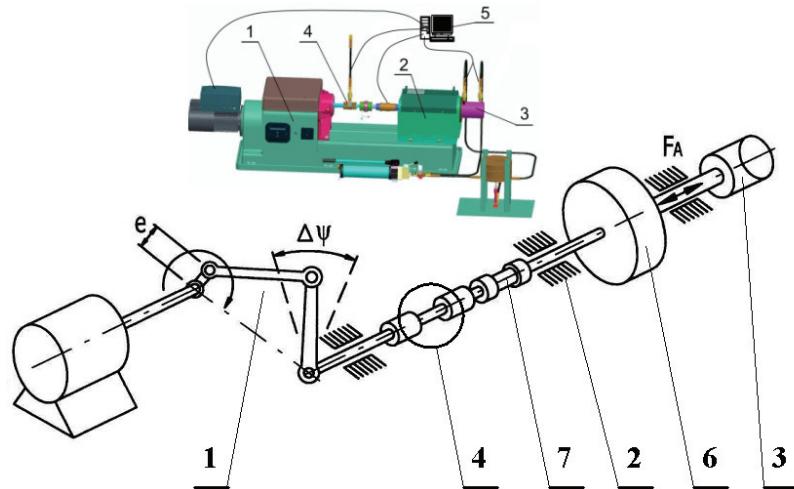
To verify multi-axial Fuxa's conjugated strength criterion the new jig was developed which generalizes the possibilities of reconstructed testing machine SHENCK type PWXN [1]. In this case the testing machine was equipped by overpressure chamber which makes possible to load the specimen by constant inner overpressure (in the range 0 to 70 MPa) in the conditions of closed vessel in addition with torque amplitude. It is also possible to add the constant tension/pressure prestress into the system.

**2 EXPERIMENTAL DEVICE**

For material testing under combined loading in the region of high-cycle fatigue was reconstructed the testing device SHENCK type PWXN [1, 2]. The new conception of the testing device changes the loading character of the specimen from deformation-controlled to force-controlled – see Fig. 1.

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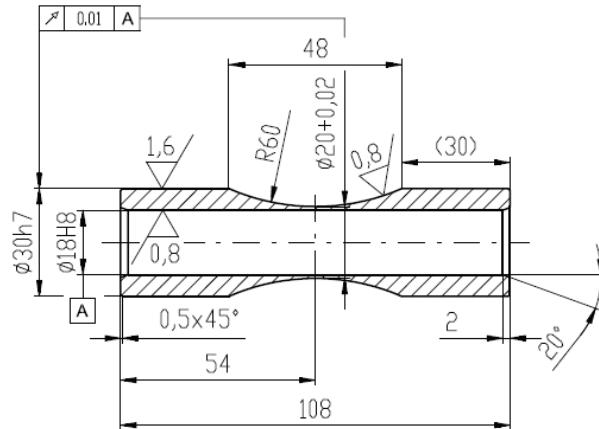
**Fig. 1** Experimental device

The base of the testing machine is the frame 1 with gear box. Required torque is gained by the acceleration of driven part and by balance wheel 6 located in measuring box 2. Axial tension/pressure force is gained by straight-line hydro motor 3 connected with hydraulic aggregate. Newly is the testing machine equipped by overpressure chamber 4 which is connected with multiplier and hydraulic aggregate. The measured values of the torque and axial force 7, number of cycles and inner overpressure are captured by computer 5 measuring cards and consequently evaluated in software Lab-VIEW.[4, 7].

The new conception of the testing device is patented as: Experimental Device for combined loading of specimens, Nr. 17286 (2007) and Experimental Device for combined loading of specimens, Nr. 18921 (2008).

### 3 EXPERIMENT – ALTERNATING TORSION

Specimens manufactured from steel 11523, melting nr. T31052 whose parameters are in Fig. 2 were loaded by amplitude of torque in the condition of alternating cycle and testing frequency of 25 Hz. The amplitude of torque was gradually decreased until the limit  $10^7$  cycles was reached.

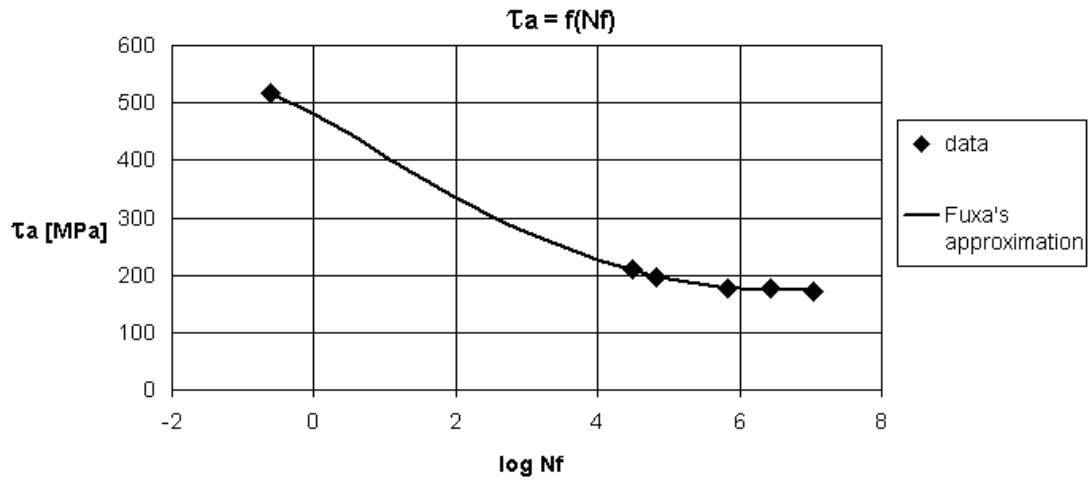


**Fig. 2** Specimen

The results are placed in Table 1. In the Fig. 3 can be seen measured values, Fuxa's approximation curves (1) [5, 6]. Point of crack initiation under static torsion [9,10] was measured by reconstructed testing machine INOVA in the institute lab. [8].

**Tab. 1** Experimental results for alternating torsion

Specimen Nr.	Stress amplitude $\tau_a$ [ MPa ]	Number of cycles	Notes
1	214,4	30800	
2	196,3	67500	
3	178,2	683620	
4	176,3	2703000	
5	172,2	10810000	No crack generated
6	516,6	0,25	Static test



**Fig. 3** W – curve for alternating torsion

Fuxa's approximation:

$$\tau_{af} = (\tau_f + \tau_c)/2 + (\tau_f - \tau_c)/2 \cdot \cos\{\pi \cdot [\log(4 \cdot N_f)/\log(4 \cdot N_c)]^{a_1}\} \quad (1)$$

for  $N_f$  in interval  $[1/4; N_c]$  and  $\tau_{af}$  in interval  $[\tau_f; \tau_c]$ .

where:

$\tau_f$  (516,6 MPa) is a value of real shear strength,  $\tau_c$  (172,9 MPa) is the stress at the fatigue limit,  $N_c$  (6400000) is number of cycles at the fatigue limit,  $a_1$  (0,631) is constant,  $\tau_{af}$  is the limit stress amplitude under alternating torsion and  $N_f$  is the limit number of cycles (until crack initiation). The mentioned values were obtained by nonlinear regression methods.

#### 4 EXPERIMENT – ALTERNATING TORSION AND TENSION PRESTRESS

For this way of testing the same specimen as in previous case were used. Every series of specimens was loaded by different constant axial tension stress. For given tension stress value was chosen the torque amplitude which was the specimen loaded until the crack initiation by. This amplitude was gradually decreased until the value when was the specimen able to endure  $10^7$  of cycles. The testing frequency was also 25 Hz.

The results of those experiments are mentioned in Table 2. The mentioned resulting tension stress was in the case of very complicated specimen shape obtained on the base of finite element method in the software ANSYS. The experimental results are also shown in Fig. 4 and approximated by lower described Fuxa's approximation (2, 3, 4, 5) which takes the influence of mean stress into account. Particular approximations are based on measured number of cycles which is mentioned in Table 2.

**Tab. 2** Experimental results for alternating torsion and tension prestress

Specimen Nr.	Tension prestress [ MPa ]	Stress amplitude $\tau_a$ [ MPa ]	Number of Cycles	$CH_F$ [%]	Notes
1	266,1	154	401000	3,82	
2	266,1	145,5	637600	1,42	
3	266,1	136,6	10487000	2,42	No crack generated
4	191,6	178,5	110300	1,69	
5	191,6	162,3	310500	3,06	
6	191,6	146,5	11300000	2,34	No crack generated

Fuxa's approximation with influence of mean stress:

$$\tau_{af2} = (\tau_f^* + \tau_c^*)/2 + (\tau_f^* - \tau_c^*)/2 \cdot \cos\left\{\pi \cdot [\log(4 \cdot N_f)/\log(4 \cdot N_c)]^{a_1}\right\}, \quad (2)$$

$$\tau_f^* = 1/\sqrt{3} \cdot ((\sqrt{3} \cdot \tau_f)^2 - 2 \cdot \sqrt{3} \cdot \tau_f \cdot B_o \cdot \sigma_t / 3 + \sigma_t^2 \cdot B_o^2 / 9 - \sigma_t^2)^{1/2}, \quad (3)$$

where (4) is the static strength condition for  $N_f = 1/4$  and constant  $B_o$  is equal to:

$$B_o = 3 \cdot (\sqrt{3} \cdot \tau_f / \sigma_f - 1), \quad (4)$$

$$\tau_c^* = \tau_c / 2 \cdot \left\{ 1 + \cos[\pi \cdot (\sigma_t / \sigma_f)^B] \right\} \text{ is the strength condition for } N_f = N_c. \quad (5)$$

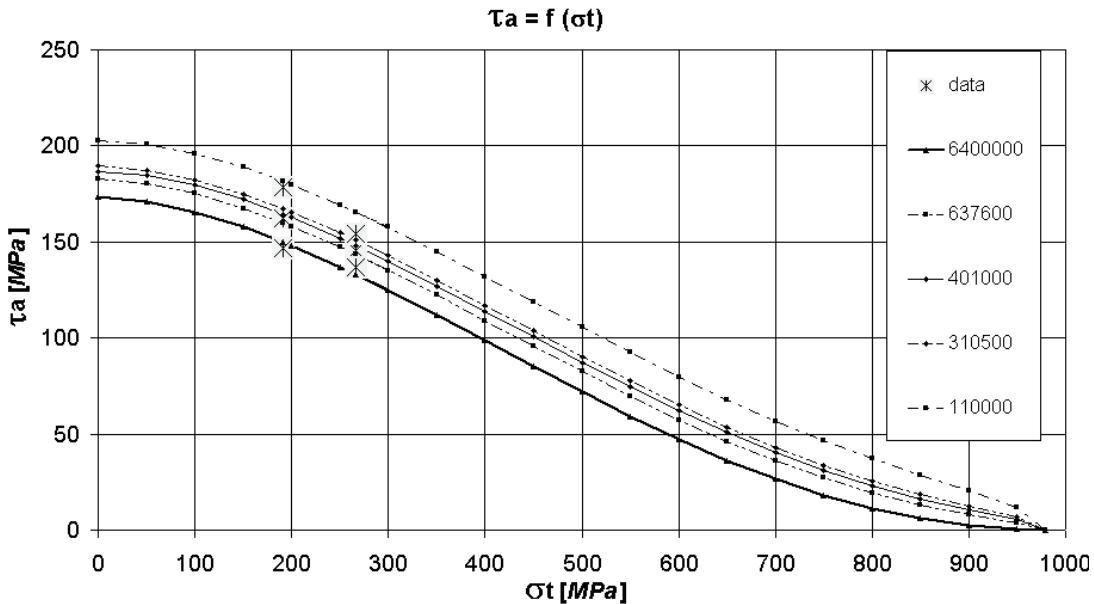
In the relations (2, 3, 4, 5) the  $\sigma_f$  is the real tension strength value,  $\tau_f$  is a value of real shear strength,  $\tau_c$  is the stress at the fatigue limit,  $N_c$  is number of cycles at the fatigue limit,  $a_1$  and  $B$  are constants,  $\tau_{af2}$  is the limit amplitude of shear stress,  $\sigma_t$  is the constant tension stress and  $N_f$  marks the (limit) number of cycles until crack initiation.

The absolute mean relative error value of used approximation is mentioned in Table 2 and can be determined according to following formula:

$$CH_F = ABS(\tau_{ai} - \tau_{af2}) / \tau_{ai} \cdot 100\%, \quad (6)$$

$\tau_{ai}$  are the measured stress amplitude values (see Table 2),

$\tau_{af2}$  are the values calculated according to the Fuxa's approximation (2).



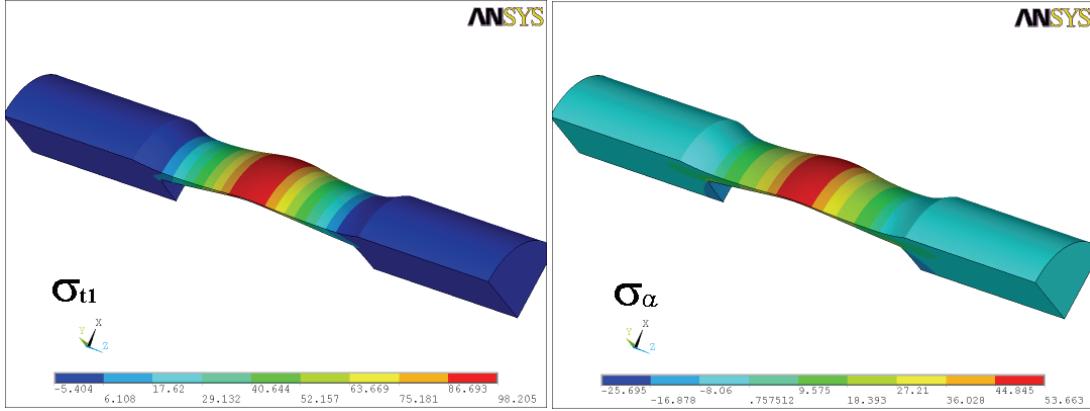
**Fig. 4** Fuxa's approximation for combined torsion – tension loading

## 5 EXPERIMENT – ALTERNATING TORSION AND INNER OVERPRESSURE

For this way of loading was the above mentioned testing machine equipped by a new type of specimen fixation which makes possible to use the overpressure chamber. This chamber is connected with multiplier and with hydraulic aggregate which serves for gaining of constant inner overpressure in the conditions of closed vessel in the range 0 - 70 MPa . For this way of testing the same specimen as in previous case were used. Every series of specimens was loaded by different constant overpressure. For given constant inner overpressure value was chosen the torque amplitude which was the specimen loaded until the crack initiation by. This amplitude was gradually decreased until the value when was the specimen able to endure  $10^7$  of cycles. The testing frequency was also 25 Hz. The results of those experiments are mentioned in Table 3.

On the base of stress state evaluation of the specimen loaded by inner overpressure (in the conditions of closed vessel) the significant circumferential  $\sigma_{t1}$  and axial  $\sigma_a$  stress can be observed on the surface. It is not easy to determine those two stresses analytically due to the complicated shape of specimen and face of acting inner overpressure. Hence this stress state had to be determined by finite element method in software ANSYS.

The static analysis was performed, where  $\frac{1}{4}$  of specimen and  $\frac{1}{4}$  of pistons of overpressure chamber (closed vessel) was modelled as one body, the SOLID element 186 was use, the material parameters was obtained on the base of tensile test. The boundary conditions are chosen so that the resting  $\frac{3}{4}$  of specimen is compensated by symmetry and further one point of specimen face is fixed in three directions (x, y, z). Opposite end of the specimen is free. On the relevant length the inner overpressure was applied. This length results from the dimensions of testing jig. Results for given overpressure are in Table 3. Results of circumferential and axial stresses in MPa, obtained from FEM analysis for inner overpressure 10 MPa, are on Fig. 5.



**Fig. 5** Circumferential and axial stresses for inner overpressure 10 MPa

Experimental results obtained from performed experiments and computation are on Fig. 6. Those results are approximated by already mentioned Fuxa's approximation (2, 3, 4, 5) which takes into account the influence of mean stress. For from inner overpressure obtained stress state it is necessary to adjust the relations (3) mentioned equation can be written as follows:

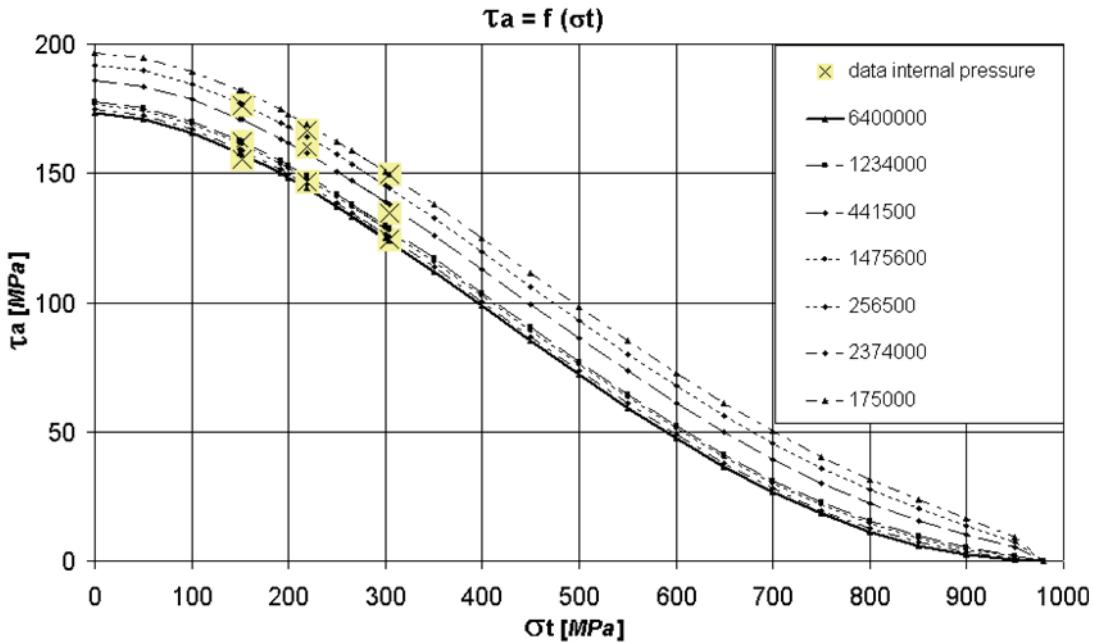
$$\tau_f^* = 1/\sqrt{3} \cdot \left( \frac{(\sqrt{3} \cdot \tau_f)^2 - 2 \cdot \sqrt{3} \cdot \tau_f \cdot B_O \cdot (\sigma_{t1} + \sigma_a)/3}{+(\sigma_{t1} + \sigma_a)^2 \cdot B_O^2 / 9 - \sigma_{t1}^2 + \sigma_{t1} \cdot \sigma_a + \sigma_a^2} \right)^{1/2} \quad (7)$$

This equation is based on reference stresses [3] and on the stress state analysis for given loading case. Particular approximations result from measured number of cycles written in Table 3.

In this case the same constants as in the case of tension/torsion combination are used for the approximation (see Fig. 4). The curves at the fatigue limit are in the case of both described problems equivalent. The absolute value of mean relative error  $CH_F$  (6) is mentioned in Table 3 as well.

**Tab. 3** Experimental results for alternating torsion and tension prestress

Specimen Nr.	Overpressure [ MPa ]	$\sigma_t = \sigma_a + \sigma_{t1}$ [ MPa ]	Stress amplitude $\tau_a$ [ MPa ]	Number of cycles	$CH_F$ [%]	Notes
1	10	151,8	176,2	256500	0,38	
2	10	151,8	162,3	1475600	0,68	
3	10	151,8	155,3	10080000	1,41	No crack generated
4	15	219	166,7	441500	5,39	
5	15	219	160,1	1234000	6,92	
6	15	219	146,7	11058000	1,74	No crack generated
7	20	303,7	149,6	175000	0,02	
8	20	303,7	134,6	2374000	6,72	
9	20	303,7	124,4	10750000	0,33	No crack generated



**Fig. 6** Fuxa's approximation for combined torsion – inner overpressure loading

## 6 CONCLUSIONS

The reconstructed testing machine equipped by overpressure chamber is briefly described in this contribution. Further are mentioned three types of fatigue experiments performed at the specimens made from steel 11523 melting T31052.

First experiment – alternating torsion. Obtained results are approximated Fuxa's approximation. The Fuxa's approximation embodies a good agreement with experiment – see Fig. 3.

Second experiment – combined loading by amplitude of torque and by constant axial tension force. The results are successfully approximated by Fuxa's approximation with the influence of mean stress (2, 3, 4, 5). Also here the Fuxa's approximation embodies the good agreement with experiment – see Fig. 4. The constants of strength criterion were tuned on this experiment.

Third experiment – combined loading by the amplitude of torque and by inner overpressure in the condition of closed vessel. Appropriate circumferential and axial stresses (mentioned in Table 3) are obtained by finite element method and the modification of Fuxa's approximation is mentioned. The experimental results are approximated by Fuxa's approximation [2] whose constants result from previous experiment. The good agreement can be seen here and hence it is possible to state the appropriate constant tuning for further possible combined loading – see Fig. 6.

Described Fuxa's approximation is a part of conjugated stress criterion which is based on the conception of reference normal and reference shear stresses, see [3] for more details.

## ACKNOWLEDGEMENTS

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